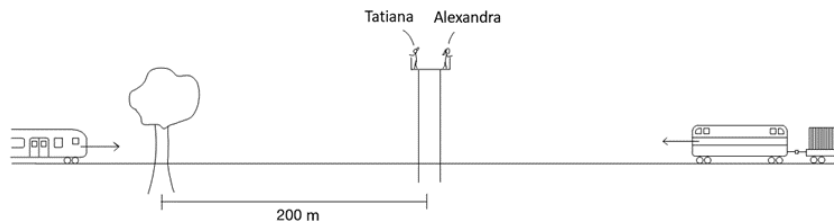




Solutions to problems for grade R8

8.1. (7 points) Tatiana and Alexandra were walking outside the city and decided to watch trains on a straight railway from a pedestrian bridge located above it. Around them there were steppes, except for a tree near the railway located 200m before the bridge. Tatiana looked in the direction of the tree and Alexandra looked in the opposite direction. From Tatiana's side a passenger train was approaching, and from Alexandra's side a freight train was approaching. At some moment the passenger train aligned with the tree, and at the same moment the freight train aligned with the bridge. Tatiana noticed that the passenger train consisted of three standard wagons of length 25.5 m each. Tatiana also measured the time interval of 7s between the moments when the train was aligned with the tree and when it was aligned with the bridge. Alexandra noticed that the freight train completely passed the bridge one second later than the passenger train. It was more difficult for her to determine the length of the freight train, since it contained wagons of different lengths. However, one of the platforms carried two standard containers, and Alexandra knew from her father, who works at a port, that their total length is 24m. This platform passed the bridge in 3s.



[1] Determine the length of the freight train if the trains move with constant velocities.

Comment. Neglect the width of the bridge.

(Laskavyi L.S.)

Answer: $l_f = 90 \text{ m}$.

Solution.

The length of the passenger train is defined as

$$l_p = nl_w$$

Let's determine the speed of the passenger train:

$$v_p = \frac{S_t}{t}$$

Over a period of time t_1 the train traveled a distance equal to the sum of its own length and the distance from the tree to the bridge:

$$S_p = S_t + l_p$$

Or, substituting the length of the train:

$$S_p = S_t + nl_w$$

The train's speed is constant, so:

$$v_p = \frac{S_t + nl_w}{t_1}$$

Equating the expressions for the speeds, express t_1 :

$$t_1 = \frac{(S_t + nl_w)t}{S_t}$$

The freight train traveled a distance equal to its own length in a time that is one second longer than t_1

$$S_f = l_f$$

Determine the speed of the freight train using the information about the containers:

$$v_f = \frac{l_c}{t_c}$$

On the other hand:

$$v_f = \frac{S_f}{t_1 + \Delta t}$$

Express:

$$S_f = v_f (t_1 + \Delta t)$$

and substitute all expressions:

$$l_f = \frac{l_c}{t_c} \left(\frac{(S_t + nl_w)t}{S_t} + \Delta t \right)$$

Substituting the given values, we end up with:

$$l_f = \frac{24}{3} \left(\frac{(200 + 3 \cdot 25.5) \cdot 7}{200} + 1 \right) = 85.42 \text{ m}$$

Since the minimum number of significant digits in the problem statement is one, round the answer so that $l_f = 90 \text{ m}$.

8.2. (9 points) Timur visited his grandmother to help her receive a delivery of firewood for heating the house. When the 0.900 m^3 of firewood was delivered, Timur decided not to carry it across the entire yard to the shed immediately and left it outside. However, it was autumn with rare but heavy rains, and the temperature remained close to $0.00 \text{ }^\circ\text{C}$. Because of the rain the firewood became wet. While gradually moving the firewood under the shed with a wheelbarrow and sorting it, Timur determined the total mass of the firewood to be 648 kg .

[2] For how many days will this firewood be sufficient to keep the house heated at the same temperature if a mass of 10 kg of dry firewood must be burned each day for this purpose?

Comment. Assume external conditions during combustion of dry and wet firewood to be unchanged and identical. The density of dry firewood is 600.00 kg/m^3 , the specific heat capacity of water is $4200.00 \text{ J/(kg} \cdot \text{K)}$, the latent heat of vaporization of water is 2.30 J/kg and the heat of combustion of dry firewood is $1.00 \cdot 10^7 \text{ J/kg}$. (Laskavyi L.S.)

Answer: $n = 51$.

Solution.

Since firewood is a heterogeneous material, it is impossible to use volumes. Therefore, we will use masses. The mass of dry firewood:

$$m_{df} = \rho_{\pi} V$$

$$m_{df} = 600 \cdot 0.9 = 540 \text{ kg}$$

Then the mass of water contained in the firewood can be found as the difference

$$m_w = m_{wf} - m_{df}$$

$$m_w = 648 - 540 = 108 \text{ kg}$$

If 540 kg of dry firewood corresponds to 108 kg of water, then 10 kg of dry firewood corresponds to the mass of water equal to

$$m_{w1} = 2 \text{ kg}$$

If we consider wet firewood, then

$$m_{wet} = m_{01} + m_{w1}$$

Moreover, from the ratio between 10 kg of firewood and 2 kg of water in wet firewood, it is clear that for any mass of wet firewood

$$m_{01} = \frac{10}{12} m_{wet}$$

$$m_{w1} = \frac{2}{12} m_{wet}$$

Let's convert the temperature to absolute values:

$$T_{in} = 273 \text{ K}$$

$$T_b = 373 \text{ K}$$

Maintaining a house at a certain temperature means that a certain amount of heat, obtained from burning firewood, is required. In the case of dry firewood, the amount of heat produced when it is burned is

$$Q_{dry} = qm_0$$

It is clear that the heat produced by wet firewood must be exactly the same as that from dry firewood. But in addition to the energy obtained from burning the firewood, it is necessary to heat and evaporate the water that has entered the firewood. Therefore, we get:

$$Q_{wet} - Q_1 - Q_2 = Q_{dry}$$

Specific heat of combustion of wet firewood:

$$Q_{wet} = q \cdot \frac{10}{12} m_{wet}$$

Amount of heat required to heat water:

$$Q_1 = c \cdot \frac{2}{12} m_{wet} (T_b - T_{in})$$

The amount of heat required to completely turn water into steam by boiling it:

$$Q_2 = L \cdot \frac{2}{12} m_{wet}$$

As a result we get:

$$q \cdot \frac{10}{12} m_{wet} - c \cdot \frac{2}{12} m_{wet} (T_b - T_{in}) - L \cdot \frac{2}{12} m_{wet} = qm_0$$

Express m_{wet} :

$$m_{wet} (10q - 2c(T_b - T_{in}) - 2L) = 12qm_0$$

$$m_{wet} = \frac{12qm_0}{10q - 2c(T_b - T_{in}) - 2L}$$

Substitute the numerical values:

$$m_{wet} = \frac{12 \cdot 10^7 \cdot 10}{10 \cdot 10^7 - 2 \cdot 4200 \cdot (373 - 273) - 2 \cdot 2.3 \cdot 10^6} = 12.6903 \text{ kg}$$

Thus, we have found the mass of wet firewood, the total heat from which will be equal to the heat from dry firewood needed for one day of heating the house. To find the total number of days, we need to divide the total mass of wet firewood by m_{wet} :

$$n = \frac{m_{wf}}{m_{wet}}$$

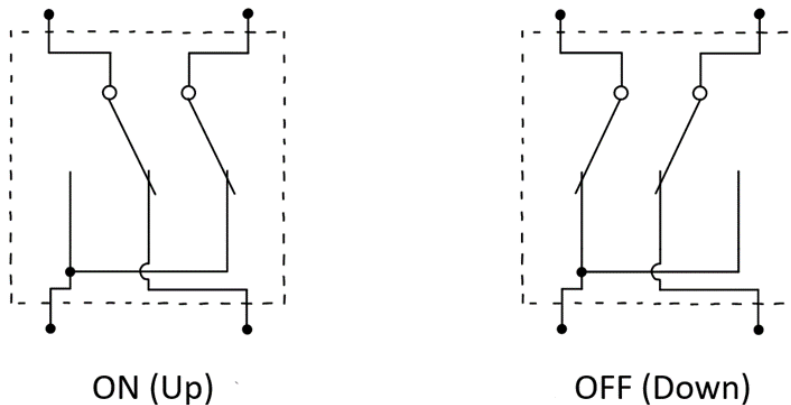
Substituting the numerical values, we get:

$$n = \frac{648}{12.6903} = 51.0624$$

The number of days must be an integer, so

$$n = 51$$

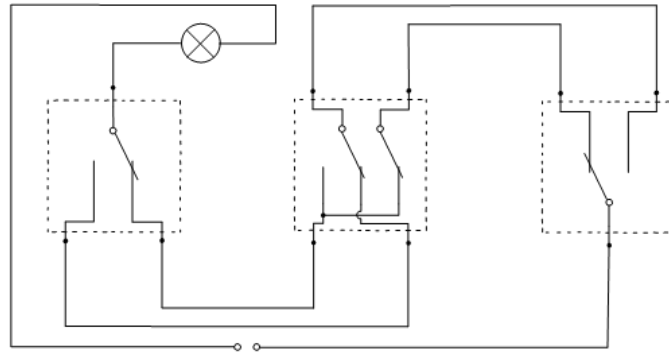
8.3. (5 points) As part of his physics club's study of electrical circuits, Artem was given the task of designing an electrical circuit so that the lighting in the laboratory could be turned on and off at any of the three doors leading into it. He knew that there exist two-way switches that operate similarly to switches in railway track systems. Alexey, who had helped his father with home repairs, told Artem that for a similar task they had bought an intermediate switch, whose circuit diagram is shown in the figure: the mechanism of the key simultaneously affects two internal branches. However, Alexey also did not know how these switches should be connected in a circuit. Both the two-way switch and the intermediate switch look exactly like ordinary wall switches and are installed in the same way.



- [3] Construct a circuit with a power source, a lamp, two-way switches and one intermediate switch so that the circuit allows Artem to achieve his goal.

(Laskavyi L.S.)

Answer:



Solution.

Each switch must break the circuit when pressed, and there must be no more than one break in the circuit. Therefore, two two-way switches must be positioned with their terminals connected to the power source and the lamp respectively, with an intermediate switch placed between them. A diagram of this circuit is shown in Figure 2.

8.4. (7 points) During a physics club session Ivan performs an experiment on thermal phenomena. A nickel cup contains 250 g of water at a temperature of 20 °C. Ivan immerses an electric heater into the cup, and after 15 minutes the water begins to boil.

[4] How long will it take for 20.0% of water to boil away?

Comment. Assume the power of the heater to be constant. Neglect heat losses to the surroundings and the heat capacity of the heater. The heat capacity of nickel is 46 J/K, the specific heat capacity of water is 4200 J/(kg · K), and the latent heat of vaporization of water is $2.3 \cdot 10^6$ J/kg.

(Laskavyi L.S.)

Answer: $\tau_2 = 1200$ s.

Solution.

Let's convert the temperature to absolute values:

$$T_1 = 293 \text{ K}$$

$$T_2 = 373 \text{ K}$$

Let's break the process down into two separate steps: first, heating the cup with water to boiling point, then boiling a portion of the water.

$$Q_0 = Q_1 + Q_2,$$

where heating the nickel requires

$$Q_1 = C_{ni} (T_2 - T_1),$$

and heating of the water requires

$$Q_2 = c_w m_w (T_2 - T_1).$$

The amount of heat released by the electric heater:

$$Q_0 = P \tau_1$$

That is

$$P\tau_1 = C_{ni}(T_2 - T_1) + c_w m_w (T_2 - T_1)$$

From where

$$P = \frac{C_{ni}(T_2 - T_1) + c_w m_w (T_2 - T_1)}{\tau_1}$$

Here, a partial substitution can be made to obtain the power of the heater. Let's consider the second process (evaporation of the water):

$$Q_2 = Lm_v$$

The ratio of the masses of water and steam:

$$m_v = 0.2m_w$$

Heat released by the heater (power is constant):

$$Q_2 = P\tau_2$$

Then

$$P\tau_2 = 0.2Lm_w$$

Wherefrom

$$\tau_2 = \frac{0.2Lm_w}{P}$$

Substitute the expression for power obtained from the first process:

$$\tau_2 = \frac{0.2Lm_w\tau_1}{C_{ni}(T_2 - T_1) + c_w m_w (T_2 - T_1)}$$

Substituting the numerical values, we get:

$$\tau_2 = \frac{0.2 \cdot 2.3 \cdot 10^6 \cdot 0.25 \cdot 900}{46 \cdot (373 - 293) + 4200 \cdot 0.25 \cdot (373 - 293)} = 1180.4288 \text{ s.}$$

Given that the minimum number of significant digits in the problem statement is two, we round the answer so that $\tau_2 = 1200 \text{ s}$.

8.5. (6 points) During one of the physics laboratory sessions Alexander assembled a block-and-tackle system using pulleys, weights, springs, and a rope, as shown in the figure. The mass of the second weight is 200 g, and the stiffness of the first, second, and third springs is 100 N/m, 90 N/m, and 150 N/m, respectively. As a result, the system constructed by Alexander is in a state of equilibrium.

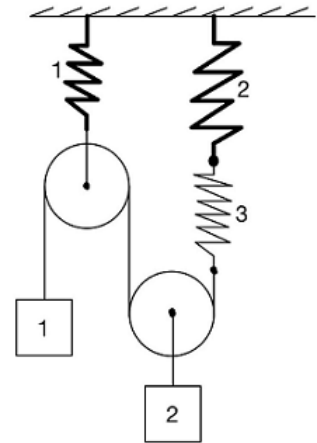
[5] Find the mass of the first weight and the elongation of the first spring, as well as the total elongation of the second and third springs.

Comment. Consider the pulleys and springs to be weightless and the rope to be ideal (weightless and inextensible). Take the acceleration due to gravity to be $g = 10.0 \text{ m/s}^2$. (Laskavy L.S.)

Answer: $m_1 = 0.1 \text{ kg}$, $\Delta l_1 = 0.02 \text{ m}$, $\Delta l_{23} = 0.02 \text{ m}$.

Solution.

Let's describe all the forces acting on each of the bodies. Let \vec{T} be the tension force of the rope; Let \vec{T}_1 be the tension force of the string, attached to the movable pulley. Therefore, we get



$$T_1 = 2T$$

Since the entire system is in equilibrium, the equilibrium equations hold for each body in the system: forces acting in one direction balance forces acting in the other direction. For the first and second bodies the equilibrium equations take the form:

$$m_1g = T$$

$$m_2g = T_1 = 2T$$

Then:

$$m_2g = 2m_1g$$

As a result:

$$m_1 = \frac{m_2}{2}$$

For springs connected in series, the following relationship holds:

$$\frac{1}{k_{23}} = \frac{1}{k_2} + \frac{1}{k_3} \quad \Rightarrow \quad k_{23} = \frac{k_2k_3}{k_2 + k_3}$$

Hooke's Law:

$$F_{el} = k\Delta l$$

The equilibrium equation for the first spring:

$$k_1\Delta l_1 = 2T = 2m_1g$$

From which

$$\Delta l_1 = \frac{2m_1g}{k_1}$$

Similarly for the second and third springs:

$$k_{23}\Delta l_{23} = T = m_1g$$

$$\frac{k_2k_3}{k_2 + k_3}\Delta l_{23} = T = m_1g$$

Express Δl_{23} :

$$\Delta l_{23} = \frac{m_1g(k_2 + k_3)}{k_2k_3}$$

Substituting the numerical values, we ultimately obtain:

$$m_1 = \frac{0.2}{2} = 0.1 \text{ kg}$$

$$\Delta l_1 = \frac{2 \cdot 0.1 \cdot 10}{100} = 0.02 \text{ m}$$

$$\Delta l_{23} = \frac{0.1 \cdot 10 \cdot (90 + 150)}{90 \cdot 150} = 0.01778 \text{ m}$$

Since the minimum number of significant digits in the problem statement is equal to one, we round only the third answer obtained and finally have: $m_1 = 0.1 \text{ kg}$, $\Delta l_1 = 0.02 \text{ m}$, $\Delta l_{23} = 0.02 \text{ m}$.



Solutions to problems for grade R9

9.1. (5 points) In a rocket-building club, Natalia and Peter built a model rocket and tested it on the school playground. After Peter launched the rocket, Natalia, who was watching the launch from the third-floor window of the school (10.0 m above the ground), noticed that the rocket reached her height in one second.

[1] What is the maximum height the rocket can reach if it has enough fuel for 7.0 seconds of engine operation?

Comment. Assume that the acceleration during engine operation is constant, ignore air resistance, and assume that the acceleration due to gravity is 10.0m/s^2 . *(Laskavyi L.S.)*

Answer: $S = 1500\text{ m}$.

Solution.

It is recommended to perform a partial substitution, but a solution in general form is also acceptable. The rocket's flight can be represented in two stages: flight while the engine is operating, and flight by inertia under the action of free-fall acceleration. While the engine is operating, the resulting acceleration (the rocket's acceleration) is the vector sum of the rocket engine's own acceleration and the acceleration of free fall.

Using the formula

$$S_1 = v_0 t_1 + \frac{at_1^2}{2}$$

and considering that $v_0 = 0$, find the rocket's acceleration during engine operation:

$$a = \frac{2S_1}{t_1^2} = \frac{2 \cdot 10}{1} = 20\text{ m/s}^2$$

Find the displacement during ascent with the engine running:

$$S_r = \frac{at_r^2}{2} = \frac{20 \cdot 7^2}{2} = 490\text{ m}$$

The final velocity at this stage of flight (which is also the initial velocity for the second stage):

$$v_1 = at_r = 20 \cdot 7 = 140\text{ m/s}$$

The rocket's velocity in the projection onto the axis:

$$v_2 = v_1 - gt_s$$

Taking into account that the velocity at the highest point is zero, we get:

$$0 = v_1 - gt_s,$$

hence

$$t_s = \frac{v_1}{g} = \frac{140}{10} = 14\text{ s}$$

The formula for the rocket's displacement under the action of free-fall acceleration is projected as

$$S_s = v_1 t_s - \frac{gt_s^2}{2} = 140 \cdot 14 - \frac{10 \cdot 14^2}{2} = 980\text{ m}.$$

As a result, the total ascent height

$$S = S_r + S_s = 490 + 980 = 1470\text{ m}.$$

Given that the minimum number of significant digits in the problem statement is two, we round the answer so that $\tau_2 = 1500 \text{ m}$.

9.2. (6 points) During a physics laboratory session, the teacher demonstrated the following experiment: he took an aquarium filled with mercury, dropped a silver ball into it, and then poured liquid on top. It turned out that the ball floats on the boundary between the two liquids, with 75.5% of its volume in mercury.

№	Liquid	Density (kg/m^3)
1	AI-98 petrol	780
2	Ethyl alcohol	789
3	Olive oil	946
4	Fresh water	1000
5	Glycerine	1260

[2] Choose from the liquids listed in the table the one that was poured on top in the experiment described. Write the name of this liquid in your answer sheet.

Comment. Assume that the boundary between the liquids is horizontal and stable, the liquids are immiscible, the density of mercury is 13600 kg/m^3 , the density of silver is 10500 kg/m^3 and the acceleration due to gravity is 10.0 m/s^2 . The ball does not touch the bottom or walls of the aquarium even when completely immersed in mercury. (Laskavyi L.S.)

Answer: Olive oil.

Solution.

The volume of the ball is divided into two components, one of which floats in mercury, the other in an unknown liquid:

$$V = V_1 + V_2$$

Then

$$V_2 = V - V_1 = V - 0.755V = 0.245V$$

Newton's second law projected onto the vertical axis (acceleration is zero, since the system is at rest):

$$\rho_m g V_1 + \rho_l g V_2 = mg$$

Substituting the mass of the ball:

$$m = \rho_s V$$

and dividing by g , we get:

$$\rho_m V_1 + \rho_l V_2 = \rho_s V.$$

From which

$$\rho_l = \frac{\rho_s V - \rho_m V_1}{V_2}$$

Expressing the volumes in terms of the total volume and dividing by it, we ultimately obtain:

$$\rho_l = \frac{\rho_s - 0.755\rho_m}{0.245}$$

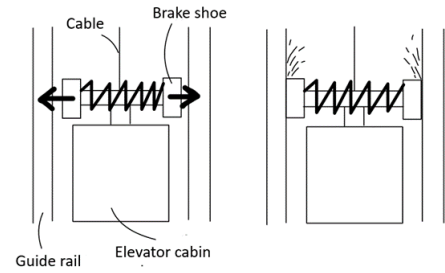
Substituting the numerical values, we get:

$$\rho_l = \frac{10500 - 0.755 \cdot 13600}{0.245} = 946.9388 \text{ kg/m}^3$$

Since the minimum number of significant digits in the problem statement is three, we round the answer so that $\rho_l = 947 \text{ kg/m}^3$.

This density corresponds to olive oil among those listed in the table.

9.3. (8 points) During a physics club session teacher asked Nikolai to understand how wedge-type elevator safety gear works. He only knew that two guide rails are installed in the elevator shaft, and when the safety system is activated, brake shoes are pressed against them. Nikolai assumed that the two brake shoes could be pressed apart by a spring between them acting as a spacer (in reality wedge-type safety gear is constructed differently) and thus stop the elevator cabin if its speed somehow exceeded the permissible value (see figure).



[3] Calculate the spring stiffness required to completely stop the elevator cabin in 1.5 s, if the total mass of the cabin with passengers is 1500 kg, the speed at which the system is activated is 1.2 m/s, the distance from each shoe to the guide rail is 0.5 cm, and the friction coefficient is 0.7.

Comment. The initial compression of the spring is equal to 20 cm. The acceleration due to gravity is 10.0m/s^2 . (Laskavyi L.S.)

Answer: $k = 60000\text{ N/m}$.

Solution.

Let's consider the action of the spring on the brake shoes. The spring, connected to the brake shoes, exerts an elastic force on each of them according to Hooke's law:

$$F_{el} = k(\Delta l_0 - 2\Delta l)$$

This force is what presses the brake shoes against the guides; therefore, by Newton's third law, the reaction force is equal to the elastic force exerted by the spring:

$$N = F_{el} = k(\Delta l_0 - 2\Delta l)$$

Newton's second law:

$$\vec{F}_r = m\vec{a}$$

Expanding the resultant force:

$$m\vec{g} + \vec{F}_{fr} + \vec{F}_{fr} = m\vec{a}$$

In the projection onto the axis pointing downward

$$mg - F_{fr} - F_{fr} = -ma$$

The drag force is determined by Coulomb–Amonton's law:

$$F_{fr} = \mu N$$

Or

$$F_{fr} = \mu k(\Delta l_0 - 2\Delta l)$$

As a result

$$mg - 2\mu k(\Delta l_0 - 2\Delta l) = -ma$$

From which

$$k = \frac{mg + ma}{2\mu(\Delta l_0 - 2\Delta l)}$$

Find the acceleration from the kinematic expression for velocity:

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

In the projection onto the same axis, directed downward:

$$v = v_0 - at$$

Taking into account that $v = 0$, we get

$$a = \frac{v_0}{t}$$

Substituting into the expression for the spring stiffness, we get:

$$k = \frac{mg + m\frac{v_0}{t}}{2\mu(\Delta l_0 - 2\Delta l)}$$

Substituting the numerical values, we end up with:

$$k = \frac{1500 \cdot 10 + 1500 \cdot \frac{1.2}{1.5}}{2 \cdot 0.7 \cdot (0.2 - 2 \cdot 0.005)} = 60902.2556 \text{ N/m}$$

Since the minimum number of significant digits in the problem statement is one, we round the answer so that $k = 60000 \text{ N/m}$.

9.4. (9 points) Tatyana and Daria arrived at a pioneer camp, which had a river on its territory. A small dam had been built on the river, so the water flowed down from it. The girls decided to rebuild it into a proper dam, leaving a slit through which the water flowed. Measurements showed that at a certain distance from the dam, the cross-sectional area of the river was $0.5m^2$, and the flow velocity of the river at the same point was 0.5 m/s . The area of the water flow through the slit was $200cm^2$. Right behind it, Tatyana installed a generator wheel with blades of the same area as the slit in the dam, and Daria connected the generator to a circuit with parallel-connected incandescent lamps rated for a voltage of 48 V and a power of 50 W .

[4] How many incandescent lamps can be in the circuit if the generator's efficiency is 5% and the power generated in the circuit is constant?

Comment. Assume that the water jet forms a single stream that immediately and entirely strikes the blades of the generator after the dam. *(Laskavyi L.S.)*

Answer: $n \geq 20$.

Solution.

The volume of water passing through each cross-section (flow rate) is the same, that is

$$Q = S_1v_1 = S_2v_2$$

From which

$$v_2 = \frac{S_1v_1}{S_2}$$

Since the water hits the blades immediately, its velocity after the slit in the dam coincides with the velocity at which it hits the blades, and the change in potential energy can be neglected. Water flow rate per unit time:

$$\mu = v_2S_2\rho$$

Then the power of the water flow striking the blade can be expressed as

$$P_w = \frac{\mu v_2^2}{2}$$

$$P_w = \frac{v_2 S_2 \rho v_2^2}{2}$$

$$P_w = \frac{S_2 \rho v_2^3}{2}$$

$$P_w = \frac{S_2 \rho S_1^3 v_1^3}{2 S_2^3}$$

The generator's efficiency is defined as the ratio of useful (electrical) power to consumed power of the water flow.

$$\eta = \frac{P}{P_w}$$

$$\eta = \frac{2 S_2^3 P}{S_2 \rho S_1^3 v_1^3}$$

Current power

$$P = \frac{U^2}{R}$$

Where U is the voltage in the circuit, R is the total resistance; for identical lamps connected in parallel we have:

$$R = \frac{R_n}{n}$$

The lamp resistance can be found using the lamp's rated specifications:

$$R_n = \frac{U_n^2}{P_n}$$

Circuit voltage:

$$U = \sqrt{PR}$$

$$U = \sqrt{\frac{PR_n}{n}}$$

$$U = \sqrt{\frac{PU_n^2}{P_n n}}$$

For the lamps to operate normally without overheating, the voltage across each of them must not exceed the rated value:

$$U \leq U_n$$

That is

$$\sqrt{\frac{PU_n^2}{P_n n}} \leq U_n$$

On the other hand, the power from the efficiency formula:

$$P = \frac{\eta S_2 \rho S_1^3 v_1^3}{2 S_2^3}$$

Substituting the power into the inequality, we get:

$$\sqrt{\frac{\eta S_2 \rho S_1^3 v_1^3 U_n^2}{2 S_2^3 P_n n}} \leq U_n$$

Substituting the values, we obtain a numerical inequality with n :

$$\sqrt{\frac{0.05 \cdot 0.02 \cdot 1000 \cdot 0.5^3 \cdot 0.5^3 \cdot 48^2}{2 \cdot 0.02^3 \cdot 50 \cdot n}} \leq 48$$

$$\sqrt{\frac{45000}{n}} \leq 48$$

$$\frac{45000}{n} \leq 2304$$

$$\frac{45000}{2304} \leq n$$

$$n \geq 19.53$$

Clearly, the number of lamps must be an integer, so for correct operation without overloading the lamps, there must be at least twenty of them.

9.5. (5 points) Alina took her grandfather's old watch, which had started to run slow, to examine it in the physics lab at school. After removing the battery, she saw the writing «3V» on it. Alina decided to measure the voltage using two different voltmeters available in the lab. One of them showed 2.6 V, the other 2.5 V, and when connected together, they both showed 2.3 V.

[5] What is the EMF of the battery?

(Laskavyy L.S.)

Answer: $\mathcal{E} = 2.9 \text{ V}$.

Solution.

It is clear that the voltmeters are connected to the battery in parallel. They show different voltages because the source has its own internal resistance, and the voltmeters are also not ideal and have resistance. Let the voltmeters have resistances of R_1 and R_2 , and the battery have an internal resistance of r . Let's consider the first case. The formula for the battery's EMF:

$$I = \frac{\mathcal{E}}{r + R_1}$$

Ohm's law for a circuit segment:

$$I = \frac{U_1}{R_1}$$

Equating these expressions for current, we ultimately obtain:

$$U_1 = \frac{\mathcal{E}}{1 + \frac{r}{R_1}}$$

Similarly, for the second case and the case of a simultaneous connection of voltmeters:

$$U_2 = \frac{\mathcal{E}}{1 + \frac{r}{R_2}}$$

$$U_{12} = \frac{\mathcal{E}}{1 + \frac{r}{R_1} + \frac{r}{R_2}}$$

Expressing the resistance ratios from the first two cases and substituting them into the last one, we ultimately obtain:

$$\mathcal{E} = \frac{1}{\frac{1}{U_1} + \frac{1}{U_2} - \frac{1}{U_{12}}}$$

Substituting the values, we end up with:

$$\mathcal{E} = \frac{1}{\frac{1}{2.6} + \frac{1}{2.5} - \frac{1}{2.3}} = 2.858508 \text{ V}$$

Since the minimum number of significant digits in the problem statement is two, we round the answer so that $\mathcal{E} = 2.9 \text{ V}$.



Solutions to problems for grade R10

10.1. (8 points) In a school laboratory Petya studies electrostatic fields produced by charged frames of unusual shapes. Using a thin insulating thread, Petya made two closed contours: a circular ring of radius $R = 20 \text{ mm}$ and a square with a side of $2a = 40 \text{ mm}$. The boy arranged the figures so that their planes are parallel, and the axis of the ring is perpendicular to these planes and passes through the center of the square. The distance between the centers of the figures is $d=10 \text{ cm}$. A charge $Q = 2\mu\text{C}$ is uniformly distributed along the ring, and a charge $q = 2\mu\text{C}$ is uniformly distributed along the square. In the center of the square Peter placed a test charge $q_0 = 2.0 \text{ nC}$ with a mass of $m=1.0 \text{ g}$ and gave it an initial velocity of $v=0.50 \text{ m/s}$, directed along the axis connecting the centers of the figures, towards the ring.

[1] Will the test charge reach the center of the ring?

(Cherenkov A.A.)

Answer: charge will not reach the center of the ring.

Solution.

1) First, let's derive the formula for the potential of the electrostatic field created by the charged ring. To do this, consider a point P on the axis of the ring at a distance x from its center O . Imagine dividing the ring into a set of point charges $Q = Q_1 + \dots + Q_n$. According to the Pythagorean theorem, each point charge is located at a distance $r = \sqrt{R^2 + x^2}$.

Using the formula for the potential of a point charge and the principle of superposition, let's express the potential φ generated by the entire ring at the point P :

$$\varphi_c = \frac{kQ_1}{r} + \dots + \frac{kQ_n}{r} = \frac{kQ}{\sqrt{R^2 + x^2}}$$

Note that due to the generality of the derivation, the resulting formula holds for any charge distribution along the ring.

2) Let us now derive the formula for the potential of the electrostatic field created by a charged square at the same point P . The distance from the center of the square M to point P is $z = d - x$. Clearly, due to the central symmetry of the problem with respect to the axis OP , to calculate the potential created by the square, it is enough to consider half of any of its sides and multiply the result by 8.

Let us mentally divide the selected side of the square into point charges $dq = \rho dl$, where $\rho = \frac{q}{8a} = \text{const}$ is the charge density distribution over the square. Let the point charge be located at a distance l from the midpoint of the side of the square. Then, according to the Pythagorean theorem, the distance from it to point P is:

$$r(l) = \sqrt{MP^2 + (a^2 + l^2)} = \sqrt{(z^2 + a^2) + l^2}$$

Finally, using the formula for the potential of a point charge and the principle of superposition, we express the potential φ_0 created by half of the side of the square at point P :

$$\varphi_0 = \int_0^a \frac{k dq}{r} = \int_0^a \frac{k \rho dl}{\sqrt{(z^2 + a^2) + l^2}} = k \rho \int_0^a \frac{dl}{\sqrt{(z^2 + a^2) + l^2}}$$

Thus, we have obtained a tabular integral; we can use it to obtain the final answer in accordance with the Newton–Leibniz formula:

$$\varphi_0 = k \rho \cdot \ln \left| l + \sqrt{l^2 + z^2 + a^2} \right| \Big|_0^a = k \rho \left(\ln \left(a + \sqrt{z^2 + 2a^2} \right) - \ln \left(\sqrt{z^2 + a^2} \right) \right) =$$

$$= k \frac{q}{8a} \ln \frac{a + \sqrt{(d-x)^2 + 2a^2}}{\sqrt{(d-x)^2 + a^2}}$$

Finally, the potential created by the entire square at point P is equal to:

$$\varphi_s = 8\varphi_0 = k \frac{q}{a} \ln \frac{a + \sqrt{(d-x)^2 + 2a^2}}{\sqrt{(d-x)^2 + a^2}}$$

3) Let us now calculate the total potential created by the charged figures at point P. Continuing to follow the principle of superposition, we obtain:

$$\varphi(x) = \varphi_c(x) + \varphi_s(x) = \frac{kQ}{\sqrt{R^2 + x^2}} + \frac{kq}{a} \ln \frac{a + \sqrt{(d-x)^2 + 2a^2}}{\sqrt{(d-x)^2 + a^2}}$$

According to the resulting formula, the potential at the center of the square ($x = d$):

$$\varphi_1 = \frac{kQ}{\sqrt{R^2 + d^2}} + \frac{kq}{a} \ln (1 + \sqrt{2})$$

And the potential at the center of the ring ($x = 0$):

$$\varphi_2 = \frac{kQ}{R} + \frac{kq}{a} \ln \left(\frac{a + \sqrt{d^2 + 2a^2}}{\sqrt{d^2 + a^2}} \right)$$

The work done when moving a test charge from the center of the square to the center of the ring:

$$\begin{aligned} A = q_0 (\varphi_1 - \varphi_2) &= -kq_0 \left[Q \left(\frac{1}{R} - \frac{1}{\sqrt{R^2 + d^2}} \right) - \frac{q}{a} \left(\ln (1 + \sqrt{2}) - \ln \left(\frac{a + \sqrt{d^2 + 2a^2}}{\sqrt{d^2 + a^2}} \right) \right) \right] = \\ &= -2.11 \cdot 10^{-4} J \end{aligned}$$

4) The test charge will reach the center of the ring if its initial kinetic energy $E = \frac{mv^2}{2}$ is not less than the magnitude of the work done by the electrostatic field during this movement. Substituting numerical values, we find:

$$E = 1.25 \cdot 10^{-4} J$$

Thus, by comparing the values of energy E and work A, we conclude that the charge will not be able to reach the center of the ring.

10.2. (5 points) During a hiking trip the students came to a long ditch filled with water that blocked their further path. In order not to go around the ditch, the students decided to build a crossing. Not far away they found the ruins of an old wooden hut made of logs with a length of $L = 4.0$ m and a rectangular cross-section of $15 \text{ cm} \times 15 \text{ cm}$. The students took one of these logs and placed it across the ditch, parallel to the water surface, so that the log only touched the water with its long face. Soon the bank of the ditch crumbled, the log became free and began to sink.

[2] Find the amount of heat that will be released until the log reaches equilibrium.

Comment. Assume that the density of wood is equal to the density of water $\rho = 1000.0 \text{ m}^3$ and take the acceleration due to gravity to be 9.8 m/s^2 . (Cherenkov A.A.)

Answer: 66 J.

Solution.

Since the density of the log is equal to the density of water, in the final state of equilibrium the log will be completely submerged in water. Initially, after the log is released, it will gain some velocity under the action of gravity, then travel a certain distance underwater by inertia, gradually slowing

down, and eventually come to rest at a certain depth below the water's surface.

Let the center of mass of the log sink a distance h upon immersion. Then the potential energy of the log decreases by

$$\Delta U_1 = mgh,$$

where $m = \rho L a^2$ is the mass of the log. After immersion the log displaced a volume of water in the shape of a rectangular parallelepiped with a mass of m to the surface. We can assume that this water spread out over the surface in a thin layer, that is, its center of mass rose to a height of $(h - a/2)$, and its potential energy increased by

$$\Delta U_2 = mg \left(h - \frac{a}{2} \right).$$

The required amount of heat is equal to the difference between these two quantities:

$$\Delta Q = \Delta U_1 - \Delta U_2 = \frac{mga}{2} = \frac{\rho g L a^3}{2} \approx 66 J$$

10.3. (7 points) A diver with a mass of $m = 90.00$ kg decided to test the capabilities of his body. For this purpose he performed a series of experiments in which he dived into the sea to a depth H without any equipment, taking a full breath of air. It turned out that the maximum depth from which he can float back to the surface without making any movements is 7.00 m.

[3] What is the volume of the diver's lungs if, after a full exhalation, the volume of his body is $V = 85.0$ L?

Comment. Assume that the density of seawater is $\rho = 1020 \text{ kg/m}^3$ and that when air is inhaled the volume of the body increases by a fraction $n = 0.900$ of the volume of inhaled air. Take the acceleration due to gravity to be 9.81 m/s^2 and the atmospheric pressure to be $p_0 = 1.00 \cdot 10^5$ Pa.

(Cherenkov A.A.)

Answer: 6.11 L.

Solution.

1) The athlete's maximum depth of immersion H is determined by the condition that at this depth the buoyant force of Archimedes is equal to the force of gravity. Thus, we have:

$$F_a = mg$$

From which

$$\rho g V_0 = mg$$

where V_0 is the athlete's volume at depth H . Thus

$$V_0 = m/\rho$$

2) When diving to a depth of H , the athlete's volume changes almost exclusively due to isothermal compression of the air in the lungs by an amount of Δv . Let the volume of the athlete's lungs be v . We apply Boyle–Mariotte's law:

$$p_0 v = (p_0 + \rho g H) (v - \Delta v),$$

From this we find:

$$\Delta v = \left(1 - \frac{p_0}{p_0 + \rho g H} \right) v$$

3) The athlete's volume is equal to $V + vn$, when his lungs fully filled with air; then, at depth H :

$$V_0 = V + (v - \Delta v)n$$

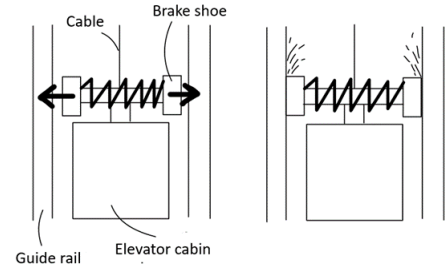
Substituting the previously found values:

$$m/\rho = V + \left(1 - 1 + \frac{p_0}{p_0 + \rho g H}\right) nv$$

Performing algebraic manipulations, we finally obtain the answer:

$$v = \frac{(m - V\rho)(p_0 + \rho g H)}{n\rho p_0} \approx 6.11 L$$

10.4. (8 points) During a physics club session teacher asked Nikolai to understand how wedge-type elevator safety gear works. He only knew that two guide rails are installed in the elevator shaft, and when the safety system is activated, brake shoes are pressed against them. Nikolai assumed that the two brake shoes could be pressed apart by a spring between them acting as a spacer (in reality wedge-type safety gear is constructed differently) and thus stop the elevator cabin if its speed somehow exceeded the permissible value (see figure).



[4] Calculate the spring stiffness required to completely stop the elevator cabin in 1.5 s, if the total mass of the cabin with passengers is 1500 kg, the speed at which the system is activated is 1.2 m/s, the distance from each shoe to the guide rail is 0.5 cm, and the friction coefficient is 0.7.

Comment. The initial compression of the spring is equal to 20 cm. The acceleration due to gravity is $10.0 m/s^2$. (Laskavyi L.S.)

Answer: $k = 60000 N/m$.

Solution.

Let's consider the action of the spring on the brake shoes. The spring, connected to the brake shoes, exerts an elastic force on each of them according to Hooke's law:

$$F_{el} = k(\Delta l_0 - 2\Delta l)$$

This force is what presses the brake shoes against the guides; therefore, by Newton's third law, the reaction force is equal to the elastic force exerted by the spring:

$$N = F_{el} = k(\Delta l_0 - 2\Delta l)$$

Newton's second law:

$$\vec{F}_p = m\vec{a}$$

Expanding the resultant force:

$$m\vec{g} + \vec{F}_{fr} + \vec{F}_{fr} = m\vec{a}$$

In the projection onto the axis pointing downward

$$mg - F_{fr} - F_{fr} = -ma$$

The drag force is determined by Coulomb–Amonton's law:

$$F_{fr} = \mu N$$

Or

$$F_{fr} = \mu k(\Delta l_0 - 2\Delta l)$$

As a result

$$mg - 2\mu k (\Delta l_0 - 2\Delta l) = -ma$$

From which

$$k = \frac{mg + ma}{2\mu (\Delta l_0 - 2\Delta l)}$$

Find the acceleration from the kinematic expression for velocity:

$$\vec{v} = \vec{v}_0 + \vec{a}t$$

In the projection onto the same axis, directed downward:

$$v = v_0 - at$$

Taking into account that $v = 0$, we get

$$a = \frac{v_0}{t}$$

Substituting into the expression for the spring stiffness, we get:

$$k = \frac{mg + m\frac{v_0}{t}}{2\mu (\Delta l_0 - 2\Delta l)}$$

Substituting the numerical values, we end up with:

$$k = \frac{1500 \cdot 10 + 1500 \cdot \frac{1.2}{1.5}}{2 \cdot 0.7 \cdot (0.2 - 2 \cdot 0.005)} = 60902.2556 \text{ N/m}$$

Since the minimum number of significant digits in the problem statement is one, we round the answer so that $k = 60000 \text{ N/m}$.

10.5. (7 points) During a physics lesson at school, the teacher assigned the students a laboratory experiment to study optical systems. The students placed a light bulb at a distance of $L = 100 \text{ cm}$ from the screen, and between them a thin converging lens with a focal length of $F = 30.0 \text{ cm}$ and a diameter of $D = 5.00 \text{ cm}$. Then students had to measure the size of the light spot formed by the bulb on the screen.

[5] What is the minimum possible measured value of the area of the spot?

(Cherenkov A.A.)

Answer: 1.99 cm^2 .

Solution.

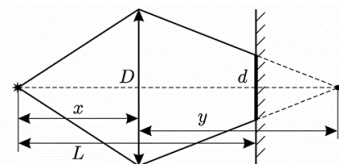


Figure. Path of rays through the lens

Let us denote the distances from the lens to the source as x and from the lens to the image as y ; let the diameter of the lens be D and the diameter of the spot on the screen be d .

Then, by triangle similarity (see Fig.):

$$d = D \cdot \frac{x + y - L}{y} = D \left(\frac{x - L}{y} + 1 \right).$$

Let's find such x that minimizes d . Note that there is no need in considering values of $x \leq F$, since in this case $d \geq D$, and the spot size clearly cannot be minimal. It is clear that d will be minimal when the function

$$f = \frac{x - L}{y}.$$

takes its minimum value. Applying the thin-lens formula, we obtain:

$$\frac{1}{y} = \frac{1}{F} - \frac{1}{x}.$$

Therefore

$$\begin{aligned} f &= \frac{(x - L)(x - F)}{Fx} = \frac{1}{F} \left(x + \frac{LF}{x} - (L + F) \right) \\ &= \frac{1}{F} \left(\left(\sqrt{x} - \frac{\sqrt{LF}}{\sqrt{x}} \right)^2 - (\sqrt{L} - \sqrt{F})^2 \right). \end{aligned}$$

The expression

$$\left(\sqrt{x} - \frac{\sqrt{LF}}{\sqrt{x}} \right)^2 - (\sqrt{L} - \sqrt{F})^2$$

is minimized when the first term is zero:

$$\sqrt{x} = \frac{\sqrt{LF}}{\sqrt{x}}, \quad x_0 = \sqrt{LF}.$$

This value corresponds to the smallest possible spot diameter on the screen:

$$d = D \cdot \frac{2\sqrt{LF} - L}{F}.$$

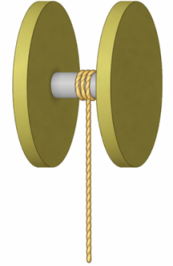
and thus the minimum spot area

$$S = \pi d^2/4 = \pi D^2 \cdot \left(\frac{2\sqrt{LF} - L}{F} \right)^2 /4 \approx 1.99 \text{cm}^2$$



Solutions to problems for grade R11

11.1. (7 points) A yo-yo toy is one of the varieties of pendulums. It consists of two identical flywheels with a radius of $R = 3$ cm, the centers of which are rigidly connected to the ends of a light rod with a radius of $r = 1$ mm. The axes of the flywheels coincide with the axis of the rod. A string with a length of $L = 1$ m is attached to the middle of the rod and is wound around it, while the free end of the string is fixed. If the toy is released, it begins to move and unwinds the string. Upon reaching the lowest point of the trajectory, continuing to rotate by inertia, it begins to wind the string back, performing oscillatory motion.

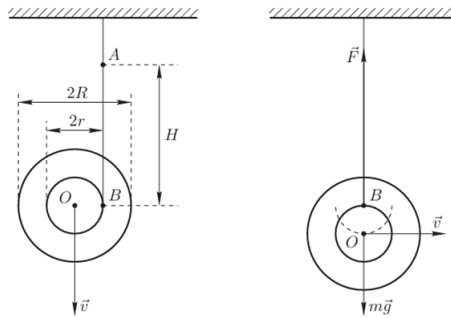


[1] Determine the tension force in the string at the lowest point of the yo-yo trajectory if initially the string was completely wound and the toy was released without initial velocity.

Comment. Assume that flywheel is a cylinder and that its entire mass $m=50$ g is concentrated at the rim. Take the acceleration due to gravity to be $g = 10m/s^2$. (Cherenkov A.A.)

Answer: 3 N.

Solution.



1) It follows from the problem statement that $r \ll R \ll L$. Figure 1 shows the pendulum before and as it passes through the lowest point of its trajectory. By virtue of the condition $r \ll L$, we can neglect the deviation of the string from the vertical and the change in the pendulum's potential energy between the specified states; that is, we will assume that the pendulum's axis O moves at a constant velocity v along a circle of radius r with a center at point B.

2) Let ω be the angular velocity of the pendulum, and u be the constant velocity of the points on the rim of the flywheel relative to its axis; then, from the equation of kinematic connection

$$\frac{v}{r} = \omega = \frac{u}{R}$$

We find that

$$v = u \cdot \frac{r}{R}. \quad (1)$$

From the condition $r \ll R$ it follows that $v \ll u$, so we can neglect the kinetic energy of the pendulum's center of mass's translational motion compared to the kinetic energy of the flywheel's rotational motion relative to the center of mass and write the law of conservation of energy as

$$2mgL = \frac{mu^2}{2} + \frac{mu^2}{2},$$

where the moment of inertia of the flywheel is calculated as $\frac{mR^2}{2}$. Thus,

$$u^2 = 2gL. \quad (2)$$

Using equations (1) and (2) together, we express

$$v^2 = 2gL \cdot \frac{r^2}{R^2}.$$

According to Newton's second law, projected onto the vertical axis, as the pendulum passes through the lower point of the circle, we have

$$2m \cdot \frac{v^2}{r} = F - 2mg,$$

from which, taking into account the expression for velocity, we find the desired force:

$$F = 2mg \left(1 + \frac{2rL}{R^2} \right) \approx 3 N.$$

11.2. (7 points) A children's camp is located near a circular quarry with a width of $L = 30.0$ m, filled with spring water to a depth of $h = 1.5$ m. On one of the banks of the quarry there is a cliff with a height of $H = 10.0$ m.

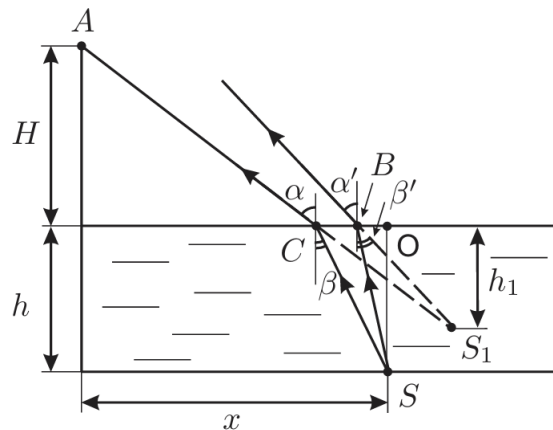
[2] How deep will the middle of the quarry appear if one stands at the very edge of the cliff?

Comment. The refractive index of water is $n = 4/3$.

(Cherenkov A.A.)

Answer: 0.40 m.

Solution.



1) A person standing at point A (see figure) estimates the depth of the water by observing two parallel rays emanating from points B and C. It appears to him that the rays originate not from point S, but from point S_1 , so the perceived depth of the water is h_1 . The change in the directions of the rays at the water's surface follows Snell's law:

$$\frac{\sin \alpha}{\sin \beta} = n, \quad \frac{\sin \alpha'}{\sin \beta'} = n,$$

where n is the refractive index of water, and the angles $\alpha, \beta, \alpha', \beta'$ are shown in the figure

2) Let Δ be the length of the segment BC . Then, from the right-angled triangles shown in the figure, it follows that

$$\Delta = h(\operatorname{tg} \beta - \operatorname{tg} \beta') = h_1(\operatorname{tg} \alpha - \operatorname{tg} \alpha').$$

The angles α and α' , β and β' differ only slightly; therefore, if $\alpha - \alpha' = \Delta\alpha$ and $\beta - \beta' = \Delta\beta$, then

$$\operatorname{tg} \beta - \operatorname{tg} \beta' \approx \frac{\Delta\beta}{\cos^2 \beta}, \quad \operatorname{tg} \alpha - \operatorname{tg} \alpha' \approx \frac{\Delta\alpha}{\cos^2 \alpha}.$$

Substituting the resulting expressions into the formula for Δ , we find:

$$h_1 = h \frac{\cos^2 \alpha}{\cos^2 \beta} \frac{\Delta\beta}{\Delta\alpha}.$$

Let's write down Snell's law for small angle increments:

$$\cos \alpha \Delta \alpha = n \cos \beta \Delta \beta,$$

wherefrom

$$h_1 = \frac{h \cos^3 \alpha}{n \cos^3 \beta} = \frac{h}{n} \frac{\cos^3 \alpha}{\left(1 - \frac{\sin^2 \alpha}{n^2}\right)^{3/2}}.$$

3) For the angle α shown in the figure, we find:

$$\sin \alpha = \frac{x - OB}{\sqrt{(x - OB)^2 + H^2}} \approx \frac{x}{\sqrt{x^2 + H^2}},$$

since $OB \ll x$ when h is small. We now substitute the expression for $\sin \alpha$ into the formula and perform the algebraic manipulations:

$$h_1 = \frac{n^2 h H^3}{((n^2 - 1)x^2 + n^2 H^2)^{3/2}}.$$

It follows from the problem statement that $x = L/2$, so the final answer:

$$h_1 \approx 0.40m$$

11.3. (6 points) In an experimental setup a long horizontal cylindrical tube with cross-sectional area $S = 100 \text{ cm}^2$ is used. Inside the tube there are two pistons with masses of $M_1 = 2 \text{ kg}$ and $M_2 = 3 \text{ kg}$, which can move along the axis of the tube almost without friction. Between the pistons there is one mole of oxygen O_2 , which can be treated as an ideal gas. The setup is placed in a chamber where the external pressure is maintained at half an atmosphere $p_0 = 0.5 p_{atm}$. Constant forces $F_1 = 600 \text{ N}$ and $F_2 = 500 \text{ N}$ are applied to the pistons from the outside along the axis of the tube toward each other. The gas is in thermal contact with a thermostat, so its temperature remains constant and equal to $T = 300 \text{ K}$.

[3] Determine the equilibrium distance between the pistons after the forces have been applied.

Comment. Take the atmospheric pressure to be $p_{atm} = 1 * 10^5 \text{ Pa}$. (*Cherenkov A.A.*)

Answer: 2 m.

Solution.

In steady state, the whole system will move with an acceleration determined by the Newton's second law

$$(M_1 + M_2 + m) a = F_1 - F_2.$$

where m is the mass of oxygen. Since the mass of the gas is much smaller than the mass of the pistons, we can assume that

$$a = \frac{F_1 - F_2}{M_1 + M_2},$$

and the pressure p of the gas is constant everywhere and is determined by the condition following from the Newton's second law

$$M_1 a = -pS + p_0 S + F_1.$$

wherefrom

$$p = \frac{M_2 F_1 + M_1 F_2}{(M_1 + M_2) S} + p_0 = \frac{M_2 F_1 + M_1 F_2 + p_0 S (M_1 + M_2)}{(M_1 + M_2) S}.$$

The volume of the gas is equal to

$$V = \frac{\nu RT}{p} = \frac{\nu RTS (M_1 + M_2)}{M_2 F_1 + M_1 F_2 + p_0 S (M_1 + M_2)},$$

where $\nu = 1$ mol, and the distance between the pistons in the steady-state is

$$x = \frac{V}{S} = \frac{\nu RT (M_1 + M_2)}{M_2 F_1 + M_1 F_2 + p_0 S (M_1 + M_2)} \approx 2 \text{ m}.$$

11.4. (8 points) In a museum, a rare manuscript had to be protected from bright illumination. For this purpose five identical thin glass protective screens were installed in front of the display case, arranged parallel to each other. Light from the lamps is incident on the surfaces of the glass plates almost normally. It is known that for a single such glass plate at normal incidence 8% of the incident energy is reflected, while 92% is transmitted through the plate; that is, the reflection and transmission coefficients are $R=0.08$ and $T=0.92$, respectively.

[4] Find the transmission coefficient of the resulting protective system.

(Cherenkov A.A.)

Answer: 0.70.

Solution.

The main difficulty in solving this problem arises from the fact that light is reflected multiple times off the surfaces of the plates within the stack. Therefore, to find the answer, we need to derive a recursive formula showing how the transmittance of the stack of n plates changes when another plate is added to it.

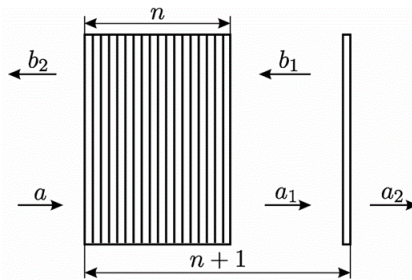


Figure. A stack of glass plates

Let a stack of n plates have a transmission coefficient T_n . Let us add one more plate to it, thus obtaining a "composite" stack of $n + 1$ plates. Let a wave with intensity a strike the resulting stack. Let us denote the intensities of the waves propagating outside and inside our "composite" stack as shown in the figure. Given that $R_n = 1 - T_n$, we obtain the following system of equations:

$$a_2 = T a_1,$$

$$b_1 = (1 - T) a_1,$$

$$a_1 = T_n a + (1 - T_n) b_1,$$

$$b_2 = (1 - T_n) a + T_n b_1.$$

Solving it, we find:

$$\frac{a}{a_2} = \frac{1}{T_{n+1}} = \frac{1}{T_n} + \frac{1 - T}{T}.$$

This is the desired recursive formula. From it, we can derive an explicit relationship between the transmission coefficient T_n and the number of plates n in the stack. Indeed, if there are only two plates in the stack ($n + 1 = 2$), then

$$\frac{1}{T_2} = \frac{1}{T} + \frac{1-T}{T}.$$

Next, for $n + 1 = 3$:

$$\frac{1}{T_3} = \frac{1}{T_2} + \frac{1-T}{T} = \frac{1}{T} + 2\frac{1-T}{T}.$$

Continuing similar calculations, for a stack of n plates, we obtain:

$$\frac{1}{T_n} = \frac{1}{T} + (n-1)\frac{1-T}{T} = \frac{T + n(1-T)}{T},$$

From which

$$T_n = \frac{T}{T + n(1-T)} = \frac{T}{T + nR}.$$

For $T = 0.92$, $n = 5$:

$$T_5 = \frac{0.92}{0.92 + 0.08 \cdot 5} \approx 0.70.$$

11.5. (8 points) After a laboratory class at school, Masha stayed behind to clean the classroom. On the desks there were measuring cylinders filled with a liquid of density $\rho_0 = 0.80 \text{ g/cm}^3$, and graduation marks were drawn on the walls of the cylinders. The girl noticed that a spherical air bubble of diameter $d_1 = 3.0 \text{ mm}$ rises in the liquid, passing one graduation mark every second, while a bubble of three times larger diameter passes three marks during the same time. When Masha dropped a spherical metal pellet with a density of $\rho_3 = 4.0 \text{ g/cm}^3$ and a diameter of $d_3 = 9.0 \text{ mm}$ into the liquid, it sank to the bottom, passing three marks every half second. Then she took a ping-pong ball with a diameter of $d_4 = 3.0 \text{ cm}$ and a density of $\rho_4 = 0.08 \text{ g/cm}^3$ and placed it at the bottom of the measuring cylinder.

[5] With what velocity will the ball rise if the graduation marks on the cylinder are spaced by $l = 5.0 \text{ cm}$?

Comment. Assume that the viscous drag force of the liquid depends on the velocity and the diameter of the ball according to a power law that is the same for all bodies. (*Cherenkov A.A.*)

Answer: 0.5 m/s.

Solution.

1) First, note that all bodies in the problem travel equal distances in equal intervals of time, which means they move uniformly. Then the speeds of the first and second bubbles and the pellet's speed, are respectively equal to:

$$v_1 = l/t \quad v_2 = 3v_1 \quad v_3 = 2v_2$$

where t equals 1 second.

2) As the ball moves through the liquid, it is acted upon by the force of gravity F_T directed downward, the buoyant force F_A directed upward and the viscous drag force F_{fr} , which, as follows from the problem's conditions, depends on the velocity and the diameter of the ball. The first two forces are volumetric. This means that their difference is proportional to the difference $\rho - \rho_0$ (ρ is the density of the ball) and the volume of the ball, that is, the cube of its diameter d :

$$|F_T - F_A| = A |\rho - \rho_0| d^3.$$

Here, A is the proportionality constant. Suppose that the viscous drag force depends on the ball's diameter d and its velocity v as follows:

$$F_{fr} = Bd^n v^m,$$

where B is the proportionality coefficient, n and m are unknown exponents. When moving at a constant speed, the difference between the gravitational force and the Archimedes' force is equal in magnitude to the viscous drag force. Then, for a bubble with a diameter of d_1 , we have:

$$A\rho_0d_1^3 = Bd_1^m v_1^m,$$

from which

$$v_1 = \sqrt[m]{\frac{A}{B}\rho_0d_1^{3-n}}.$$

Similarly, for a bubble with a diameter three times larger, we obtain

$$v_2 = \sqrt[m]{\frac{A}{B}\rho_0(3d_1)^{3-n}} = 3v_1 = 3\sqrt[m]{\frac{A}{B}\rho_0d_1^{3-n}},$$

from which the following equation follows

$$3^{(3-n)/m} = 3^1 \quad \text{or} \quad 3 - n = m.$$

Next, for a pellet with a diameter of $d_3 = 3d_1$, considering that $v_3 = 2v_2$ and $\rho_3 = 5\rho_0$, we obtain:

$$v_3 = \sqrt[m]{\frac{A}{B}(\rho_3 - \rho_0)d_3^{3-n}} = \sqrt[m]{\frac{A}{B} \cdot 4\rho_0d_3^{3-n}} = 2\sqrt[m]{\frac{A}{B}\rho_0(3d_1)^{3-n}},$$

From this, we find that $m = 2$. Then it follows that $n = 1$. Therefore, the relationship for $F_{fr}(d,v)$ takes the following form:

$$F_{fr} = Bdv^2.$$

Now we can calculate the speed u at which the ping-pong ball will pop up. Given that its density is $\rho_4 = \frac{1}{10}\rho_0$ and its diameter is $d_4 = 10d_1$, we get:

$$u = \sqrt{\frac{A}{B}(\rho_0 - \rho_4)d_4^2} = \sqrt{\frac{A}{B}\left(\rho_0 - \frac{1}{10}\rho_0\right)(10d_1)^2} = \sqrt{90\frac{A}{B}\rho_0d_1^2} = \sqrt{90}v_1 = \sqrt{90}\frac{l}{t} \approx 47.4 \text{ cm/s}.$$