Solutions to problems for grade R8

8.1. (5 points) Dmitry went with his father to a city amusement park, where he saw a shooting gallery arranged in an original way. In particular, there was a lead ball suspended by a cord, the two parts of which were connected by a target. When a shooter hits the target, it is destroyed and the ball falls down. The father posed two questions to Dmitry:

[1] What is the speed of the ball at the end of its fall, if the height at which the ball is suspended is 5 m? Take the free-fall acceleration to be 9.8 N/kg.

[2] By how many degrees will the ball heat up if half of all the mechanical energy of the ball goes into heating? The specific heat capacity of lead is 140 $J/(kg \cdot K)$. Assume the cord is weightless and inextensible, the target's destruction is instantaneous, and air resistance is negligible.

(*Laskavyy L.S.*)

Answer: 10.
Answer: 0.2.
Solution. When the target breaks, the sphere starts falling with zero initial speed. Since we neglect mechanical energy losses due to the absence of air resistance, we use the law of conservation of mechanical energy:

$$E_{k1} + E_{p1} = E_{k2} + E_{p2} = const,$$

where the kinetic energy is

$$E_k = \frac{mv^2}{2}/2,$$

and the potential energy is

$$E_p = mgh.$$

Let state 1 be the moment when the target breaks, and state 2 be the end of the fall. Since at the beginning of the fall the speed is zero, we have $E_{k1} = 0$. Taking the surface on which the sphere lands as the zero level of height, we obtain that at the end of the fall the height is zero, hence $E_{p2} = 0$. As a result, the conservation law takes the form

$$E_{p1} = E_{k2}.$$

Substituting the expressions for the kinetic and potential energies, we obtain:

$$mgh = \frac{mv^2}{2}/2 \quad \Rightarrow \quad 2gh = \frac{v^2}{2}.$$

Thus:

$$v = \sqrt{2gh} = \sqrt{2 \cdot 9.8 \cdot 5} = \sqrt{98} \approx 9.89 \ m/s.$$

Taking into account the minimal number of significant digits in the statement of the problem, which is one, we round and obtain $v = 10 \ m/s$.
To find the temperature increase of the sphere after its fall, note that half of its mechanical energy goes into heating:

$$\eta = \frac{Q}{E_{mech}}.$$

Since there are no mechanical energy losses, we can take the energy in the initial state:

$$E_{mech} = E_p = mgh.$$

The amount of heat gained by a body upon heating is

$$Q = cm\Delta T.$$

Substituting these expressions into the efficiency formula, we obtain:

$$\eta = \frac{cm\Delta T}{mgh}.$$

Thus:

$$\Delta T = \frac{\eta gh}{c} = \frac{0.5 \cdot 9.8 \cdot 5}{140} = 0.175\ K.$$

Taking into account the minimal number of significant digits in the problem statement, which is one, we round and obtain $\Delta T = 0.2\ K$.
Answer: $v = 10\ m/s$, $\Delta T = 0.2\ K$.

8.2. (6 points) Aleksey went on a winter hiking trip to the Altai with his school team. During a break Aleksey was tasked with preparing tea by melting ice. Aleksey used a primus stove and a stainless steel cup of mass 200.0 g, placing 1.5 kg of ice into it. The process was not monitored, and 150.0 g of water boiled away. The stove's efficiency is 45.0%. As a result, the process consumed 93.7 ml of AI-98 gasoline, which density is 780.0 kg/m^3 and specific combustion heat is $4.50 \cdot 10^7$ J/kg.
[3] Find the initial temperature of the ice. Give the answer in degrees Celsius.

Comment. The specific heat capacities of water, ice, and steel are 4200.0 $J/(kg{\cdot}K)$, 2100.0 $J/(kg{\cdot}K)$, and 500.0 $J/(kg \cdot K)$ respectively. The latent heat of fusion of ice is $3.30 \cdot 10^5$ J/kg, and the latent heat of vaporization is $2.30 \cdot 10^6$ J/kg. (*Laskavyy L.S.*)
Answer: -16.
Solution. The primus stove converts the internal energy of the fuel into the amount of heat received by the bodies in the thermal processes. The efficiency is given by

$$\eta = \frac{Q_u}{Q_c},$$

where Q_u is the useful energy and Q_c is the consumed energy.
Let us expand each term. The consumed energy is

$$Q_c = Q_b = qm_f.$$

Taking into account that

$$m_f = \rho_f V,$$

we obtain:

$$Q_c = Q_b = q\rho_f V.$$

The useful energy consists of the amount of heat for heating the ice (Q_1), melting the ice (Q_2), heating the water to the boiling point (Q_3), boiling part of the water (Q_4), and heating the steel cup from the initial temperature to 100 degrees (Q_5), that is:

$$Q_u = Q_1 + Q_2 + Q_3 + Q_4 + Q_5.$$

Let us expand each term:

$$Q_1 = c_i m (T_m - T_0),$$
$$Q_2 = \lambda m,$$
$$Q_3 = c_w m (T_b - T_m),$$
$$Q_4 = L m_v,$$
$$Q_5 = c_s m_s (T_b - T_0).$$

Substituting these expressions into the formula for the useful energy, we obtain:

$$Q_u = c_i m (T_m - T_0) + \lambda m + c_w m (T_b - T_m) + L m_v + c_s m_s (T_b - T_0).$$

Then the efficiency formula becomes:

$$\eta = \frac{c_i m (T_m - T_0) + \lambda m + c_w m (T_b - T_m) + L m_v + c_s m_s (T_b - T_0)}{q \rho_f V}.$$

Solving for the initial temperature, we obtain:

$$T_0 = \frac{c_i m T_m + \lambda m + c_w m (T_b - T_m) + L m_v + c_s m_s T_b - \eta q \rho_f V}{c_i m + c_s m_s}.$$

Substituting the numerical values, we obtain:

$$T_0 \approx 256.9646\ K.$$

Taking into account the minimal number of significant digits in the problem statement, which is two, we round and obtain $T_0 = 257\ K$.
Converting this value to degrees Celsius, we obtain:

$$t_0 = T_0 - 273 \approx -16\ °C.$$

Answer: $t_0 = -16\ °C$.

8.3. (7 points) Two classes of schoolchildren went on a excursion to the Hermitage by buses shaped as rectangular parallelepipeds, differing only in length by a factor of two. As usual, it was raining in St. Petersburg that day. On the way to the museum, $1.0 \cdot 10^5$ and $1.8 \cdot 10^5$ raindrops fell on the buses, which were traveling at equal speeds.

[4] How many raindrops fell on the long bus on the return trip, if it was moving at half the speed as on the way to the museum?

Comment. Assume that the wind does not affect or deflect the raindrops, and that the rainfall intensity is constant. (*Cherenkov A.A.*)

Answer: 340000.

Solution. 1) Short bus on the way to the museum.
The number of raindrops hitting the bus is proportional to the travel time t, concentration n of droplets in the air, their velocity v_r relative to the bus, and the effective area S exposed to rain:

$$N = n t v_r S. \qquad (1)$$

Let L be the distance from school to museum, v — the speed of the bus, u — speed of the droplets. Then

$$t = \frac{L}{v}. \qquad (2)$$

The effective area S is the projection of the bus onto the plane perpendicular to the droplet velocity. Let d,b,h be length, width and height of the bus respectively, α — angle of inclination of the droplet flow in the reference system associated with the bus, then the total area of the projections from the roof and front surface of the bus is given by

$$S = b\left(d\cos\alpha + h\sin\alpha\right). \qquad (3)$$

Substitute (2) and (3) in (1), using the expressions $\cos\alpha = \frac{u}{v_r}$ and $\sin\alpha = \frac{v}{v_r}$:

$$N = Lnb\left(\frac{du}{v} + h\right). \qquad (4)$$

2) Long bus on the way to the museum.
Reasoning similarly, or simply replacing N with 1.8N and d with 2d in formula (4), we obtain

$$1{,}8N = Lnb\left(\frac{2du}{v} + h\right). \qquad (5)$$

From (4) and (5) we get:

$$\frac{2du}{v} : h = 8 : 1.$$

3) Long bus on the way back to school
If v is replaced with v/2 in formula (5), then the first term, which is 8/9 of the sum, will double, so the number of raindrops will be:

$$n = 17/9 \cdot 1{,}8N = 3.4 \cdot 10^5$$

Answer: $3.4 \cdot 10^5$

8.4. (6 points) Masha went on an excursion with her class to the city of Gus-Khrustalny, where they were shown the process of manufacturing glass products. In one of the workshops Masha noticed that there was a fairly large air bubble in the form of a sphere in a glass block that had been sent for processing. When the block was weighed on a dynamometer, its weight was $3.0 \cdot 10^3$ N, and when submerged in water it was 1300 N.
[5] Determine the volume of the bubble in the block.

Comment. The mass of the air in the bubble can be considered negligibly small, and the effect of air during weighing can be ignored. Take the density of glass to be $2.5 \cdot 10^3$ kg/m^3. (*Laskavyy L.S.*)
Answer: 0.05.
Solution. When the block is weighed in air, its weight equals the gravity force, from which we can find the mass of the block :

$$P_1 = mg \quad \Rightarrow \quad m = \frac{P_1}{g}.$$

Taking into account that

$$m = \rho_{gl} V_m,$$

the volume occupied by the material (that is, the glass) is

$$V_m = \frac{m}{\rho_{gl}} = \frac{P_1}{\rho_{gl} g}.$$

When the block is weighed while immersed in water, the gravity force is balanced by the elastic force from the dynamometer (which is equal to the indicated weight) and the Archimedes' force from the liquid, that is:

$$mg = P_2 + F_A.$$

The total volume of the block is the sum of the volume of the material (glass) and the volume of the bubble:

$$V = V_m + V_b.$$

Substituting the expression for the Archimedes' force, we obtain:

$$mg = P_2 + \rho_w g\left(V_m + V_b\right).$$

Solving for the bubble volume, we obtain:

$$V_b = \frac{mg - P_2}{\rho_w g} - V_m.$$

Replacing the volume of the material and the mass by their expressions, we finally get:

$$V_b = \frac{P_1 - P_2}{\rho_w g} - \frac{P_1}{\rho_{gl} g}.$$

Substituting the numerical values:

$$V_b = \frac{3000 - 1300}{1000 \cdot 10} - \frac{3000}{2500 \cdot 10} = 0.05\ m^3.$$

Taking into account the minimal number of significant digits in the problem statement, which is one, the answer remains unchanged.
Answer: $V_b = 0.05\ m^3$.

8.5. (6 points) One day Sasha caught a cold and did not want to tell his parents about it. He decided to cure himself and found a folk method for brewing medicinal tea on the internet. One needs to take 5.0 g of green tea with chamomile, pour it into a pot and sequentially add 50.0 ml portions of water. Each subsequent portion is 1.0^o C hotter than the previous one, and the first portion is taken at a temperature of 20.0^o C.

[6] What was the temperature of the resulting medicinal tea, if Sasha used a rectangular parallelepiped-shaped pot with dimensions of 13.0 cm × 15.0 cm × 9.00 cm, which initial temperature was 10.0^o C and its heat capacity is 540 J/K?

Comment. Assume the density of water is $1.0 \cdot 10^3\ kg/m^3$ and its specific heat capacity is c = 4200 $J/(kg \cdot K)$. (*Cherenkov A.A.*)

Answer: 310.

Solution. 1)Since the volume of the pot is $V = 15 \cdot 13 \cdot 9 = 1755$ ml, then it follows that Sasha added $n = [V/\Delta V] = [1755/50] = 35$ portions of water and the remaining space in pot he filled with 5 g of tea. Compute the average temperature of the poured water:

$$t_{cp} = \frac{t_1 + t_n}{2} = \frac{t_1 + (t_1 + (n-1))}{2} = 37°C = 310°K$$

2)According to the heat balance equation:

$$Ct_0 + cn\rho\Delta V t_{cp} = Q = Ct + cn\rho\Delta V t$$

where t_o is the initial temperature of the pot, C is its heat capacity, and t is the final temperature. Solving for t gives:

$$t = \frac{Ct_0 + cn\rho\Delta V t_{cp}}{C + cn\rho\Delta V} \approx 308K$$

Taking into account the minimal number of significant digits in the problem statement, which is two, we obtain answer 310 K.
Answer: 310 K

8.6. (6 points) In the school chemistry lab, Nadya was tasked with determining the density of an unknown gas in a sealed laboratory flask. Nadya weighed the flask and found that the weight of the flask with the gas is 58.3 N. The flask is spherical, with a constant wall thickness of 0.20 cm and an external diameter of 60.0 cm.

[7] Determine the density of the unknown gas.

Comment. The density of the glass is 2500 kg/m^3. Assume the density of air is 1.224 kg/m^3.

(Laskavyy L.S.)

Answer: 3.2.

Solution. When the flask is weighed, the forces acting on it are the elastic force (equal to the weight indicated by the scales), the gravity force acting on the glass and on the gas in the flask, and the Archimedes' force acting on the flask from the air. The equilibrium of these forces is expressed as

$$P + F_A = m_s g + m_g g.$$

The Archimedes' force is

$$F_A = \rho_a g V_{out} = \rho_a g \cdot \frac{4}{3}\pi R^3.$$

The inner radius of the flask is

$$r = R - d.$$

The mass of the glass is

$$m_s = \rho_{gl} V_{gl} = \rho_{gl}\left(V_{out} - V_{in}\right) = \rho_{gl} \cdot \frac{4}{3}\pi\left(R^3 - r^3\right).$$

The mass of the gas is

$$m_g = \rho_g V_{in} = \rho_g \cdot \frac{4}{3}\pi r^3.$$

Substituting all this into the force balance equation, we obtain:

$$P + \rho_a g \cdot \frac{4}{3}\pi R^3 = \rho_{gl} \cdot \frac{4}{3}\pi\left(R^3 - (R-d)^3\right) g + \rho_g \cdot \frac{4}{3}\pi (R-d)^3 g.$$

Solving for the gas density, we finally obtain:

$$\rho_g = \frac{3P}{4\pi (R-d)^3 g} + \frac{\rho_a R^3}{(R-d)^3} - \rho_{gl}\frac{R^3 - (R-d)^3}{(R-d)^3}.$$

Substituting the values (converting centimeters to meters: $R = 0.15\ m$, $d = 0.002\ m$):

$$\rho_g = \frac{3 \cdot 58.3}{4 \cdot 3.14 \cdot (0.3 - 0.002)^3 \cdot 10} + \frac{1.224 \cdot 0.3^3}{(0.3-0.002)^3} - 2500 \cdot \frac{0.3^3 - (0.3-0.002)^3}{(0.3-0.002)^3} \approx 3.195\ kg/m^3.$$

Taking into account the minimal number of significant digits in the problem statement, which is two, we round and obtain $\rho_g = 3.2\ kg/m^3$.

Answer: $\rho_g = 3.2\ kg/m^3$.

8.7. (7 points) In the school laboratory, Natalya is conducting an experiment to study the processes of heating and cooling of bodies. She places a heated steel cylinder of radius 5.0 cm and height of 5.0 cm with its base on a thick layer of ice at the melting temperature. As a result, a cylindrical pit of radius 5.5 cm is formed in the ice. The temperature of the cylinder is 26.2^o C.

[8] Calculate how much water spills out from the formed pit.

Comment. Assume that no energy is lost to the atmosphere and all the energy of the cylinder goes into heating the ice. The specific heat capacity of steel is 0.50 $kJ/(kg \cdot K)$, the latent heat of fusion of ice is $3.3 \cdot 10^5$ J/kg, the densities of steel, ice and water are 7800 kg/m^3, $0.90 \cdot 10^3$ kg/m^3 and $1.0 \cdot 10^3$ kg/m^3, respectively. (*Laskavyy L.S.*)

Answer: 0.098.

Solution. We neglect energy losses to the atmosphere. Let us write the heat balance equation:

$$Q_1 + \ldots + Q_n = 0.$$

The melting of ice is provided by the amount of heat given by the cylinder, so that:

$$Q_1 = \lambda m_i,$$

$$Q_2 = c_s m_s (t_m - t_s).$$

Thus, the heat balance equation has the form:

$$\lambda m_i + c_s m_s (t_m - t_s) = 0.$$

Let us express the mass of the cylinder. Its volume is

$$V = Sh = \pi r^2 h.$$

Then its mass is

$$m_s = \rho_s \pi r^2 h.$$

Hence the heat balance equation becomes:

$$\lambda m_i + c_s \rho_s \pi r^2 h (t_m - t_s) = 0.$$

Solving for the mass of ice (which is equal to the mass of water), we obtain:

$$m_i = -\frac{c_s \rho_s \pi r^2 h (t_m - t_s)}{\lambda}.$$

Now we determine the depth of the cavity. To do this, we find the volume of the melted ice and, using the volume of the cylinder, find its height. The pit volume is

$$V_{pit} = \pi R^2 H.$$

On the other hand,

$$m_i = \rho_i V_{pit}, \quad V_{pit} = \frac{m_i}{\rho_i}.$$

Thus, the depth of the pit

$$H = \frac{m_i}{\rho_i \pi R^2}.$$

Substituting the expression for the mass of ice, we obtain:

$$H = -\frac{c_s \rho_s r^2 h (t_m - t_s)}{\lambda \rho_i R^2}.$$

Computing the depth of the cavity:

$$H = -\frac{500 \cdot 7800 \cdot 0.05^2 \cdot 0.05 \cdot (0 - 26.2)}{3.3 \cdot 10^5 \cdot 900 \cdot 0.055^2} \approx 0.01422\ m.$$

We see that the depth of the pit is smaller than the height of the cylinder.
The volume of a part of the cylinder which is in a pit is a cylinder in radius r and height H:

$$V_{cyl\_in\_pit} = \pi r^2 H$$

Volume in the pit with which water can occupy:

$$V_{water\_in\_pit} = V_{pit} - -V_{cyl\_in\_pit} = \pi H(R^2 - r^2)$$

We find volume of the water which have turned out from ice

$$V_{water} = m_i/\rho_w$$

The difference between it and volume of the remained water is a volume of the spilled out water.

$$V_{spill} = V_{water} - -V_{water\_in\_pit} = V_{water} - \pi H(R^2 - r^2)$$

Then

$$M_{spill} = \rho_w V_{spill}$$

We substitute expressions for volume and obtaine

$$m_{spill} = \rho_w \left(\frac{m_i}{\rho_w} - \frac{\pi m_i \left(R^2 - r^2\right)}{\rho_i \pi R^2} \right) = m_i \left(1 - \frac{\rho_w}{\rho_i} \frac{\left(R^2 - r^2\right)}{R^2} \right)$$
$$= -\frac{\rho_s c_s \pi r^2 h \left(t_m - t_s\right)}{\lambda} \left(1 - \frac{\rho_w}{\rho_i} \left(1 - \left(\frac{r}{R} \right)^2 \right) \right)$$

$$m_{spill} = -\frac{7800 \cdot 500 \cdot \pi \cdot 0.05^2 \cdot 0.05 \left(0 - 26.2\right)}{3.3 \cdot 10^5} \left(1 - \frac{1000}{900} \left(1 - \left(\frac{0.05}{0.055} \right)^2 \right) \right) \approx 0.09815$$

Taking into account the minimal number of significant digits in the problem statement, which is two, we round and obtain $m_{spill} = 0.098\ kg$.
Answer: $m_{spill} = 0.098\ kg$

Solutions to problems for grade R9

9.1. (5 points) Masha went on an excursion with her class to the city of Gus-Khrustalny, where they were shown the process of manufacturing glass products. In one of the workshops Masha noticed that there was a fairly large air bubble in the form of a sphere in a glass block that had been sent for processing. When the block was weighed on a dynamometer, its weight was $3.0 \cdot 10^3$ N, and when submerged in water it was 1300 N.

[1] Determine the volume of the bubble in the block.

Comment. The mass of the air in the bubble can be considered negligibly small, and the effect of air during weighing can be ignored. Take the density of glass to be $2.5 \cdot 10^3$ kg/m^3. (*Laskavyy L.S.*)
Answer: 0.05.
Solution. When the block is weighed in air, its weight equals the gravity force, from which we can find the mass of the block :

$$P_1 = mg \quad \Rightarrow \quad m = \frac{P_1}{g}.$$

Taking into account that

$$m = \rho_{gl} V_m,$$

the volume occupied by the material (that is, the glass) is

$$V_m = \frac{m}{\rho_{gl}} = \frac{P_1}{\rho_{gl} g}.$$

When the block is weighed while immersed in water, the gravity force is balanced by the elastic force from the dynamometer (which is equal to the indicated weight) and the Archimedes' force from the liquid, that is:

$$mg = P_2 + F_A.$$

The total volume of the block is the sum of the volume of the material (glass) and the volume of the bubble:

$$V = V_m + V_b.$$

Substituting the expression for the Archimedes' force, we obtain:

$$mg = P_2 + \rho_w g \left(V_m + V_b\right).$$

Solving for the bubble volume, we obtain:

$$V_b = \frac{mg - P_2}{\rho_w g} - V_m.$$

Replacing the volume of the material and the mass by their expressions, we finally get:

$$V_b = \frac{P_1 - P_2}{\rho_w g} - \frac{P_1}{\rho_{gl} g}.$$

Substituting the numerical values:

$$V_b = \frac{3000 - 1300}{1000 \cdot 10} - \frac{3000}{2500 \cdot 10} = 0.05\ m^3.$$

Taking into account the minimal number of significant digits in the problem statement, which is one, the answer remains unchanged.
Answer: $V_b = 0.05\ m^3$.

9.2. (7 points) In the school physics club during winter, students were studying reactive motion. The teacher decided to demonstrate the physical principles in action. He took a barrel with a mass of M = 5 kg and base area of S = 2 m^2, placed it on an ice-covered track, and filled it with water up to a level of H = 1 m. In the barrel, near the base, there was a hole closed with a cork, with a cross-sectional area of 15 cm^2. Immediately after the cork was removed, under the pressure of the water, the barrel started moving with acceleration of a = 1 mm/s^2.
[2] Estimate the coefficient of friction between the barrel and the surface of the track.

Comment. Assume the density of water is 1000 kg/m^3. (*Cherenkov A.A.*)
Answer: 0.001.
Solution. 1) The acceleration a of the barrel occurs due to the difference between the reactive thrust force F_t of the water jet flowing out of the hole and the sliding friction force F_f of the barrel against the track surface.
2) In a short period of time Δt water with a mass of $\Delta m = \rho\sigma v\Delta t$. will pour out through the hole. Neglecting energy losses due to viscosity and changes in the kinetic energy of the water remaining in the barrel, we write the law of conservation of energy of the system:

$$\Delta mgH = \frac{\Delta m}{2}v^2,$$

hence

$$v^2 = 2gH.$$

Using the law of momentum change of the outflowing water $\Delta mv = F\Delta t$ and substituting the expression for Δm, we find the pressure force $F = \rho\sigma v^2 = 2\rho gH\sigma$, which, according to Newtonś third law, is equal to the desired reactive thrust force, i.e. $F_t = F = 2\rho gH\sigma$.
3)Immediately after the cork pops out, the mass of water in the vessel is $m = \rho HS$, and the friction force between the barrel and the table is $F_f = \mu(M+m)g$. From Newtonś second law

$$(M+m)a = F_t - F_f$$

we get

$$\mu = \frac{2\rho H\sigma}{M+\rho HS} - \frac{a}{g} \approx 0.001.$$

Answer: 0.001

9.3. (6 points) In the school laboratory, Natalya is conducting an experiment to study the processes of heating and cooling of bodies. She places a heated steel cylinder of radius 5.0 cm and height of 5.0 cm with its base on a thick layer of ice at the melting temperature. As a result, a cylindrical pit of radius 5.5 cm is formed in the ice. The temperature of the cylinder is 26.2^o C.
[3] Calculate how much water spills out from the formed pit.

Comment. Assume that no energy is lost to the atmosphere and all the energy of the cylinder goes into heating the ice. The specific heat capacity of steel is 0.50 $kJ/(kg \cdot K)$, the latent heat of fusion of ice is $3.3 \cdot 10^5$ J/kg, the densities of steel, ice and water are 7800 kg/m^3, $0.90 \cdot 10^3\ kg/m^3$ and $1.0 \cdot 10^3\ kg/m^3$, respectively. (*Laskavyy L.S.*)
Answer: 0.098.
Solution. We neglect energy losses to the atmosphere. Let us write the heat balance equation:

$$Q_1 + \ldots + Q_n = 0.$$

The melting of ice is provided by the amount of heat given by the cylinder, so that:

$$Q_1 = \lambda m_i,$$

$$Q_2 = c_s m_s \left(t_m - t_s\right).$$

Thus, the heat balance equation has the form:

$$\lambda m_i + c_s m_s \left(t_m - t_s\right) = 0.$$

Let us express the mass of the cylinder. Its volume is

$$V = Sh = \pi r^2 h.$$

Then its mass is

$$m_s = \rho_s \pi r^2 h.$$

Hence the heat balance equation becomes:

$$\lambda m_i + c_s \rho_s \pi r^2 h \left(t_m - t_s\right) = 0.$$

Solving for the mass of ice (which is equal to the mass of water), we obtain:

$$m_i = -\frac{c_s \rho_s \pi r^2 h \left(t_m - t_s\right)}{\lambda}.$$

Now we determine the depth of the cavity. To do this, we find the volume of the melted ice and, using the volume of the cylinder, find its height. The pit volume is

$$V_{pit} = \pi R^2 H.$$

On the other hand,

$$m_i = \rho_i V_{pit}, \quad V_{pit} = \frac{m_i}{\rho_i}.$$

Thus, the depth of the pit

$$H = \frac{m_i}{\rho_i \pi R^2}.$$

Substituting the expression for the mass of ice, we obtain:

$$H = -\frac{c_s \rho_s r^2 h \left(t_m - t_s\right)}{\lambda \rho_i R^2}.$$

Computing the depth of the cavity:

$$H = -\frac{500 \cdot 7800 \cdot 0.05^2 \cdot 0.05 \cdot (0 - 26.2)}{3.3 \cdot 10^5 \cdot 900 \cdot 0.055^2} \approx 0.01422\ m.$$

We see that the depth of the pit is smaller than the height of the cylinder.
The volume of a part of the cylinder which is in a pit is a cylinder in radius r and height H:

$$V_{cyl\_in\_pit} = \pi r^2 H$$

Volume in the pit with which water can occupy:

$$V_{water\_in\_pit} = V_{pit} - -V_{cyl\_in\_pit} = \pi H (R^2 - r^2)$$

We find volume of the water which have turned out from ice

$$V_{water} = m_i/\rho_w$$

The difference between it and volume of the remained water is a volume of the spilled out water.

$$V_{spill} = V_{water} - -V_{water\_in\_pit} = V_{water} - \pi H(R^2 - r^2)$$

Then

$$M_{spill} = \rho_w V_{spill}$$

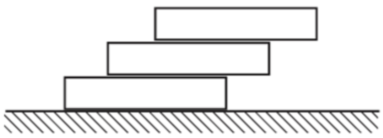
We substitute expressions for volume and obtaine

$$m_{spill} = \rho_w \left(\frac{m_i}{\rho_w} - \frac{\pi m_i (R^2 - r^2)}{\rho_i \pi R^2} \right) = m_i \left(1 - \frac{\rho_w}{\rho_i} \frac{(R^2 - r^2)}{R^2} \right)$$
$$= -\frac{\rho_s c_s \pi r^2 h (t_m - t_s)}{\lambda} \left(1 - \frac{\rho_w}{\rho_i} \left(1 - \left(\frac{r}{R} \right)^2 \right) \right)$$

$$m_{spill} = -\frac{7800 \cdot 500 \cdot \pi \cdot 0.05^2 \cdot 0.05 (0 - 26.2)}{3.3 \cdot 10^5} \left(1 - \frac{1000}{900} \left(1 - \left(\frac{0.05}{0.055} \right)^2 \right) \right) \approx 0.09815$$

Taking into account the minimal number of significant digits in the problem statement, which is two, we round and obtain $m_{spill} = 0.098\ kg$.
Answer: $m_{spill} = 0.098\ kg$

9.4. (7 points) One day a magician came to a school physics lesson to advertise the circus in which he performs. The magician took several wooden blocks with a density of 520 kg/m^3, length of L = 10 cm, width of B = 5 cm, height of H = 3 cm, and began stacking them on each other as shown in the figure. Each new block he shifted by the maximum distance along the previous block without disturbing the equilibrium of the entire structure. As a result, the top block was offset by 10 cm relative to the bottom one.



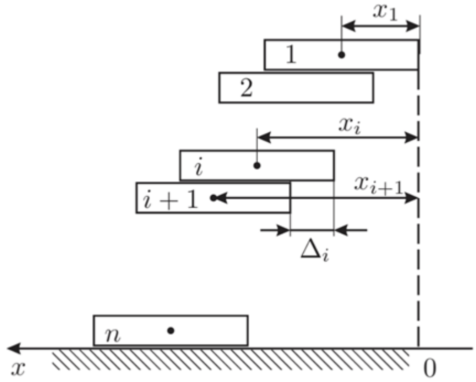
[4] How many blocks did the magician need for this?

(Cherenkov A.A.)

Answer: 5.

Solution. A stack of blocks will not fall if the center of mass of all blocks lying above each block does not shift beyond its edge to the right. To find the allowed shifts, denote the coordinates x_i of the centers of mass of the blocks. The top block corresponds to $i = 1$.

Coordinates are measured from the edge of the top block. Blocks with numbers i and $(i + 1)$ are shifted by Δ_i relative to each other. Coordinate x_{ci} of the center of mass of the stack above $(i + 1)$-st block is



$$x_{ci} = \frac{1}{i} (x_1 + x_2 + \ldots + x_i),$$

since all blocks have the same mass. The condition for equilibrium of the blocks for limiting shifts can be written as:

$$x_{ci} = x_{i+1} - \frac{L}{2}, \quad or \quad x_{i+1} = \frac{L}{2} + \frac{1}{i}\left(x_1 + x_2 + \ldots + x_i\right).$$

We find the displacement of i-th block:

$$\Delta_i = \frac{1}{i}\left(x_1 + x_2 + \ldots + x_i\right) - \frac{1}{i-1}\left(x_1 + x_2 + \ldots + x_{i-1}\right)$$

Let us transform this expression using the expression for x_{i+1}, obtained above:

$$\Delta_i = -\frac{1}{(i-1)\,i}\left(x_1 + x_2 + \ldots + x_{i-1}\right) + \frac{x_i}{i} =$$

$$= \frac{1}{i}\left(x_i - \frac{1}{i-1}\left(x_1 + x_2 + \ldots + x_{i-1}\right)\right) =$$

$$= \frac{1}{i}\left(\frac{L}{2} + \frac{1}{i-1}\left(x_1 + x_2 + \ldots + x_{i-1}\right) - \frac{1}{i-1}\left(x_1 + x_2 + \ldots + x_{i-1}\right)\right) = \frac{L}{2i}.$$

Thus, the displacements of the blocks starting from the upper one form the sequence

$$\Delta_i = \frac{L}{2i} = \frac{L}{2}, \frac{L}{4}, \frac{L}{6}, \ldots$$

find the displacement of the lower $n-th$ block relative to the upper one:

$$\Delta(n) = \Delta_1 + \Delta_2 + \ldots + \Delta_n.$$

According to the problem statement, it is necessary to find such n that $\Delta(n) \geqslant L$. Finding the sums for $n = 2{,}3{,}4$, we get

$$\Delta(2) = 0{,}75L; \quad \Delta(3) = 0{,}92L; \quad \Delta(4) = 1{,}04L > L.$$

Thus, a stack of five bars satisfies the requirement of the problem, with the last bar being shifted by a distance less than its maximum possible shift.
Answer: 5

9.5. (7 points) On a windless day, Pasha went to a lake to go fishing. Having reached the middle of the lake in a boat with a length of L = 2.0 m and a mass of m = 10.0 kg, he decided to walk from the bow to the stern. The boat then began to move, experiencing a resistance force from the water $F = -ku$, where u is the speed of the boat and k is a known proportionality coefficient. Pasha's mass is M = 80.0 kg.

[5] Find the displacement S_0 of the boat by the time it comes to a complete stop in the absence of resistance force ($k = 0$).

[6] Find the displacement S of the boat by the time it comes to a complete stop in the presence of resistance force ($k = 0.50$ kg/s).

Comment. Assume that the boat moves in only one direction. (*Cherenkov A.A.*)

Answer: 1.8.

Answer: 0.

Solution. 1. Let us use the property of conservation of the center of mass of an initially stationary system in the absence of external forces (internal forces do not change the position of the center of mass due to Newton's third law). The displacement of the center of mass of a system of two bodies with masses m_1,m_2 is determined by their corresponding displacements Δx_1,Δx_2, as follows:

$$\Delta x_c = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}.$$

Let's apply this formula to our case:

$$\Delta x_c = \frac{m \cdot (-S_0) + M \cdot (L - S_0)}{M + m} = 0,$$

From which we get

$$S_0 = \frac{ML}{M + m} \approx 1.8m$$

2. Let v and u be the velocities of the person and the boat relative to the ground at a certain moment in time, and let x be the coordinate of the boat at the same moment in time. Then the law of momentum change ($\Delta p/\Delta t = F$) for the system consisting of the person and the boat is

$$\frac{\Delta}{\Delta t}(Mv + mu) = -ku,$$

from which, after substituting $u = \Delta x/\Delta t$, we find

$$\frac{\Delta}{\Delta t}(Mv + mu + kx) = 0.$$

The last equation means that the value in brackets does not change over time, i.e. its initial and final values are equal to each other:

$$Mv_{in} + mu_{in} + kx_{in} = const = Mv_f + mu_f + kx_f.$$

Since $v_{in} = u_{in} = v_f = u_f = 0$, after substituting the velocities and dividing by k, we obtain $x_f = x_{in}$, i.e. the platform will return to its previous position, and the desired displacement

$$S = x_f - x_{in} = 0$$

Answer:1.8 m; 0 m

9.6. (6 points) During a physics lesson on electrical circuits, students were performing a lab experiment. Vasya received a set of equipment in which an electrical circuit with two terminals had already been assembled, consisting only of DC voltage sources and resistors. After Vasya connected a voltmeter to the terminals of the circuit, it showed a voltage of $U_0 = 20.0$ V. Then he connected a resistor of resistance of R = 5.0 Ω to the terminals of the circuit, and the voltmeter then showed a voltage of $U_1 = 10.0$ V.

[7] What voltage will the voltmeter show if another identical resistor is connected to the circuit?

Comment. Assume the voltmeter is ideal. (*Cherenkov A.A.*)

Answer: 6.7.

Solution. 1)Any circuit with two terminals consisting only of DC voltage sources and resistors is equivalent to a single imperfect DC voltage source with EMF $\mathcal{E}$ and internal resistance r.
When only an ideal voltmeter is connected to the circuit, no current flows through the equivalent source, so $\mathcal{E} = U_0$.
2) Now let's connect one resistor to the circuit. Let's apply Ohm's law to the complete circuit and find the current:

$$I_1 = \frac{\mathcal{E}}{R + r}$$

Then, according to Ohm's law for the section of the circuit, the voltage on the resistor is:

$$U_1 = I_1 \cdot R = \frac{\mathcal{E}}{R + r} \cdot R = \frac{U_0 R}{R + r}, \quad wherefrom \quad r = \frac{U_0 - U_1}{U_1} \cdot R;$$

2) Now let's consider the case where a second resistor is connected to the circuit. Since the resistors are connected in parallel and their resistances are equal, their total resistance is R/2. Thus, according

to Ohm's law, the current for the complete circuit can be found as:

$$I_2 = \frac{\mathcal{E}}{\frac{R}{2} + r}$$

Then, according to Ohm's law for the section of the circuit, we find the voltage on the resistors:

$$U_2 = I_2 \cdot \frac{R}{2} = \frac{\mathcal{E}}{\frac{R}{2} + r} \cdot \frac{R}{2} =$$

$$= \frac{U_0}{\frac{R}{2} + \frac{U_0 - U_1}{U_1} \cdot R} \cdot \frac{R}{2} = \frac{U_0 U_1}{2U_0 - U_1} \approx 6.7V$$

Answer: 6.7 V

9.7. (7 points) In the school laboratory, optical systems were studied. The teacher assembled a setup consisting of two plane mirrors sharing a common side and forming a right dihedral angle. A thin converging lens with a focal length of f = 10 cm and diameter of d = 20 cm is positioned between the mirrors. The lens touches both mirrors, and its principal optical axis passes through the line of intersection of the mirrors, perpendicular to that line.

[8] Find the distance from the lens at which the image of a small bulb will be formed, if the bulb is located on the lens's principal optical axis at a distance of l = 15 cm from its center.

(*Cherenkov A.A.*)

Answer: 0.05.

Solution. 1) From geometric considerations it follows that object S is located in front of the front focus F of the lens, and its rear focus coincides with the point of contact of the mirrors C (figure 1). The light rays we are interested in first pass through the lens, then reflect off the pair of mirrors, and then pass through the lens again in the opposite direction. To find the image S', calculate the positions of the intermediate images produced separately by the lens and mirrors as the rays pass through the system.

To illustrate the reasoning, Fig. 2 shows the continuations of the rays that create the intermediate images S_1, S_2 and S_3 as well as auxiliary constructions for finding image. Points O, A and B are, respectively, the optical centre of the lens and its points of contact with each of the mirrors.

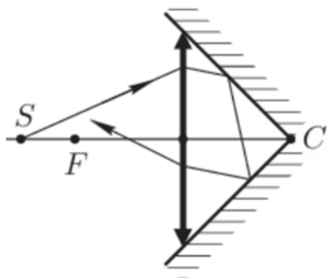


Fig. 1

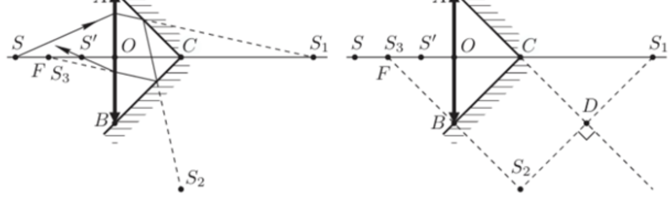


Fig. 2

2) Let a and b be the distances from the lens to the object and to the image, respectively, then according to the thin lens formula

$$\frac{1}{a} + \frac{1}{b} = \frac{1}{f},$$

wherefrom

$$b = \frac{af}{a-f},$$

Point S_1 is image of source S in the lens without taking mirrors into account, therefore, using $a = SO = 1{,}5f$ we obtain $OS_1 = b = 3f$.
3) A mirror produces an image that is symmetrical to the object relative to the plane of the mirror, and this property applies to both real and imaginary objects. Point S_2 is the image of object S_1in mirror AC without taking into account the second mirror, which means that by constructing a perpendicular S_1D in the plane of mirror AC and continuing it for the same distance, we find S_2.
4) Reasoning similarly, we construct point S_3 — the image of object S_2 in mirror BC without taking the lens into account. The sequential application of two reflections relative to mutually perpendicular lines is equivalent to a single reflection relative to their point of intersection (recall the property of central symmetry of the graph of an odd function), therefore $S_3C = CS_1$, from which

$$S_3O = S_3C - OC = CS_1 - OC = OS_1 - 2OC = 3f - 2f = f,$$

that is, the image S_3 coincides with the front focus F of the lens.
5) The desired point S' is the actual image of the imaginary object S_3, so from the thin lens formula, using $a = -S_3O = -f$, we get $S'O = b = f/2$.
At the very beginning of the solution, a ray was chosen that first reflects from mirror AC and then from BC, but there are rays that reflect in the reverse order. In this case, it is clear from symmetry that they will intersect at the same point, but with a different arrangement of mirrors, two finite images could arise. Thus, the final answer is that the image will be at a distance of $f/2 = 5$ cm in front of the lens.
Answer: 5 cm

Solutions to problems for grade R10

10.1. (7 points) Experimenter Vasily performed an unusual cycle on a monoatomic ideal gas in his laboratory: in the PV diagram (at the chosen scale on the axes), it consists of two circular arcs of different radii with centers at points (2V, 2P) and (V, (2+$\sqrt{3}$)P), where P = 300 Pa, V = 5 m^3. Segment 1–2 of the cycle is one third of a circle.

[1] Find the thermodynamic efficiency of the cycle in percent.

(*Cherenkov A.A.*)

Answer: 6.

Solution. 1) Let the centers of the circles be O_1(V, (2+$\sqrt{3}$)P) and O_2(2V, 2P), and denote point 1 as A and point 2 as B. From the problem statement it follows that A has coordinates (V, 2P), the angle adjacent to angle $\angle AO_2B$ equals 60°, hence point B has coordinates (2.5V,(2+$\sqrt{3}$/2)P). Thus we draw segment AB. Considering the isosceles triangles AO_1B and AO_2B we find that $\angle AO_1B = 60°$, which means that segment 2–1 represents one-sixth of a circle.

2) The work done by the gas on each segment 1–2 and 2–1 equals the area under the corresponding part of the PV-diagram. Compute the works:

$$A_{12} = 1/3\pi PV + 1/2 \cdot \sqrt{3}/2P \cdot 1/2V + 2P \cdot 3/2V = \left(8\pi + 3\sqrt{3} + 72\right)/24 \cdot PV$$

$$A_{21} = -\left(\sqrt{3}P \cdot 3/2V - 1/2 \cdot 3/2V \cdot \sqrt{3}/2P - 3\pi PV/6 + 3/2V \cdot 2P\right) = -\left(9\sqrt{3} - 4\pi + 24\right)/8 \cdot PV$$

Thus, the work of the gas per cycle is:

$$A = A_{12} + A_{21} = (5\pi/6 - \sqrt{3})PV$$

3) Compute the change in internal energy on segment 1–2:

$$\Delta U = 3/2\nu R\Delta T = 3/2\Delta\left(PV\right) = 3/2\left(5/2V \cdot \left(2+\sqrt{3}\right)P - 2PV\right) = \left(18 + 15\sqrt{3}\right)/4 \cdot PV)$$

According to the first law of thermodynamics, on segment 1–2 the gas receives heat:
$Q = \Delta U + A_{12} = \left(8\pi + 93\sqrt{3} + 180\right)/24 \cdot PV$
Similarly, on segment 2–1 the gas releases heat, since $\Delta U_{21} = -\Delta U_{12} < 0$,$A_{12} < 0$, hence $Q_{21} = \Delta U_{21} + A_{21} < 0$

4) Therefore, the efficiency of the gas over the cycle is:

$$\eta = \frac{A}{Q} * 100\% \approx 6\%$$

Answer: 6%

10.2. (8 points) In one laboratory, engineers are calculating a model of a building's dome in the form of a truncated cone with base radii of R = 2 m and r = 1 m and a height of H = 2 m, which will be used in construction of underwater structures. The engineers are interested in the case when the dome is placed under water with a density of 1000 kg/m^3 so that the center of its smaller base is at a depth of L = 10 m, and the axis is tilted at an angle of 30^o to the water surface. The larger base of the dome is submerged deeper than the smaller base.

[2] Determine the force acting on the lateral surface of the dome from the water side, if the air pressure above the water surface is 10^5 Pa. Give the answer in meganewtons.

(*Cherenkov A.A.*)

Answer: 2.

Solution. 1) The area of the base of the cone is $S = \pi R^2$, its volume is $V = SH/3 = \pi R^2 H/3$. The volume of a truncated cone can be found as the difference between the volumes of the whole cone and the truncated part:

$$V = V_R - V_r = \frac{1}{3}\pi \left(R^2 (H + h) - r^2 h\right)$$

where h - is the height of the truncated part. Let us find h. Consider a cross-section of the cone formed by a plane passing through its axis. Then, from similarity of the triangles, representing the cross-sections of the full cone and the truncated part, we obtain:

$$\frac{h}{H+h} = \frac{r}{R}$$

wherefore

$$h = \frac{rH}{R-r}$$

Substituting this expression for h into the formula for the volume of the truncated cone, we obtain:

$$V = \frac{1}{3}\pi H \left(R^2 + rR + r^2\right)$$

Then the Archimedes force acting on the cone:

$$F_A = \rho g V = \rho g \pi H \left(R^2 + rR + r^2\right)/3.$$

From geometric considerations we also find that the lower base is submerged to a depth
$L + H\sin\alpha$

2) By its nature, Archimedes' force is equal to the vector sum of the forces of liquid pressure on the surface of a body placed in it, that is, in our case

$$\overrightarrow{F}_A = \overrightarrow{F} + \overrightarrow{F}_r + \overrightarrow{F}_R,$$

where F is the required force, F_r and F_R — are the pressure forces of the liquid on the smaller and larger bases of the cone respectively.

By symmetry of the bases and homogeneity of the gravitational field, the mean pressure P_{cp} on a base equals the pressure at the level of the center of the base, therefore

$$F_r = P^r_{cp} S = (P_0 + \rho g L)\,\pi r^2, \quad F_R = P^R_{cp} S = (P_0 + \rho g\,(L + H\sin\alpha))\,\pi R^2$$

The force F_A is directed vertically upward, the forces F_r *and* F_R are directed perpendicular to the bases of the cone, i.e. along its axis, at an angle α to the horizontal. Let us denote $\overrightarrow{F_0} = \overrightarrow{F_r} + \overrightarrow{F_R}$. Since the pressure forces on the bases have the same direction, then $F_0 = -F_r + F_R$. Applying the law of cosines to the triangle formed by the vectors $\overrightarrow{F}_A, \overrightarrow{F}, \overrightarrow{F_0}$, we get

$$F^2 = F_A^2 + F_0^2 - 2F_A F_0 \cos\left(\frac{\pi}{2} + \alpha\right),$$

wherefrom

$$F = \sqrt{F_A^2 + F_0^2 + 2F_A F_0 \sin\alpha} \approx 2MN.$$

Answer: 2 MN

10.3. (6 points) Masha and Dasha were playing with tennis balls on a sports ground. They simultaneously threw their balls with equal initial velocities of v = 10.0 m/s at angles of 30.0^o and

60.0^o to the horizontal.

[3] After how much time from the start of the motion will the velocities of the balls have the same direction?

Comment. Assume that the initial velocities of the balls lie in one vertical plane and take the acceleration due to gravity as g = 9.8 m/s^2. (*Cherenkov A.A.*)

Answer: 1.4.

Solution. 1) Let's denote $\alpha = 30°$. Both balls considered in the problem move with constant acceleration equal to the acceleration of free fall. The projections of their velocities onto the horizontal x and vertical y axes are as follows:

$$v_{1x} = v_0 \cos\alpha, \quad v_{1y} = v_0 \sin\alpha - g\tau,$$

$$v_{2x} = v_0 \cos 2\alpha, \quad v_{2y} = v_0 \sin 2\alpha - g\tau.$$

Index 1 refers to Masha's ball, and index 2 refers to Dasha's ball. Since the velocities at time τ were in the same direction, we obtain

$$\frac{v_{1y}}{v_{1x}} = \frac{v_{2y}}{v_{2x}},$$

Substitute the expressions for the velocity projections:

$$tg\alpha - \frac{g\tau}{v_0 \cos\alpha} = tg2\alpha - \frac{g\tau}{v_0 \cos 2\alpha}.$$

From this equation express the time:

$$\tau = \frac{v_0}{g} \cdot \frac{tg2\alpha - tg\alpha}{\frac{1}{\cos 2\alpha} - \frac{1}{\cos\alpha}},$$

Let's perform trigonometric transformations:

$$tg2\alpha - tg\alpha = \frac{\sin 2\alpha}{\cos 2\alpha} - \frac{\sin\alpha}{\cos\alpha} = \frac{\sin\alpha}{\cos 2\alpha \cos\alpha},$$

$$\frac{1}{\cos 2\alpha} - \frac{1}{\cos\alpha} = \frac{2sin\,(\alpha/2)\sin\,(3\alpha/2)}{\cos 2\alpha \cos\alpha},$$

Then we finally get:

$$\tau = \frac{v_0 \cos\,(\alpha/2)}{g \sin\,(3\alpha/2)} \approx 1.4s$$

2)The problem can be solved in another way that does not require trigonometric transformations. To do this, we need to consider the motion of one ball relative to the other. From the law of addition of accelerations (similar to the law of addition of velocities), it follows that the balls move relative to each other at a constant speed. The velocity of Masha's ball $\vec{v}_{12}$ relative to Dasha's ball is

$$\vec{v}_{12} = \vec{v}_1 - \vec{v}_2 = \vec{v}_{10} - \vec{v}_{20},$$

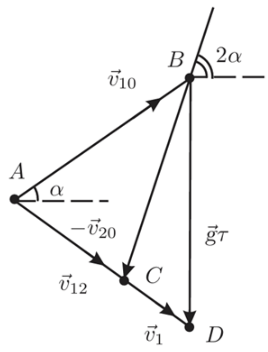
where the index 0 denotes the initial values of the velocities. At time τ the velocities $\vec{v}_1$, $\vec{v}_2$ and $\vec{v}_{12}$ are collinear. With this in mind, let us draw (see fig.) the velocity triangle ABC , corresponding to the formula for $\overrightarrow{v_{12}}$, as well as the triangle ABD, reflecting the equality

$$\vec{v}_1 = \vec{v}_{10} + \vec{g}\tau.$$

Since $AB = BC = v_0$, we have $\angle BAC = 90° - \frac{\alpha}{2}$, and $\angle ADB = \frac{3\alpha}{2}$.
Apply the sine theorem to $\triangle ABD$:

$$\frac{v_0}{\sin\left(\frac{3\alpha}{2}\right)} = \frac{g\tau}{\cos\left(\frac{\alpha}{2}\right)}.$$

This immediately gives the answer obtained earlier. Thus, the transition to another coordinate system made it possible to use geometric methods instead of trigonometric transformations.



Answer: 1.4 s

10.4. (5 points) In the microclimate laboratory, the behavior of humid air is studied. For this, two reservoirs, which have volumes of $V_1 = 5.0$ l and $V_2 = 2.0$ l, are connected by a short pipe with a valve, that was initially closed. The first reservoir is filled with humid air at pressure of $P_1 = 3.0$ atm and relative humidity of $\phi = 80.0\%$, and the second with water vapor at pressure of $P_2 = 0.75$ atm. Heat insulating coating maintains a constant temperature of $T = 100.0^o$ C in the vessels at all times. The valve is opened and after some time thermodynamic equilibrium is established in the vessels.

[4] Determine the pressure in atmospheres and the relative humidity in the vessels in percent after equilibrium is reached, if the saturated vapor pressure at 100^o C is equal to 1.0 atm.

(*Cherenkov A.A.*)

Answer: 2.4;78.6.

Solution. 1) Write down the Mendeleev-Clapeyron equation for gases in vessels before opening the valve:

$$P_1V_1 = \nu_1 RT, \quad P_2V_2 = \nu_2 RT$$

Add the equations:

$$P_1V_1 + P_2V_2 = (\nu_1 + \nu_2)\, RT$$

2) Let's write down the Mendeleev-Clapeyron equation for gases in vessels after opening the valve:

$$P\,(V_1 + V_2) = (\nu_1 + \nu_2)\, RT$$

Thus, we obtain:

$$P\,(V_1 + V_2) = P_1V_1 + P_2V_2$$

from which we find the desired pressure:

$$P = \frac{P_1V_1 + P_2V_2}{V_1 + V_2} = 2{,}4\ atm.$$

3) To calculate the steady-state humidity in the vessels, find the partial pressure of water vapor. From the Mendeleev-Clapeyron equation for water vapor in the second vessel before opening the valve, express its amount:

$$\nu_2 = \frac{P_2V_2}{RT}$$

Use the definition of relative humidity to calculate the partial pressure of water vapor in the first vessel before opening the valve:

$$P_{01} = \varphi P_0$$

Thus, the amount of water vapor in the first vessel before opening the valve is:

$$\nu_{01} = \frac{P_{01}V_1}{RT} = \frac{\varphi P_0 V_1}{RT}$$

Write down the Mendeleev-Clapeyron equation after opening the valve for water vapor:

$$P_{02}\left(V_1 + V_2\right) = \left(\nu_{01} + \nu_2\right) RT$$

From which find the partial pressure of water vapor:

$$P_{02} = \frac{\left(\nu_{01} + \nu_2\right) RT}{V_1 + V_2} = \frac{\varphi P_0 V_1 + P_2 V_2}{V_1 + V_2}$$

4) Thus, calculate the relative humidity in the vessels after opening the valve:

$$\phi = \frac{P_{02}}{P_0} \cdot 100\% = \frac{\varphi P_0 V_1 + P_2 V_2}{P_0\left(V_1 + V_2\right)} \cdot 100\% \approx 78.6\%$$

Answer: 2.4; 78.6%

10.5. (6 points) On a windless day, Pasha went to a lake to go fishing. Having reached the middle of the lake in a boat with a length of L = 2.0 m and a mass of m = 10.0 kg, he decided to walk from the bow to the stern. The boat then began to move, experiencing a resistance force from the water $F = -ku$, where u is the speed of the boat and k is a known proportionality coefficient. Pasha's mass is M = 80.0 kg.

[5] Find the displacement S_0 of the boat by the time it comes to a complete stop in the absence of resistance force ($k = 0$).

[6] Find the displacement S of the boat by the time it comes to a complete stop in the presence of resistance force ($k = 0.50$ kg/s).

Comment. Assume that the boat moves in only one direction. (*Cherenkov A.A.*)

Answer: 1.8.

Answer: 0.

Solution. 1. Let us use the property of conservation of the centre of mass of an initially stationary system in the absence of external forces (internal forces do not change the position of the centre of mass due to Newton's third law). The displacement of the centre of mass of a system of two bodies with masses m_1,m_2 is determined by their corresponding displacements Δx_1,Δx_2, as follows:

$$\Delta x_c = \frac{m_1 \Delta x_1 + m_2 \Delta x_2}{m_1 + m_2}.$$

Let's apply this formula to our case:

$$\Delta x_c = \frac{m \cdot (-S_0) + M \cdot (L - S_0)}{M + m} = 0,$$

From which we get

$$S_0 = \frac{ML}{M + m} \approx 1.8m$$

2. Let v and u be the velocities of the person and the boat relative to the ground at a certain moment in time, and let x be the coordinate of the boat at the same moment in time. Then the law of momentum change ($\Delta p/\Delta t = F$) for the system consisting of the person and the boat is

$$\frac{\Delta}{\Delta t}\left(Mv + mu\right) = -ku,$$

from which, after substituting $u = \Delta x/\Delta t$, we find

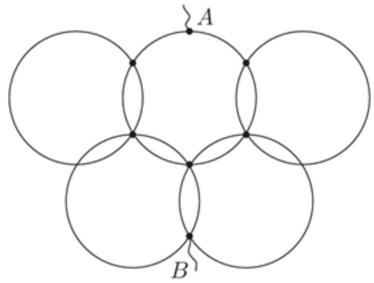
$$\frac{\Delta}{\Delta t}\left(Mv + mu + kx\right) = 0.$$

The last equation means that the value in brackets does not change over time, i.e. its initial and final values are equal to each other:

$$Mv_{in} + mu_{in} + kx_{in} = const = Mv_f + mu_f + kx_f.$$

Since $v_{in} = u_{in} = v_f = u_f = 0$, after substituting the velocities and dividing by k, we obtain $x_f = x_{in}$, i.e. the platform will return to its previous position, and the desired displacement
$S = x_f - x_{in} = 0$.
Answer:1.8 m; 0 m

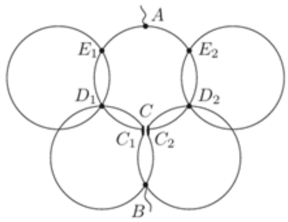
10.6. (6 points) For the opening of the Olympic Games, an amateur electrician assembled a luminous garland in the shape of the Olympic emblem from five identical rings with a resistance of r = 33 Ω. The rings were soldered together at the points marked by bold dots in the figure. The solder joints and point A divide the central ring into 6 equal parts.

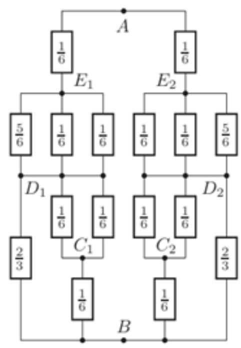


[7] What is the resistance of the garland between points A and B?

Comment. Neglect any resistance at the solder joints. (*Cherenkov A.A.*)
Answer: 7.
Solution. 1) Due to the symmetry of the circuit relative to the axis AB, the point C can be divided into two (see fig.). Let us draw an equivalent circuit (see fig.), in which the corresponding points are marked with letters, and the numerical values of the resistances are given in units of r. Using the properties of series and parallel connections, find the resistances of the sections of the circuit.

$$R_{AE_1} = R_{AE_2} = \frac{1}{6}r,$$





$$R_{E_1D_1} = R_{E_2D_2} = \frac{1}{\frac{1}{\frac{1}{6}r} + \frac{1}{\frac{1}{6}r} + \frac{1}{\frac{5}{6}r}} = \frac{5}{66}r$$

Note that two resistances of 1/6r connected in parallel have a total resistance of 1/12r. Then

$$R_{D_1B} = R_{D_2B} = \frac{1}{\frac{1}{\frac{2}{3}r} + \frac{1}{\frac{1}{12}r + \frac{1}{6}r}} = \frac{2}{11}r.$$

Find the desired total resistance:

$$R = \frac{1}{2}\left(R_{AE_1} + R_{E_1D_1} + R_{D_1B}\right) = \frac{7}{33}r = 7\Omega.$$

Answer: 7 Ω

10.7. (7 points) During one physics lesson students were performing a laboratory experiment. Vasya finished the assigned task first and, so as not to be bored, began experimenting. He took two inclined planes with angles of $a_1 = 45.0^o$ and $a_2 = 30.0^o$ and masses of $m_1 = 100.0$ g and $m_2 = 500.0$ g respectively. He placed small weights on a lightweight rod and laid the rod horizontally on the planes. His task was to choose a mass for the weight such that when the rod falls onto the table, it touches the table with both ends simultaneously.

[8] For what mass of the weight is this possible, if the weight divides the rod in the ratio 2:1 measured from the left end?

Comment. Neglect friction. (*Cherenkov A.A.*)
Answer: 0,0282.
Solution. 1) The ends of the rod will hit the surface simultaneously only if the rod falls horizontally

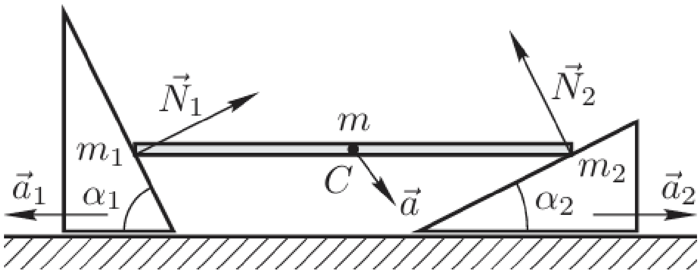
at all times. For this, it is necessary and sufficient that there be no rotation of the rod in the vertical plane in which the rod lies, i.e., the sum of the moments of the forces acting on the rod is zero (see figure):

$$N_1 \cos\alpha_1 \cdot 2 \cdot L - N_2 \cos\alpha_2 \cdot L = 0,$$

From which we get

$$\frac{N_2}{N_1} = \frac{2\cos\alpha_1}{\cos\alpha_2}$$

where L is the distance from weight C to the right inclined plane, and the moments are calculated relative to the point to which the weight is attached.



Rod on inclined planes

2) Using Newton's second law, express the accelerations a_1 and a_2 of the inclined planes and the horizontal component of the rod's acceleration a_x:

$$a_1 = \frac{N_1 \sin\alpha_1}{m_1}, \quad a_2 = \frac{N_2 \sin\alpha_2}{m_2}, \quad a_x = \frac{N_1 \sin\alpha_1 - N_2 \sin\alpha_2}{m}.$$

write down the horizontal displacements of the inclined planes relative to the rod at the moment t:

$$\Delta x_1 = \frac{(a_1 + a_x)\, t^2}{2}, \quad \Delta x_2 = \frac{(a_2 - a_x)\, t^2}{2}.$$

3) The vertical displacement Δy of the rod is the same for both ends:

$$\Delta x_1 tg\alpha_1 = \Delta y = \Delta x_2 tg\alpha_2,$$

from which, after substituting the expressions for Δx_1,Δx_2 and simplifying, we obtain

$$(a_1 + a_x)\, tg\alpha_1 = (a_2 - a_x)\, tg\alpha_2.$$

Thus, substitute the expressions for the accelerations:

$$\left(\frac{N_1 \sin\alpha_1}{m_1} + \frac{N_1 \sin\alpha_1 - N_2 \sin\alpha_2}{m}\right) tg\alpha_1 = \left(\frac{N_2 \sin\alpha_2}{m_2} - \frac{N_1 \sin\alpha_1 - N_2 \sin\alpha_2}{m}\right) tg\alpha_2,$$

Let us divide the entire equation by N_1, substitute the expression for N_2/N_1 and regroup the terms:

$$\frac{1}{m}\left(\sin\alpha_1 - 2\cos\alpha_1 tg\alpha_2\right)\left(tg\alpha_1 + tg\alpha_2\right) = \frac{2\cos\alpha_1 tg^2\alpha_2}{m_2} - \frac{\sin\alpha_1 tg\alpha_1}{m_1}$$

Finally, divide the equation $\cos\alpha_1$ by and express the desired mass:

$$m = \frac{(tg\alpha_1 - 2tg\alpha_2)\,(tg\alpha_1 + tg\alpha_2)}{\frac{2tg^2\alpha_2}{m_2} - \frac{tg^2\alpha_1}{m_1}} \approx 28.2\ g$$

Answer:28.2 g

Solutions to problems for grade R11

11.1. (9 баллов) Pupils were having a snowball fight at a playground. Petya, standing on a hill of height H = 3.00 m, threw a snowball with an initial velocity of v = 10.0 m/s at an angle of $\phi = 60.0^o$ to the horizontal. At that same moment, from ground level Yuri threw his own snowball after Petya's, being at a distance of L = 5.00 m from the hill.

[1] With what minimum speed and at what angle should Yuri have thrown his snowball so that the two snowballs collided?

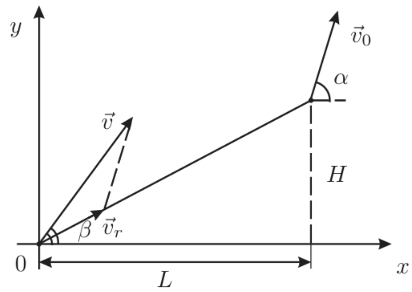
Comment. Assume the initial velocities of the snowballs lie in one vertical plane and ignore air resistance. (*Cherenkov A.A.*)

Answer: 8.25;67.0.

Solution. 1) Both snowballs move with constant acceleration equal to the acceleration of free fall $\overrightarrow{g}$. To describe their motion, let's introduce a Cartesian coordinate system as shown in the figure. Let Yura throw the snowball at an angle β to the horizontal with a speed v. Then the equations of motion of the snowballs in projection on the coordinate axes are:

$$\begin{cases} x_1(t) = L + v_0 t\cos\alpha \\ y_1(t) = H + v_0 t\sin\alpha - \frac{gt^2}{2} \end{cases}$$

$$\begin{cases} x_2(t) = vt\cos\beta \\ y_2(t) = vt\sin\beta - \frac{gt^2}{2} \end{cases}$$



2) If after a time interval t the snowballs collide, then their coordinates can be equated:

$$vt\cos\beta = L + v_0 t\cos\alpha,$$

$$vt\sin\beta - \frac{gt^2}{2} = H + v_0 t\sin\alpha - \frac{gt^2}{2}.$$

From these equations, find

$$v = v_0\frac{\sin\alpha - \frac{H}{L}\cos\alpha}{\sin\beta - \frac{H}{L}\cos\beta},$$

$$t = \frac{L}{v_0\cos\alpha}\left(\frac{\frac{H}{L} - tg\alpha}{\frac{H}{L} - tg\beta} - 1\right)^{-1}.$$

The time of motion before the collision can be conveniently expressed in terms of the relative velocity $\overrightarrow{v}_r$ of one snowball relative to the other. Since the accelerations of the snowballs are the same, the relative velocity is constant, and the time is expressed as follows:

$$t = \frac{\sqrt{L^2 + H^2}}{v_r}.$$

With fixed length and direction of the vector $\overrightarrow{v}_0$ and a given direction of the vector $\overrightarrow{v}$ from the vector triangle of velocities shown in the figure, it can be seen that a smaller v_r corresponds to a smaller v. Thus, the minimum speed v corresponds to the minimum value of v_r and, therefore, the maximum time t. Another condition limits the possibility of collision between snowballs. The snowball, which Yuri threw, must move away from the origin horizontally by a distance greater than

L. Its flight range is equal to $v^2 \sin 2\beta / g$. Therefore, the following condition must be satisfied

$$v^2 \sin 2\beta \geqslant Lg.$$

So, we need to find values for v and β, that satisfy equality (1), inequality (3), and ensure the maximum value of (2). Substituting expression (1) into (3), we get the inequality

$$2\frac{v_0^2}{Lg}\left(\sin\alpha - \frac{H}{L}\cos\alpha\right)^2 tg\beta - \left(tg\beta - \frac{H}{L}\right)^2 \geqslant 0.$$

denoting

$$\frac{v_0^2}{Lg}\left(\sin\alpha - \frac{H}{L}\cos\alpha\right)^2 = a$$

And $tg\beta = \xi$, we get

$$\xi^2 - 2\left(\frac{H}{L} + a\right)\xi + \frac{H^2}{L^2} \leqslant 0.$$

This inequality is satisfied if

$$\frac{H}{L} + a - \sqrt{a^2 + 2a\frac{H}{L}} \leqslant \xi \leqslant \frac{H}{L} + a + \sqrt{a^2 + 2a\frac{H}{L}}.$$

As can be seen from formula (2) if $\alpha = 60°$, then larger values of t correspond to larger values of $\xi = tg\beta$. Therefore, we need to choose the largest value for ξ. Therefore, we obtain

$$tg\beta = \frac{H}{L} + a + \sqrt{a^2 + 2a\frac{H}{L}} \quad if \quad \alpha = 60°.$$

Thus, substituting numerical values into the derived formulas, we find

$$\beta \approx 67.0°, \quad v \approx 8.25\ m/s.$$

Answer: $8.25; 67.0°$

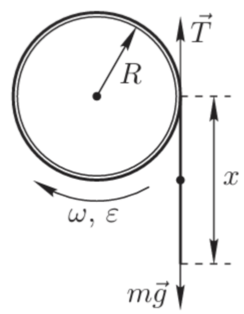
11.2. (6 баллов) At a construction site, workers were instructed to run a cable of linear density λ = 1.0 kg/m from the upper floors down to the lower floors. The cable is wound on a reel with a radius of R = 20.0 cm, which has a moment of inertia of I = 0.20 $kg \cdot m^2$ about its axis. On the top floor, the reel is fixed so that it can freely rotate around its axis without friction as the cable unwinds.

[2] What acceleration will the cable acquire after it is released without initial velocity and it unwinds by L = 3.0 m?

Comment. Assume the thickness of the cable is negligible. (*Cherenkov A.A.*)

Answer: 3.7.

Solution.



Let T be the tension force of the cable at the top point of its vertical part (see fig.), ε be the angular acceleration of the reel, $m = \lambda x$ be the mass of the hanging part of the cable, and $I_x = \lambda(L - x)R^2$ and be the moment of inertia of the cable wound on the reel. Then Newton's second law for the hanging part of the cable, the equation of rotational motion of the reel with the cable wound on it, and the kinematic relationship are respectively

$$ma = mg - T, \qquad (I + I_x)\,\varepsilon = TR, \qquad a = \varepsilon R.$$

Solving the system, we obtain

$$a\left(x\right)=\frac{gR^2\lambda x}{I+R^2\lambda l}.$$

Then, by the time the cable unwinds by L meters, its acceleration will be:

$$a\left(l\right)=\frac{gR^2\lambda l}{I+R^2\lambda l}\approx 3.7\ m/s$$

Answer: 3.7 m/s

11.3. (5 баллов) The Ivanov family has a cozy country house where they often spend winter weekends. On one of those days, when the outside temperature was t_1 = –25^o C, they noticed that the indoor thermometer showed temperature of $T_1 = 10^o$ C. The family decided that they will move to live in the house when it gets warmer and the house can maintain a constant indoor temperature of $T_2 = 20^o$ C. The heating in the house works steadily: the radiators always heat up to $\theta = 80^o$ C, regardless of the weather outside.

[3] What must the outside temperature be for the Ivanovs to move into the house?

(*Cherenkov A.A.*)

Answer: -10.
Solution. Let T and t be the current temperatures inside the house and outside, respectively, then the heat flow powers from the radiators into the room and from the house to the outside are

$$P=\alpha\left(\tau-T\right)\qquad and \qquad N=\beta\left(T-t\right),$$

where α and β are some constants. In steady state, these powers are equal to each other, so for the first and second cases we can write

$$\alpha\left(\tau-T_1\right)=\beta\left(T_1-t_1\right)\qquad \text{и} \qquad \alpha\left(\tau-T_2\right)=\beta\left(T_2-t_2\right),$$

From which we get

$$t_2=\tau-\frac{\left(\tau-T_2\right)\left(\tau-t_1\right)}{\tau-T_1}=-10°C$$

Answer: $-10°C$

11.4. (7 баллов) One day Vova went into a hat shop and tried on a funny tall hat. When he put it on, it turned out that the top edge of the hat was above Vova's eye level by h = 25 cm. He approached a flat mirror standing on the floor and noticed that he could not see the floor in the reflection, even though he could see his whole body.

[4] At what distance from the floor does Vova see the top of the hat, given that he no longer saw his eyes in the mirror when he stepped back to a distance 2.5 times further away? Give answer in centimeters.

(*Cherenkov A.A.*)

Answer: 0.17.
Solution.

1) Vova did not see the floor in the mirror, which means that the ray coming from the bottom of his shoes and hitting his eye was reflected from the lower edge of the mirror and traveled horizontally at a very low height above the floor before being reflected. This is only possible if the plane of the mirror forms an angle φ with the vertical. Based on this, construct the images of Vova AB and a hat BC in the mirror (Fig. 1). The image $B'C'$ of the hat forms an angle 2φ with the vertical, therefore

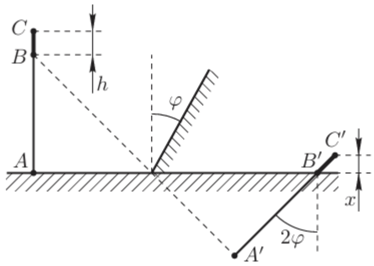


Figure 1

$$x = h\cos 2\varphi = h \cdot \frac{1 - tg^2\varphi}{1 + tg^2\varphi}.$$

2)To find the angle φ, let us consider the additional position when Vova stopped seeing himself below eye level (Fig. 2), moving away from the mirror by a distance $L = nl$, where l is the distance from him to the mirror in the first case, n = 2.5. In the second position, the ray coming from the eye and hitting the eye was reflected from the lower edge of the mirror, since otherwise the boy would have seen his image below eye level. Since this ray returns to its starting point, it is perpendicular to the mirror. Let H be the height of Vova's eyes above the floor level, then from geometric considerations in the first and second positions we express

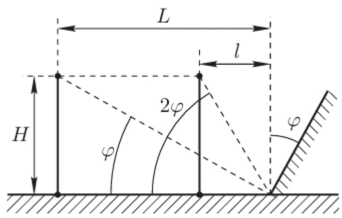


Figure 2

$$\frac{H}{l} = tg2\varphi = \frac{2tg\varphi}{1 - tg^2\varphi}, \qquad tg\phi = \frac{H}{L} = \frac{H}{nl},$$

From which

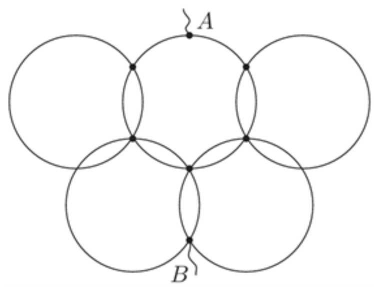
$$tg^2\varphi = 1 - \frac{2}{n}.$$

Substituting this expression into the formula for x, we find

$$x = \frac{h}{n - 1} \approx 17\ cm.$$

Answer: 17 cm

11.5. (6 points) For the opening of the Olympic Games, an amateur electrician assembled a luminous garland in the shape of the Olympic emblem from five identical rings with a resistance of r = 33 Ω. The rings were soldered together at the points marked by bold dots in the figure. The solder joints and point A divide the central ring into 6 equal parts.



[5] What is the resistance of the garland between points A and B?

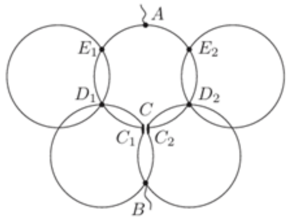
Comment. Neglect any resistance at the solder joints. (*Cherenkov A.A.*)

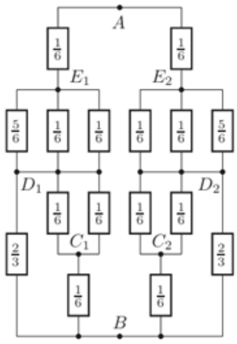
Answer: 7.

Solution. Due to the symmetry of the circuit relative to the axis AB, the point C can be divided into two (see fig.). Let us draw an equivalent circuit (see fig.), in which the corresponding points are marked with letters, and the numerical values of the resistances are given in units of r. Using the properties of series and parallel connections, find the resistances of the sections of the circuit.

$$R_{AE_1} = R_{AE_2} = \frac{1}{6}r,$$

$$R_{E_1D_1} = R_{E_2D_2} = \frac{1}{\frac{1}{\frac{1}{6}r} + \frac{1}{\frac{1}{6}r} + \frac{1}{\frac{5}{6}r}} = \frac{5}{66}r$$





Note that two resistances of 1/6r connected in parallel have a total resistance of 1/12r. Then

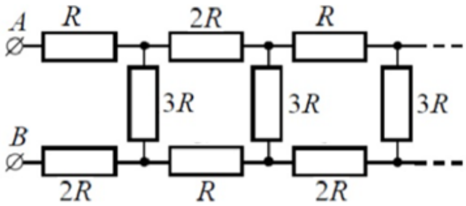
$$R_{D_1B} = R_{D_2B} = \frac{1}{\frac{1}{\frac{2}{3}r} + \frac{1}{\frac{1}{12}r + \frac{1}{6}r}} = \frac{2}{11}r.$$

Find the desired total resistance:

$$R = \frac{1}{2}\left(R_{AE_1} + R_{E_1D_1} + R_{D_1B}\right) = \frac{7}{33}r = 7\Omega.$$

Answer: 7 Ω

11.6. (6 баллов) At an extracurricular physics class, Alexey studied a problem concerning the determination of the resistance between points A and B of an infinite electrical circuit (see the figure). He decided to verify the obtained result experimentally by assembling an actual circuit using only resistors of 10 Ω resistance.



[6] What is the minimum number of such resistors he must solder together so that the experimental result differs from the theoretical value by no more than 20%?

(*Yakovlev A.B.*)

Answer: 12.

Solution. 1) Note that the circuit consists of blocks of resistors $R - 3R - 2R$ connected in series. Then each subsequent block is connected in parallel to the resistor $3R$. Let the resistance of a chain of n blocks be R_n. Then, when adding a new block to the circuit, we obtain that the resistances R_n and $3R$ are connected in parallel, and their total resistance is

$$R_0 = \frac{3RR_n}{3R + R_n}.$$

This section is connected in series with resistances R and $2R$, hence

$$R_{n+1} = R + 2R + R_0 = 3R\left(1 + \frac{R_n}{3R + R_n}\right).$$

2) With an unlimited increase in the number of blocks connected to the circuit, we obtain

$$R_n, R_{n+1} \underset{n\to\infty}{\to} R_\infty,$$

where R_∞ is the resistance of the entire circuit. Then, letting n tend to infinity in (1), we obtain an equation for determining R_∞:

$$R_\infty = 3R\left(1 + \frac{R_\infty}{3R + R_\infty}\right).$$

Divide the entire equation by $3R$, divide the numerator and denominator of the fraction in parentheses by R_∞, and introduce the substitution

$$x = \frac{3R}{R_\infty} > 0.$$

We obtain

$$\frac{1}{x} = 1 + \frac{1}{1+x}.$$

Multiply the equation by $x(x+1) \neq 0$:

$$x + 1 = x^2 + x + x.$$

After rearranging, we obtain a quadratic equation for determining x:

$$x^2 + x - 1 = 0.$$

Taking into account that $x > 0$, we get

$$x = \frac{-1+\sqrt{1+4}}{2} = \frac{-1+\sqrt{5}}{2}.$$

By definition, $x = \frac{3R}{R_\infty}$, therefore

$$R_\infty = \frac{3R}{x} = \frac{3R}{\frac{-1+\sqrt{5}}{2}} = \frac{6R}{\sqrt{5}-1}.$$

Multiplying numerator and denominator by $\left(\sqrt{5}+1\right)$, we have

$$R_\infty = \frac{3R}{2}\left(\sqrt{5}+1\right).$$

3) We use the recurrence relation for R_n to find the equivalent resistance of a circuit consisting of n blocks. For $n = 1$ we have one block, whose resistance is $R_1 = 6R$. Note that $R_1/R_\infty \approx 1.23$, that is, the error is greater than 20%. Next we find that $R_2 = 5R$, and $R_2/R_\infty \approx 1.03$, that is, the error is 3%. Thus, two blocks are sufficient to satisfy the condition of the problem. Each such block consists of resistors $R - 3R - 2R$, which are equivalent to 6 resistors of resistance R. Hence, a total of 12 resistors is required.
Answer: 12.

11.7. (7 баллов) Perpendicular lines are drawn through the vertices of an equilateral triangle ABC with side length 20 cm, along which three coloured light bulbs—red, green, and blue—can move. Initially, all the bulbs were located at the vertices of triangle ABC and had the same velocity of 0.5 m/s directed in the same direction. After that, the red bulb continued moving at a constant velocity, while the green and blue bulbs moved with uniform accelerations of 2 m/s^2 and –3 m/s^2, respectively.

[7] Determine the velocity of the centre of mass of the triangle formed by the bulbs at the moment t = 3 seconds.

(*Yakovlev A.B.*)

Answer: -0.5.

Solution. 1) Let the mass of each bulb be m. All the bulbs move strictly vertically, so the center of mass is at rest horizontally, and we are interested only in the vertical component of its velocity.
2) First, all three bulbs rose to a height h_0; at the end of the ascent all three bulbs have the same speed $v_0 = 0.5\ m/s$, directed upward.
3) Let us describe the further motion of the bulbs. To do this, we start the time count from zero and direct the Oz axis vertically upward. The equations of motion of the bulbs are:

$$\begin{cases} z_1(t) = h_0 + v_0 t, \\ z_2(t) = h_0 + v_0 t + \frac{a_2 t^2}{2}, \\ z_3(t) = h_0 + v_0 t + \frac{a_3 t^2}{2}, \end{cases}$$

where the indices 1,2,3 refer to the red, blue, and green bulb respectively.
4) We write the expression for the coordinate z_c of the center of mass:

$$z_c(t) = \frac{mz_1(t) + mz_2(t) + mz_3(t)}{3m} = \frac{z_1(t) + z_2(t) + z_3(t)}{3}.$$

The speed of the center of mass is the time derivative of its coordinate:

$$v_c(t) = \dot{z}_c(t) = \frac{\dot{z}_1(t) + \dot{z}_2(t) + \dot{z}_3(t)}{3} = \frac{1}{3}\left(v_0 + (v_0 + a_2 t) + (v_0 + a_3 t)\right) = v_0 + \frac{(a_2 + a_3)\, t}{3}.$$

From the condition of the problem, we are interested in the speed at $t = 3$ s:

$$v_c(3) = v_0 + \frac{3(a_2 + a_3)}{3} = v_0 + (a_2 + a_3) = 0.5 + 2 - 3 = -0.5\ m/s.$$

Answer: -0.5 m/s.