



International Mathematical Olympiad
 “Formula of Unity” / “The Third Millennium”
 Year 2026–2027. Qualifying round

Problems for grade R5



Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan),
 more details are at the page formulo.org/en/olymp/2026-math-en/.

Your paper should be sent until **23:59:59 UTC, 5 November 2026**.

Please solve the problems by yourself. The use of calculators, programming tools and neural networks is prohibited.

Remember that the majority of the problems require not only an answer but also its full proof.

The paper should not contain your personal data, so **please do not sign your paper**.

1. Arthur is playing “Battleship” with a friend. In this game, one player places 10 ships (four 1×1 , three 1×2 , two 1×3 and one 1×4) on a 10×10 board so that the ships do not touch each other, not even at corners, and the other player names cells one at a time in order to “sink” these ships. Arthur has already made several moves (see the figure; ● — hit, ⊗ — miss). How many valid ship placements are there? (P. Mullenko)

	A	B	C	D	E	F	G	H	I	J
1	●	●	⊗						⊗	●
2					●					
3	⊗				●					●
4										●
5	●					●	⊗			●
6	⊗		●	●		⊗				
7	●									●
8										
9		●	⊗							
10		⊗								

2. A machine takes as input a natural number greater than 10 and finds the sum of the squares of all its digits. Then, if the resulting number has at least 2 digits, the machine finds the sum of the squares of its digits, and repeats these operations until it gets a one-digit number, which it returns as the result (for example, 24 turns into $2^2 + 4^2 = 20$, which turns into $2^2 + 0^2 = 4$, and the machine returns 4). Is there a number consisting only of even digits for which the machine returns an odd one-digit number? (M. Karlukova)

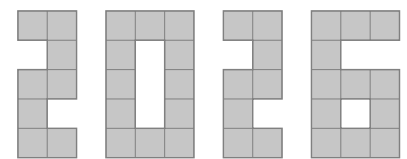
3. In a country inhabited only by knights, who always tell the truth, and liars, who always lie, people are preparing to celebrate Independence Day. Two residents saw a poster showing the two-digit age of the country, and told you the following.

First resident: 1) “The age is divisible by 5.” 2) “The age is not divisible by 2.”

Second resident: 1) “The age is divisible by 10.” 2) “The sum of the digits of the age is not 7.”

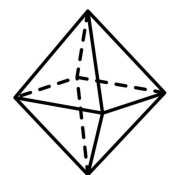
How old can the country be? Find all possibilities. (P. Mullenko)

4. The number 2026 has been cut out of grid paper along the grid lines (see the figure). Cut one or more of the digits along the grid lines and assemble a 5×8 rectangle from all the resulting pieces. The total length of all cuts must not exceed 4 (the side of a cell is 1). Draw how the digits are cut into pieces, and then how the rectangle is assembled from these pieces. (S. Pavlov)



5. The digits 1, 2 and 3 are written on a board. Remarkably, their sum is divisible by each of them. How can three more digits be written on the board so that all six digits are different and their sum is divisible by each of them? Find all ways and prove that there are no others. (P. Mullenko, S. Pavlov)

6. A regular octahedron is a polyhedron made up of 8 triangles with equal sides. An octahedron with edge 1 was cut along several edges and unfolded onto the plane in one piece so that the faces do not overlap and no two faces touch along a segment unless they remained glued along it. What can the perimeter of the net be? Find all possibilities and prove that there are no others. (P. Mullenko)



7. What is the smallest number of cells of a 3×2026 board that must be colored so that every rectangle of 4 cells contains at least one colored cell? Do not forget to prove that this number is indeed the smallest. (S. Pavlov)



International Mathematical Olympiad
 "Formula of Unity" / "The Third Millennium"
 Year 2026–2027. Qualifying round

Problems for grade R6



Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan),
 more details are at the page formulo.org/en/olymp/2026-math-en/.

Your paper should be sent until **23:59:59 UTC, 5 November 2026**.

Please solve the problems by yourself. The use of calculators, programming tools and neural networks is prohibited.
 Remember that the majority of the problems require not only an answer but also its full proof.

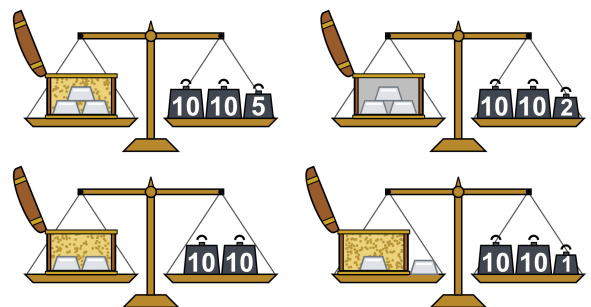
The paper should not contain your personal data, so **please do not sign your paper**.

1. Arthur is playing "Battleship" with a friend. In this game, one player places 10 ships (four 1×1 , three 1×2 , two 1×3 and one 1×4) on a 10×10 board so that the ships do not touch each other, not even at corners, and the other player names cells one at a time in order to "sink" these ships. Arthur has already made several moves (see the figure; ● — hit, ⊗ — miss). How many valid ship placements are there? (P. Mulenko)

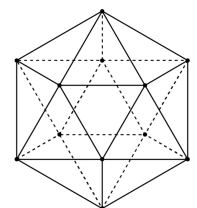
	A	B	C	D	E	F	G	H	I	J
1	●	●							⊗	●
2					●					
3					●					●
4										●
5	●					●	⊗			●
6	⊗		●	●		⊗				
7	●									●
8										
9		●	⊗							
10		⊗								

2. Is it possible to cut an 8×8 chessboard into several congruent pieces, so that in each piece the number of black cells is different from the number of white cells, and one 2×2 square? (P. Mulenko, S. Pavlov)
3. The digits 1, 2 and 3 are written on a board. Remarkably, their sum is divisible by each of them. How can several more digits be written on the board so that all the digits are different and their sum is divisible by each of them? Find all ways and prove that there are no others. (P. Mulenko, S. Pavlov)
4. In a country inhabited only by knights, who always tell the truth, and liars, who always lie, people are preparing to celebrate Independence Day. Two residents saw a poster showing the three-digit age of the country, and told you the following.
- 1st resident: 1) "The age is divisible by 25." 2) "The sum of the digits of the age is not 9."
- 2nd resident: 1) "The sum of the digits of the age is 18." 2) "The last digit is not 8."
- How old can the country be? Find all possibilities. (P. Mulenko)

5. Pirate Jack hides identical silver bars in a chest. Sometimes he fills the chest to the top with sand, so that when it is opened, it seems that there is nothing valuable inside. While choosing how many bars to hide, he made four weighings on a balance scale (see the figure; the weights show their mass in pounds). How many pounds of sand fit into the empty chest? (A. Tesler)



6. A regular icosahedron is a polyhedron made up of 20 triangles with equal sides. An icosahedron with edge 1 was cut along several edges and unfolded onto the plane in one piece so that the faces do not overlap and no two faces touch along a segment unless they remained glued along it. What can the perimeter of the net be? Find all possibilities and prove that there are no others. (P. Mulenko)



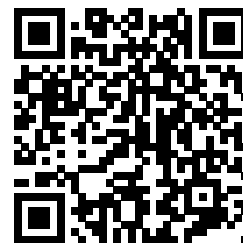
7. For what smallest natural number k is the following statement true: "If the product of 6 arbitrary natural numbers is divisible by 960, then the product of some k of them is divisible by 240"?

(A. Tesler)



International Mathematical Olympiad
 “Formula of Unity” / “The Third Millennium”
 Year 2026–2027. Qualifying round

Problems for grade R7



Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan),
 more details are at the page formulo.org/en/olymp/2026-math-en/.

Your paper should be sent until **23:59:59 UTC, 5 November 2026**.

Please solve the problems by yourself. The use of calculators, programming tools and neural networks is prohibited.

Remember that the majority of the problems require not only an answer but also its full proof.

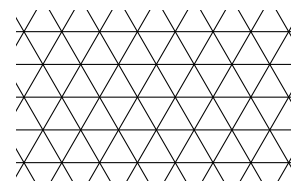
The paper should not contain your personal data, so **please do not sign your paper**.

1. Arthur is playing “Battleship” with a friend. In this game, one player places 10 ships (four 1×1 , three 1×2 , two 1×3 and one 1×4) on a 10×10 board so that the ships do not touch each other, not even at corners, and the other player names cells one at a time in order to “sink” these ships. Arthur has already made several moves (see the figure; ● — hit, ⊗ — miss). How many valid ship placements are there? (P. Mullenko)

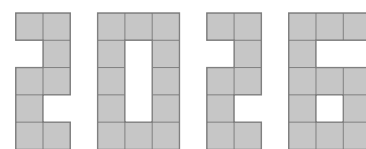
	A	B	C	D	E	F	G	H	I	J
1	●	●								●
2					●					
3					●					●
4										●
5	●					●	⊗			●
6	⊗		●	●		⊗				
7	●									●
8										
9	⊗	●	⊗							
10		⊗								

2. For what smallest n is it possible to choose ten pairwise distinct numbers among the numbers from 1 to n so that the product of five of them equals the product of the other five? (S. Pavlov)
3. A $1 \times 1 \times 2$ cuboid was cut along several edges and unfolded onto the plane in one piece so that the faces do not overlap and no two faces touch along a segment unless they remained glued along it. What can the perimeter of the net be? Find all possibilities and prove that there are no others. (P. Mullenko)
4. In a country inhabited only by knights, who always tell the truth, and liars, who always lie, people are preparing to celebrate Independence Day. Two residents saw a poster showing the four-digit age of the country, and told you the following about this age.
- First resident:* 1) “The remainder when divided by 101 is 80.” 2) “The remainder when divided by 99 is not 40.”
- Second resident:* 1) “The remainder when divided by 99 is 18.” 2) “The remainder when divided by 101 is not 92.”
- How old can the country be? Find all possibilities. (P. Mullenko)

5. A grasshopper is sitting at a vertex of one of the tiles of a plaza paved with equilateral triangles. With each jump, it moves along a side of a tile to an adjacent vertex. At how many different vertices can it end up after 60 jumps? (E. Voronetskiy)



6. The number 2026 has been cut out of grid paper along the grid lines (see the figure). Cut one or more of the digits along the grid lines and assemble a 5×8 rectangle from all the resulting pieces. The total length of all cuts must not exceed 3 (the side of a cell is 1). Draw how the digits are cut into pieces, and then how the rectangle is assembled from these pieces. (S. Pavlov)

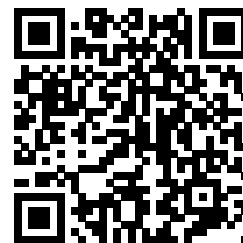


7. Once, 25 rolls of wallpaper were bought to paper a rectangular hall, but they were not enough, and one more roll had to be bought. Now it has been decided to divide the hall into four rectangular rooms by two perpendicular partitions running from wall to wall. Each room will be papered with wallpaper of its own color: red, yellow, green, and blue. It turned out that for three of the rooms, 10 rolls of red wallpaper, 12 of yellow, and 16 of green are enough (one roll fewer would no longer be enough, though). What is the smallest number of rolls of blue wallpaper that must be bought to be sure that they will be enough for the fourth room? Assume that the wallpaper consumption is proportional to the total length of the walls; ignore windows and doors. (P. Mullenko)



International Mathematical Olympiad
 “Formula of Unity” / “The Third Millennium”
 Year 2026–2027. Qualifying round

Problems for grade R8



Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan),
 more details are at the page formulo.org/en/olymp/2026-math-en/.

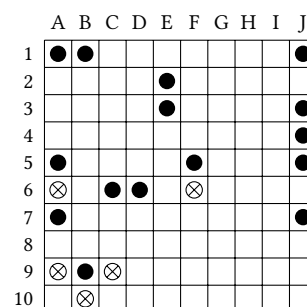
Your paper should be sent until **23:59:59 UTC, 5 November 2026**.

Please solve the problems by yourself. The use of calculators, programming tools and neural networks is prohibited.

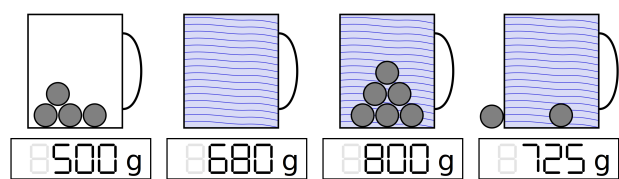
Remember that the majority of the problems require not only an answer but also its full proof.

The paper should not contain your personal data, so **please do not sign your paper**.

1. Arthur is playing “Battleship” with a friend. In this game, one player places 10 ships (four 1×1 , three 1×2 , two 1×3 and one 1×4) on a 10×10 board so that the ships do not touch each other, not even at corners, and the other player names cells one at a time in order to “sink” these ships. Arthur has already made several moves (see the figure; ● — hit, ⊗ — miss). How many valid ship placements are there? (P. Mullenko)

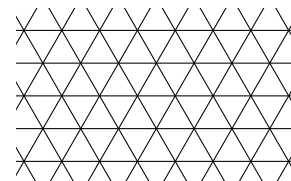


2. For what smallest n is it possible to choose ten pairwise distinct numbers among the numbers from 1 to n so that the product of five of them equals the product of the other five? (S. Pavlov)
3. Once, 25 rolls of wallpaper were bought to paper a rectangular hall, but they were not enough, and one more roll had to be bought. Now it has been decided to divide the hall into four rectangular rooms by two perpendicular partitions running from wall to wall. Each room will be papered with wallpaper of its own color: red, yellow, green, and blue. It turned out that for three of the rooms, 10 rolls of red wallpaper, 12 of yellow, and 16 of green are enough (one roll fewer would no longer be enough, though). What is the smallest number of rolls of blue wallpaper that must be bought to be sure that they will be enough for the fourth room? Assume that the wallpaper consumption is proportional to the total length of the walls; ignore windows and doors. (P. Mullenko)
4. Archimedes has a mug, several identical metal balls, a water tap, accurate scales, and plenty of free time. He made several weighings and obtained the results shown in the figure (each time there is water in the mug, it is filled to the brim). What is the mass of the empty mug?



(A. Tesler)

5. A $1 \times 2 \times 3$ cuboid was cut along several edges and unfolded onto the plane in one piece so that the faces do not overlap and no two faces touch along a segment unless they remained glued along it. What can the perimeter of the net be? Find all possibilities and prove that there are no others. (P. Mullenko)
6. An ant found itself on a plaza paved with equilateral triangles. It can run parallel to the sides of the tiles at a speed of 1 cm/s. What is the perimeter of the set of points that the ant can reach in 1 minute? (E. Voronetskiy)

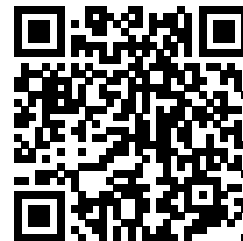


7. Let us call a natural number k *green* if there exists a natural number n such that the decimal representation of $n!$ ends in exactly k zeros, and *purple* otherwise. Which numbers are there more of in the range from 1 to 20262026: green or purple? (A. Tesler)



International Mathematical Olympiad
 “Formula of Unity” / “The Third Millennium”
 Year 2026–2027. Qualifying round

Problems for grade R9



Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan),
 more details are at the page formulo.org/en/olymp/2026-math-en/.

Your paper should be sent until **23:59:59 UTC, 5 November 2026**.

Please solve the problems by yourself. The use of calculators, programming tools and neural networks is prohibited.

Remember that the majority of the problems require not only an answer but also its full proof.

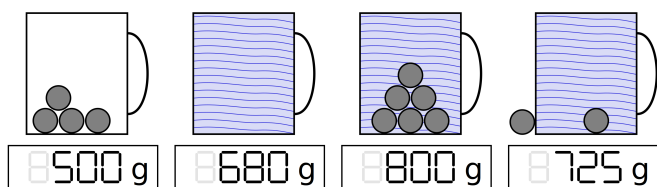
The paper should not contain your personal data, so **please do not sign your paper**.

1. Arthur is playing “Battleship” with a friend. In this game, one player places 10 ships (four 1×1 , three 1×2 , two 1×3 and one 1×4) on a 10×10 board so that the ships do not touch each other, not even at corners, and the other player names cells one at a time in order to “sink” these ships. Arthur has already made several moves (see the figure; ● — hit, ⊗ — miss). How many valid ship placements are there? (P. Mulenko)

	A	B	C	D	E	F	G	H	I	J
1	●	●							⊗	●
2					●					
3					●					●
4										●
5	●					●	⊗			●
6	⊗		●	●		⊗				
7	●									●
8										
9		●								
10										

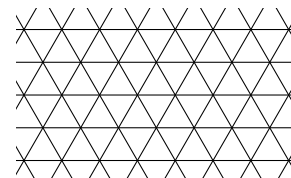
2. For what smallest $n > 1$ can all natural numbers from 1 to n be written in a row so that the sum of any two adjacent numbers is a perfect square? (S. Pavlov)
3. The “football” polyhedron is made up of 12 regular pentagons and 20 regular hexagons with side 1. It was cut along several edges and unfolded onto the plane in one piece so that the faces do not overlap and no two faces touch along a segment unless they remained glued along it. What can the perimeter of the net be? Find all possibilities and prove that there are no others. (P. Mulenko)

4. Archimedes has a mug, several identical metal balls, a water tap, accurate scales, and plenty of free time. He made several weighings and obtained the results shown in the figure (each time there is water in the mug, it is filled to the brim). What is the mass of the empty mug?



(A. Tesler)

5. An ant found itself on a plaza paved with equilateral triangles. It can run parallel to the sides of the tiles at a speed of 1 cm/s. What is the area of the set of points that the ant can reach in 1 minute? (E. Voronetskiy)

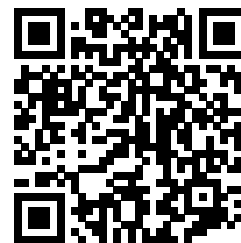


6. TikTok Johnny cut a square with side 2 into a triangle and a quadrilateral with one straight cut. If the perimeter of the triangle is 6, what is the perimeter of the quadrilateral? (P. Mulenko)
7. Prove that among the numbers from 1 to 10^{1000} , more than two million numbers contain only twos and fives in their prime factorization. (A. Tesler)



International Mathematical Olympiad
 “Formula of Unity” / “The Third Millennium”
 Year 2026–2027. Qualifying round

Problems for grade R10



Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan),
 more details are at the page formulo.org/en/olymp/2026-math-en/.

Your paper should be sent until **23:59:59 UTC, 5 November 2026**.

Please solve the problems by yourself. The use of calculators, programming tools and neural networks is prohibited.

Remember that the majority of the problems require not only an answer but also its full proof.

The paper should not contain your personal data, so **please do not sign your paper**.

1. Arthur is playing “Battleship” with a friend. In this game, one player places 10 ships (four 1×1 , three 1×2 , two 1×3 and one 1×4) on a 10×10 board so that the ships do not touch each other, not even at corners, and the other player names cells one at a time in order to “sink” these ships. Arthur has already made several moves (see the figure; ● — hit, ⊗ — miss). How many valid ship placements are there? (P. Mulenko)

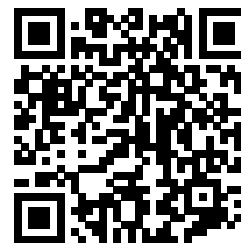
	A	B	C	D	E	F	G	H	I	J
1	●	●								●
2					●					
3					●					●
4										●
5	●					●				●
6	⊗			●	●					
7	●									●
8										
9	⊗	●	⊗							
10		⊗								

2. Once, 25 rolls of wallpaper were bought to paper a rectangular hall, but they were not enough, and one more roll had to be bought. Now it has been decided to divide the hall into four rectangular rooms by two perpendicular partitions running from wall to wall. Each room will be papered with wallpaper of its own color: red, yellow, green, and blue. It turned out that for three of the rooms, 10 rolls of red wallpaper, 12 of yellow, and 16 of green are enough (one roll fewer would no longer be enough, though). What is the smallest number of rolls of blue wallpaper that must be bought to be sure that they will be enough for the fourth room? Assume that the wallpaper consumption is proportional to the total length of the walls; ignore windows and doors. (P. Mulenko)
3. For what smallest natural number k is the following statement true: “If the product of 10 arbitrary natural numbers is divisible by 61440, then the product of some k of them is divisible by 240”? (A. Tesler)
4. A country has 2026 airports, numbered from 1 to 2026. There is a flight between two airports if the sum of their numbers is the square of a natural number. Is it true that one can get from any airport to any other, possibly changing planes at intermediate airports along the way? (S. Pavlov)
5. TikTok Johnny cut a square into a triangle and a quadrilateral with one straight cut. The ratio of the perimeter of the triangle to the perimeter of the quadrilateral is 6 : 7. What is the ratio of the area of the triangle to the area of the quadrilateral? (P. Mulenko, A. Tesler)
6. Prove that among the numbers from 1 to 10^{1000} , more than two million numbers contain only twos and fives in their prime factorization. (A. Tesler)
7. A point P is marked inside a convex quadrilateral $ABCD$ so that the triangles PAB , PBC , PCD , and PDA have equal areas. It is also known that $PB = PC$ and $PA = PD$. Prove that P is the center of the circumcircle of the quadrilateral $ABCD$. (P. Mulenko)



International Mathematical Olympiad
 “Formula of Unity” / “The Third Millennium”
 Year 2026–2027. Qualifying round

Problems for grade R11



Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan),
 more details are at the page formulo.org/en/olymp/2026-math-en/.

Your paper should be sent until **23:59:59 UTC, 5 November 2026**.

Please solve the problems by yourself. The use of calculators, programming tools and neural networks is prohibited.

Remember that the majority of the problems require not only an answer but also its full proof.

The paper should not contain your personal data, so **please do not sign your paper**.

1. Arthur is playing “Battleship” with a friend. In this game, one player places 10 ships (four 1×1 , three 1×2 , two 1×3 and one 1×4) on a 10×10 board so that the ships do not touch each other, not even at corners, and the other player names cells one at a time in order to “sink” these ships. Arthur has already made several moves (see the figure; ● — hit, ⊗ — miss). How many valid ship placements are there? (P. Mulenko)

	A	B	C	D	E	F	G	H	I	J
1	●	●								●
2					●					
3					●					●
4										●
5	●					●				●
6	⊗			●	●					
7	●									●
8										
9	⊗	●								
10		⊗								

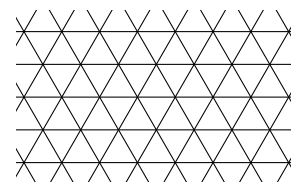
2. Lily wrote her phone number, consisting of 11 digits, on a piece of paper. Is it possible to write one digit at the beginning, one at the end and one between every two adjacent digits of the phone number so that the resulting 23-digit number is divisible by 2026? (A. Tesler)
3. A country has 2026 airports, numbered from 1 to 2026. There is a flight between two airports if the sum of their numbers is the square of a natural number. Is it true that even if one of the airports closes, one can still get from any of the remaining airports to any other, possibly changing planes at intermediate airports (but not at the closed one)? (S. Pavlov)
4. A point with coordinates (k, ℓ, m) is marked on the unit sphere, and

$$m = \frac{\ell}{\sqrt{k^2 + \ell^2}}.$$

Prove that the numbers $1, \frac{mk}{\ell}, k^2 + \ell^2, \frac{k\ell}{m}$ (in this order) form a geometric progression. (P. Mulenko)

5. An ant found itself on a plaza paved with equilateral triangles (see the figure). It can run parallel to the sides of the tiles at a speed of 1 cm/s. The ant wants to get to the point 30 cm to the right of its current position, and it has 1 minute to do it. What is the area of the set of points it can visit along the way?

(E. Voronetskiy)



6. Prove that among the numbers from 1 to 10^{1000} , more than two million numbers contain only twos and fives in their prime factorization. (A. Tesler)
7. A point P is marked inside a convex quadrilateral $ABCD$ so that the triangles PAB , PBC , PCD , and PDA have equal areas. It is also known that $PB = PC$. Prove that the quadrilateral $ABCD$ has a right angle. (P. Mulenko)