International Mathematical Olympiad
“Formula of Unity” / “The Third Millennium”
Year 2025/2026. Qualifying round

Problems for grade R5

Please hand in your paper in electronic form (e. g. as a doc-file with text or as a scan), some details are at the page formulo.org/en/olymp/2025-math-en/.

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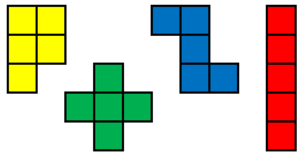
1. Is it possible to place all nine nonzero digits $1, 2, \ldots, 9$ into the equality $* + * + * + * + * + * + * = **$ so that the equation is true? (On the left, there are seven one-digit numbers; on the right, it is a two-digit number.) (*S. Pavlov*)

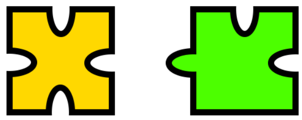
2. Russian programmers invented a new game called “Tetris-5”. The pieces (shown in the figure) can be placed on a square grid in any orientation so that they do not overlap (as in classical “Tetris”, pieces can be rotated but not flipped).



a) Prove that it is possible to cover an 8×8 board if the four corner cells are removed.
b) Prove that it is impossible to cover a 2026×2026 board if the four corner cells are removed. (*L. Koreshkova*)

3. If you prepend the digit 2 to a natural number (that is, write it in front of the number, for example, $13 \longrightarrow 213$), the result is the square of that number. What can this number be? Find all possible solutions and prove that no others exist. (*P. Mulenko*)

4. Little Mary received a rectangular puzzle for her birthday, made up of several equally sized square pieces, each having indentations or protrusions on its sides (the overall puzzle outline is rectangular without any gaps).



It is known that this set contains the two pieces shown in the figure. What is the smallest number of pieces that the puzzle can have? Don’t forget to provide an example and explain why there cannot be fewer pieces. (*P. Mulenko*)

5. An ant crawls through a tunnel from the left edge of an ant farm to the right (the width of the farm is 28 cm). The tunnel consists of horizontal sections, climbs, and descents (all climbs and descents have the same slope). On a climb, the ant crawls at 3 cm/min, on horizontal sections at 4 cm/min. The entire journey took the ant 7 minutes. The tunnel is not completely horizontal, but at the end the ant is at the same height as at the start (i.e., the total ascent equals the total descent). Find the ant’s rate on a descent (in cm/min). (*P. Mulenko*)

6. The parrot Roza knows all four sounds of her name (R, O, Z, and A) and can pronounce “words” consisting of 1 to 3 sounds. However, it cannot pronounce the same sound twice in a row. How many different “words” can Roza pronounce? (*O. Tretyakova*)

7. All ten digits were divided into 5 pairs, and for each pair the difference was calculated (subtracting the smaller digit from the larger one). What is the highest power of two that the product of all these differences can equal? (A “power of two” is a number obtained by multiplying two by itself several times; for example, the fifth power of two is $2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 = 32$.) (*S. Pavlov*)

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Problems for grade R6

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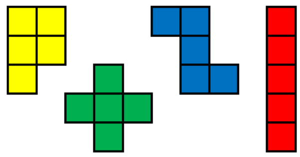
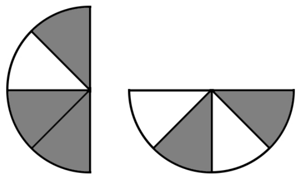
1. Is it possible to place all nine nonzero digits $1, 2, \ldots, 9$ into the equality $* + * + * + * + * + * + * = **$ so that the equation is true? (On the left, there are seven one-digit numbers; on the right, it is a two-digit number.) (*S. Pavlov*)

2. Russian programmers invented a new game called “Tetris-5”. The pieces (shown in the figure) can be placed on a square grid in any orientation so that they do not overlap (as in classical “Tetris”, pieces can be rotated but not flipped).

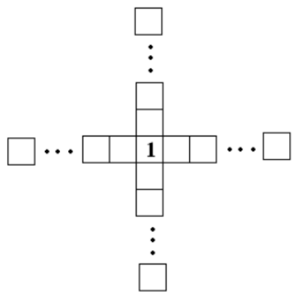


a) Prove that it is possible to cover an 8×8 board if the four corner cells are removed.
b) Prove that it is impossible to cover a 2026×2026 board if the four corner cells are removed.
(*L. Koreshkova*)

3. A transparent circular disk is divided into 8 equal sectors. Some of the sectors are shaded. If the disk is folded in half along the vertical axis, three shaded sectors are visible. If the disk is folded in half along the horizontal axis, two shaded sectors are visible. How many sectors are shaded in total? Find all possible answers and prove that there are no others. (*P. Mulenko*)



4. On a cross-shaped game board made of square cells (see the figure), two players take turns making moves. At the beginning of the game, only the central cell is occupied, and it contains the number 1. A move consists in writing the next four consecutive natural numbers in the four cells adjacent to the already occupied ones — one number in each direction of the cross — so that any two numbers placed in neighboring cells are coprime (that is, they do not have common divisors greater than 1). The first player writes the numbers 2, 3, 4, 5; then the second player writes 6, 7, 8, 9; and so on. If one of the players cannot make a move he or she loses. Which player can guarantee a win with perfect play? (*S. Pavlov*)



5. The parrot Roza knows all four sounds of her name (R, O, Z, and A) and can pronounce “words” consisting of 1 to 4 sounds. However, it cannot pronounce the same sound twice in a row. How many different “words” can Roza pronounce? (*O. Tretyakova*)

6. Irene has 289 coins from several countries. She distributed them equally into several boxes, and in each box there are coins (more than one) from only one country. It is known that Turkish coins make up more than 6% of the total, Spanish coins more than 12%, Ecuadorian coins more than 24%, and Russian coins more than 36%. How many Chinese coins can Irene have? Find all possibilities. (*L. Koreshkova*)

7. In the Land of Oz, all cities are numbered from 1 to N, where N is even but not divisible by 4. Each pair of cities is connected either by a yellow brick road or by a green emerald road. The Great Wizard decided to renumber the cities so that pairs of numbers originally connected by a yellow road would now be connected by a green road, and vice versa. Is it possible for the Wizard to achieve this? (*L. Koreshkova*)

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Problems for grade R7

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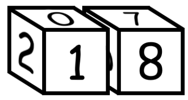
1. An ant crawls through a tunnel from the left edge of an ant farm to the right (the width of the farm is 28 cm). The tunnel consists of horizontal sections, climbs, and descents (all climbs and descents have the same slope). On a climb, the ant crawls at 3 cm/min, on horizontal sections at 4 cm/min. The entire journey took the ant 7 minutes. The tunnel is not completely horizontal, but at the end the ant is at the same height as at the start (i.e., the total ascent equals the total descent). Find the ant’s rate on a descent (in cm/min). (*P. Mulenko*)

2. Each square of a 3×5 grid contains a positive integer. All numbers are distinct, but the sums of the numbers in all rows are equal, and the sums of the numbers in all columns are also equal. What is the smallest possible sum of all numbers in the grid? (*A. Tesler*)

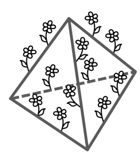
3. Pauly has many wooden cubes and sticker digits. Using two cubes, one can make a souvenir calendar: each face of both cubes must be covered with a sticker so that, by arranging the cubes appropriately, any day of the month can be displayed (that is, any number from 01 to 31; an example of the number 18 is shown in the picture). Pauly wants to make a special version of such a calendar for each of his friends. How many different calendars can he make?


Pauly considers two calendars different if in one of them there is a cube with a certain set of stickers, while in the other there is no cube with the same set of stickers. The arrangement of digits on the cube faces is not taken into account. (*M. Karlukova*)

4. Irene has 289 coins from several countries. She distributed them equally into several boxes, and in each box there are coins (more than one) from only one country. It is known that Turkish coins make up more than 6% of the total, Spanish coins more than 12%, Ecuadorian coins more than 24%, and Russian coins more than 36%. How many Chinese coins can Irene have? Find all possibilities. (*L. Koreshkova*)

5. If an integer $n > 1$ is entered into a magic machine, it builds a square grid of size $n \times n$, removes a single 1×1 square from it, and then adds 1×2 dominoes until the total area of the figures becomes equal to the area of some square with an integer side. The machine then returns the side length of this new square. Kate exchanged cards with the machine one hundred times and received 2025. What number did she start with? (*P. Mulenko*)

6. In the Land of Oz, all cities are numbered from 1 to N, where N is even but not divisible by 4. Each pair of cities is connected either by a yellow brick road or by a green emerald road. The Great Wizard decided to renumber the cities so that pairs of numbers originally connected by a yellow road would now be connected by a green road, and vice versa. Is it possible for the Wizard to achieve this? (*L. Koreshkova*)

7. A hill has the shape of a regular triangular pyramid with all edges equal to 3 m. On each edge there are two flowers growing at the points that divide the edge into three equal parts. A bee lands on one of the flowers and wants to visit all 12 flowers along the shortest possible route. What is the length of this route? (*L. Koreshkova*)


International Mathematical Olympiad
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Year 2025/2026. Qualifying round

Problems for grade R8

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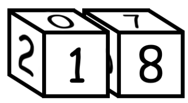
1. Peter drew a rhombus on a square sheet of paper; the rhombus is not a square. Will Victor always be able to draw, on the same sheet, a square such that two adjacent vertices of the square coincide with two adjacent vertices of the rhombus? (*A. Tesler*)

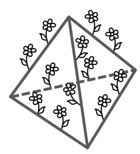
2. In a $n \times n$ grid, some squares contain checkers (no more than 1 checker in a square) in such a way that each row, each column, and each of the two main diagonals contains exactly two checkers. For which values of n is this possible? (*S. Pavlov*)

3. Paul has many wooden cubes and sticker digits. Using two cubes, one can make a souvenir calendar: each face of both cubes must be covered with a sticker so that, by arranging the cubes appropriately, any day of the month can be displayed (that is, any number from 01 to 31; an example of the number 18 is shown in the picture). Paul plans to start a business producing souvenir calendars, and he wants each product to be unique. How many different calendars can he make?



   Paul considers two calendars identical if for every cube in the first calendar there exists an identical cube in the second one. Two cubes are considered identical if they can be placed side by side so that each corresponding face bears the same digit sticker, possibly with the digit rotated on the face. (*M. Karlukova*)

4. A hill has the shape of a regular triangular pyramid with all edges equal to 3 m. On each edge there are two flowers growing at the points that divide the edge into three equal parts. A bee lands on one of the flowers and wants to visit all 12 flowers along the shortest possible route. What is the length of this route? (*L. Koreshkova*)



5. If you prepend a single digit to a natural number (that is, write one digit in front of the number, for example, 13 $\to$ 213), the result is the square of that number. Find the largest such number. (*P. Mulenko*)

6. Consider an obtuse triangle ABC with distinct integer side lengths. Through the vertex of the obtuse angle A, a line is drawn parallel to BC, and the points of intersection of this line with the angle bisectors of B and C are marked as P and Q. If $BC = 4$, what is the length of PQ? (*P. Mulenko*)

7. A quiz consists of several questions (more than one), and each question has the same number of answer choices. If you choose a wrong answer on any question, you lose. However, you have one "extra life" for the entire quiz: the first time you choose a wrong answer, you do not get eliminated and can try again on the same question (if the second answer is also wrong, you lose). The probability of passing the quiz by guessing answers at random is $2/81$. How many questions are in the quiz, and how many answer choices does each question have? (*P. Mulenko*)

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Problems for grade R9

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1. Irene has 289 coins from several countries. She distributed them equally into several boxes, and in each box there are coins (more than one) from only one country. It is known that Turkish coins make up more than 6% of the total, Spanish coins more than 12%, Ecuadorian coins more than 24%, and Russian coins more than 36%. How many Chinese coins can Irene have? Find all possibilities. (*L. Koreshkova*)

2. Each square of a 3×5 grid contains a positive integer. All numbers are distinct, but the sums of the numbers in all rows are equal, and the sums of the numbers in all columns are also equal. What is the smallest possible sum of all numbers in the grid? (*A. Tesler*)

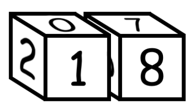
3. On an island with a radius of 40 km, there are several wells. A well is called *remote* if there is no sea or other well within 25 km of it. What is the maximum number of remote wells that can be located on the island? (*A. Tesler*)

4. The GCD (greatest common divisor) of $2025n + 1$ and $5202n + 1$, where n is a natural number, is odd. Find all possible values of this CGD, and prove that no other values are possible. (*S. Pavlov*)

5. Consider an obtuse triangle ABC with distinct integer side lengths. Through the vertex of the obtuse angle A, a line is drawn parallel to BC, and the points of intersection of this line with the angle bisectors of B and C are marked as P and Q. If $BC = 4$, what is the length of PQ? (*P. Mulenko*)

6. A quiz consists of several questions (more than one), and each question has the same number of answer choices. If you choose a wrong answer on any question, you lose. However, you have one "extra life" for the entire quiz: the first time you choose a wrong answer, you do not get eliminated and can try again on the same question (if the second answer is also wrong, you lose). The probability of passing the quiz by guessing answers at random is $2/81$. How many questions are in the quiz, and how many answer choices does each question have? (*P. Mulenko*)

7. Mr. Paul owns a factory that produces souvenir calendars. Each calendar is made of two cubes and digit stickers: each face of both cubes must be covered with a sticker so that, by arranging the cubes appropriately, any day of the month can be displayed (that is, any number from 01 to 31; an example of the number 18 is shown in the picture). The factory prides itself on the fact that every product is different from the others. How many distinct calendars can the factory produce?



Two calendars are considered identical if for each cube in the first calendar there is an identical cube in the second. Two cubes are considered identical if they can be placed side by side so that each corresponding face bears the same digit sticker in the same position. Stickers with the digits 0 and 8 have a center symmetry, while all others do not. (*M. Karlukova*)

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Problems for grade R10

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1. List all years of the current decade (from 2021 to 2030) that can be represented as the sum of a one-digit, a two-digit, a three-digit, and a four-digit number, using each digit exactly once. (*S. Pavlov*)

2. A 13×13 chessboard consists of 169 unit squares. Serge placed four queens on it so that none of them attack each other. It turned out that the centers of the squares containing the queens form a rhombus. Is it necessary for this rhombus to be a square? (*S. Pavlov*)

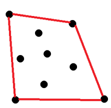
3. Each square of a 4×5 grid contains a positive integer. All numbers are distinct, but the sums of the numbers in all rows are equal, and the sums of the numbers in all columns are also equal. What is the smallest possible sum of all numbers in the grid? (*A. Tesler*)

4. In a circle ω, a hexagon $ABCDEF$ is inscribed such that $AB = BC = CD$. Segment BE intersects CF and DF at points G and H, respectively, and segment AE intersects CF and BF at points J and K, respectively. It turns out that lines GK and HJ intersect at point L on ω, and also intersect the circle at points M and N. Prove that $MN = AB$. (*O. Pyayve*)

5. The parrot Roza knows all four sounds of her name (R, O, Z, and A) and can pronounce "words" of length 1 to n sounds. However, it cannot pronounce the same sound twice in a row. How many different "words" can Roza pronounce? Write the answer in closed form (without ellipses and $\sum$ sign). (*O. Tretyakova*)

6. Three distinct acute-angled triangles (with no shared vertices) are inscribed in a circle. Prove that it is possible to choose one vertex from each triangle such that the triangle formed by these three points is not obtuse. (*L. Koreshkova*)

7. Paul and Barbara are playing the following game. On each turn, a player marks a point on the plane, until 2025 points have been marked (Paul starts and also makes the last move). Then Barbara must pay Paul as many dollars as there are vertices of the convex hull of the resulting set of points. For which maximum N does Paul have a strategy that guarantees him at least \$$N$, regardless of how Barbara plays?
   Convex hull of a finite set of points is the minimal (by inclusion) convex polygon containing all these points. At the picture, a 9-point set and its quadrilateral convex hull are shown. (*A. Tesler*)



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Problems for grade R11

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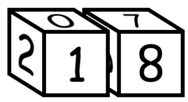
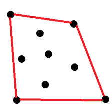
1. All ten digits were divided into 5 pairs, and for each pair the difference was calculated (subtracting the smaller digit from the larger one). What is the highest power of two that the product of all these differences can equal? (*S. Pavlov*)
2. In a circle ω, a hexagon $ABCDEF$ is inscribed such that $AB = BC = CD$. Segment BE intersects CF and DF at points G and H, respectively, and segment AE intersects CF and BF at points J and K, respectively. It turns out that lines GK and HJ intersect at point L on ω, and also intersect the circle at points M and N. Prove that $MN = AB$. (*O. Pyayve*)
3. One rectangle is a cross-section of two different cubes. What is the greatest possible ratio of the volumes of these cubes? (*A. Tesler*)
4. Each square of a 4×2025 grid contains a positive integer. All numbers are distinct, but the sums of the numbers in all rows are equal, and the sums of the numbers in all columns are also equal. What is the minimal possible sum of all numbers in the grid? (*A. Tesler*)
5. If an integer $n > 1$ is entered into a magic machine, it builds a square grid of size $n \times n$, removes a single 1×1 square from it, and keeps drawing such figures until the total area of all drawn figures becomes equal to the area of some square with an integer side. The machine then returns the side length of this new square. For example, when given the number 5, the machine builds a 5×5 square without one square, repeats this figure 6 times (obtaining 144 squares in total, which equals 12^2), and outputs the number 12. If the machine performed this operation 10 times, could the number of digits on the final card be 1024 times greater than on the original one? (*P. Mulenko, A. Tesler*)
6. Mr. Paul owns a factory that produces souvenir calendars. Each calendar is made of two cubes and digit stickers: each face of both cubes must be covered with a sticker so that, by arranging the cubes appropriately, any day of the month can be displayed (that is, any number from 01 to 31; an example of the number 18 is shown in the picture). The factory prides itself on the fact that every product is different from the others. How many distinct calendars can the factory produce?
   

   Two calendars are considered identical if for each cube in the first calendar there is an identical cube in the second. Two cubes are considered identical if they can be placed side by side so that each corresponding face bears the same digit sticker in the same position. Stickers with the digits 0 and 8 have a center symmetry, while all others do not. (*M. Karlukova*)
7. Paul and Barbara are playing the following game. On each turn, a player marks a point on the plane, until 2025 points have been marked (Paul starts and also makes the last move). Then Barbara must pay Paul as many dollars as there are vertices of the convex hull of the resulting set of points. For which maximum N does Paul have a strategy that guarantees him at least \$$N$, regardless of how Barbara plays?
   Convex hull of a finite set of points is the minimal (by inclusion) convex polygon containing all these points. At the picture, a 9-point set and its quadrilateral convex hull are shown.
   
   (*A. Tesler*)