Solutions to problems for grade R8

8.1. (9 points) A cube with an edge length of a = 5.00 cm and a mass of m = 200.0 g floats in a rectangular vessel (length 20.0 cm, width 10.0 cm, height 20.0 cm) filled with a liquid of density 2 g/cm^3.

[1] How much water (density 1.00 g/cm^3) needs to be added to the vessel for the cube to start sinking?

Comment. The initial amount of liquid is 2.45 liters. Assume that the acceleration of free fall is 10.0 m/s^2. Give the answer in liters. (*Yakovlev A.B.*)

Answer: a) If the liquids are miscible, the problem has no solution. б) If the liquids are immiscible, 0.175 l of water must be added.

Solution. 1) Two cases are possible: the liquids are either miscible or immiscible. Let's consider both cases and perform some preliminary calculations:

Density of the cube: $\rho = \frac{m}{a^3} = \frac{0.200}{(0.0500)^3} = 1.60 g/cm^3$.

Volume of the vessel: $V_v = 20{,}0 \times 10{,}0 \times 20{,}0 = 4000 cm^3 = 4{,}00 l$

Area of the base of the vessel: $S = 20{,}0 \times 10{,}0 = 200 cm^2$

2) Miscible liquids. The cube will sink, if the average density of the liquid mixture is less than the cube's density:

$$\frac{\rho_1 V_0 + \rho_2 V}{V_0 + V} < \rho.$$

where ρ_1,ρ_2 - densities of liquid and water respectively, V_0,V - initial volume occupied by liquid and volume of water to be added to it. Solving this inequality for V:

$$V > \frac{\rho_1 - \rho}{\rho - \rho_2} V_0 = 1{,}63 l.$$

Note that in this case the total volume of liquids is greater than the volume of the vessel. Therefore this case is not possible.

3) Immiscible liquids. The cube will sink if enough water is added so that the liquid level reaches the top face of the cube.

From Archimedes' principle we get:

$$\rho_1 g V_3 = mg$$

From where we obtain the volume of the immersed part of the cube:

$$V_3 = \frac{m}{\rho_1}$$

So the immersion depth is equal to:

$$h = \frac{V}{a^2} = \frac{m}{\rho_1 a^2}$$

Height of protruding part:

$$H = a - h = a - \frac{m}{\rho_1 a^2}$$

Then the required volume of water, taking into account that the cube occupies part of the volume:
$V = SH - a^2 H = (S - a^2)(a - \frac{m}{\rho_1 a^2}) = 175\ cm^3 = 0{,}175$ l

8.2. (8 points) A group of light sport aircraft must take off from point A at the same time, drop a pennant at point B, and return back. The distance from A to B is 450 km. Each aircraft has a fuel tank capacity of 240 liters, and 1 liter of fuel allows it to travel 2.5 km at a cruising (most fuel-efficient) speed of 300 km/h.

[2] What is the minimum number of aircraft in the group required for all of them to complete the mission and return to A, assuming instantaneous fuel transfer between aircraft is possible?

[3] What is the total flight time of all aircraft?

(*Gamov G.A., Yakovlev A.B.*)

Answer: 3.
Answer: 6 h.
Solution. 1) Let's define the variables:
Fuel tank capacity:V = 240 l
Fuel consumption: $\alpha = \frac{1}{2{,}5}$ l/km
Aircraft speed: v = 300 km/h
Distance from A to B and back: S = 900 km
Let's solve the problem iteratively, first considering one aircraft, then two, and so on.
2) Let one plane fly. The maximum distance it can cover:

$$S_0 = \frac{V}{\alpha} = 600km$$

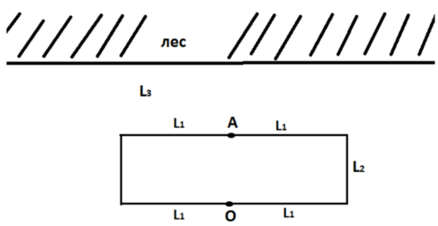
Таким образом, самолет не долетит $\Delta S = S - S_0 = 300km$ more, for what he needs $\Delta V = \Delta S\alpha = 120l$ of additional fuel, i.e. half a tank.
3) Let two aircraft fly. It is clear that one of the aircraft must fly ΔS, transfer half a tank of fuel to the other aircraft and turn around. Otherwise, if it flies less or more than ΔS, it will not be able to transfer the necessary amount of fuel to the second aircraft to complete the flight to A. However, by this moment it will have already used up half a tank of fuel, so if it gives the second half of the tank, it will not be able to return to A itself.
4) Let three aircraft fly. Consider the following strategy. Let the aircraft fly $\Delta S/2$, then they use up a quarter of their tank each. At this moment, one of the planes pumps its fuel to the other planes up to a full tank. Then it will have exactly a quarter of a tank left, and this is enough to return back. Next, the remaining planes fly another $\Delta S/2$, again using a quarter of a tank each. One of the planes pumps its fuel to the second plane until it has a full tank. Then it will have exactly half a tank left, which is just enough for it to return back to A. The second aircraft will have a full tank, on which it will fly the remaining distance.
5) Let's calculate the total number of hours the aircraft will be in the air. Total distance travelled:

$$L = 2 * (\Delta S/2 + \Delta S + S/2) = 3\Delta S + S$$

Which means the total time in the air:

$$T = \frac{L}{v} = \frac{3\Delta S + S}{v} = 6h$$

8.3. (8 points) Sasha, Yura, Alexey, and Anya are playing hide and seek on the outskirts of the village (see the figure). The given parameters in the figure are: $L_1 = 10$ m, $L_2 = 5$ m, $L_3 = 8$ m. The seeker, Sasha, starts behind the barn at point O, while Yura, Alexey, and Anya are on the other side of the barn at point A. How fast should Sasha count to ten before he starts searching to



[4] see all players before they manage to hide in the forest ("лес" on the figure),

[5] see at least one of the players before they manage to hide in the forest?

Comment. It is known, that players can run with the following velocities: Sasha — 5 m/s, Yura —

3.5 m/s, Alexey — 3 m/s, Anya — 2 m/s. (*Yakovlev A.B., Cherenkov A.A.*)

Answer: Sasha won't be able to see all the players.

Answer: 1,2 s (1 s).

Solution. 1) The corners of the garage, starting from the top left and going clockwise, are labelled K,L,M,N, the point of the forest directly opposite to point A is labelled P. In order for Sasha to see the other players he needs to choose the right strategy. Since the other players do not know in which direction Sasha will start moving, it is advantageous for them not to deviate from the straight-line route to the forest, i.e. they will run along the segment AP. Thus, Sasha needs to see the point P as soon as possible. Due to the symmetry of the problem, we can assume that Sasha will start moving to the left. Let's find the blind spot to the left of the garage, from which point P is not visible. To do this, let's extend PK to intersection with ON at point R. Then the triangle KRN defines the blind spot. It is clear that in any case Sasha will start moving along ON, and when he is at point N, he needs to get out of the blind spot as soon as possible. Therefore, Sasha will move along the shortest distance from the point N to the line KR - along the height NH of the triangle KRN.

2) Let's now calculate the distance Sasha must cover. From the similarity of triangles APK and KRN, we get

$$\frac{AP}{AK} = \frac{KN}{NR}$$

so

$$NR = \frac{KN * AK}{AP} = l_1\frac{l_2}{l_3}$$

In triangle KRN, by the Pythagorean Theorem:

$$RK = \sqrt{NR^2 + NK^2} = l_2\sqrt{1 + \frac{l_1^2}{l_3^2}}$$

Let's find NH by calculating the area of triangle KRN in two ways:

$$S = \frac{1}{2}NH * RK = \frac{1}{2}NR * NK$$

From this, we derive:

$$NH = \frac{NR * NK}{RK} = \frac{l_1 l_2}{\sqrt{l_1^2 + l_3^2}}$$

So, the total distance Sasha must cover is:

$$S = ON + NH = l_1(1 + \frac{l_2}{\sqrt{l_1^2 + l_3^2}})$$

The time it takes for Sasha to cover this distance is:

$$t_1 = \frac{S}{v_s} = 2{,}8s$$

3) For Sasha to see all the players, he must cover distance S before the fastest player — Yura — reaches the forest, covering distance $PQ = l_3$. Thus, Yura will reach the forest in:

$$t_2 = \frac{l_3}{v_y} = 8/3{,}5 = 2{,}3s$$

Since $t_1 > t_2$, even if Sasha doesn't count and immediately starts running, he still won't be able to see all the players because Yura will manage to hide in the forest.

4) For Sasha to see at least one player, he must cover distance S before the slowest player — Anya

— reaches the forest, also covering distance $PQ = l_3$. Thus, Anya will reach the forest in:

$$t_3 = \frac{l_3}{v_a} = 8/2 = 4s$$

Thus, Sasha has $\Delta t = t_3 - t_1 = 1{,}2s$, so he should count no longer than 1.2 s.
Note: An alternative, more straightforward (but non-optimal) solution is also accepted, where Sasha moves entirely along the garage.
1) For Sasha to see all the players, he must cover distance $S = ON + NK = l_1 + l_2$ before the fastest player — Yura — reaches the forest by the shortest route, covering distance l_3. Thus, the time it takes Sasha to walk the distance S:

$$t_1 = \frac{S}{v_s} = 15/5 = 3s$$

Yura reaches the forest in:

$$t_2 = \frac{l_3}{v_y} = 8/3{,}5 = 16/7s$$

Since $t_1 > t_2$, even if Sasha doesn't count and immediately starts running, he still won't be able to see all the players because Yura will manage to hide in the forest.
2) For Sasha to see at least one player, he must cover distance S before the slowest player — Anya — reaches the forest, also covering distance $PQ = l_3$. Thus, Anya will reach the forest in:

$$t_3 = \frac{l_3}{v_a} = 8/2 = 4s$$

Thus, Sasha has $\Delta t = t_3 - t_1 = 1s$, so he should count no longer than 1s.

8.4. (5 points) When heating a body from 20^{o}C to 50^{o}C, it is required to supply 1200 J of heat, from 50^{o}C to 100^{o}C 2000 J of heat are required , from 100^{o}C to 140^{o}C — 800 J, and from 140^{o}C to 165^{o}C — 500 J.
[6] What is the minimum amount of heat must be supplied to the body to heat it from 90^{o}C to 130^{o}C, assuming that the heat capacity changes in the simplest possible way? (*Yakovlev A.B.*)

Answer: 1000 J.
Solution. 1) To determine the dependence of heat capacity on temperature let's calculate average heat capacities at each of the given temperature intervals:

$$\begin{cases} C_1 = \frac{Q_1}{\Delta T_1} = \frac{1200}{50-20} = 40J/K, \quad T \in [20{,}50) \\ C_2 = \frac{Q_2}{\Delta T_2} = \frac{2000}{100-50} = 40J/K, \quad T \in [50{,}100) \\ C_3 = \frac{Q_3}{\Delta T_3} = \frac{800}{140-100} = 20J/K, \quad T \in [100{,}140) \\ C_4 = \frac{Q_4}{\Delta T_4} = \frac{500}{165-140} = 20J/K, \quad T \in [140{,}165) \end{cases}$$

Since the average heat capacities coincide at the first two temperature intervals and at the last two, we can conclude that at these intervals the heat capacity is constant (by the condition it changes in the simplest way). We obtain:

$$C = \begin{cases} C_{01} = 40J/K, \quad T \in [20{,}100) \\ C_{02} = 20J/K, \quad T \in [100{,}165) \end{cases}$$

2) Thus, to calculate the amount of heat required to heat a body from 90 to 130 degrees, let's break the process into two steps - heating from 90 to 100 and heating from 100 to 130 degrees. We obtain that:

$$Q = Q_{90\to100} + Q_{100\to130} = C_{01}\Delta T_{90\to100} + C_{02}\Delta T_{100\to130} = 400 + 600 = 1000J.$$

8.5. (6 points) group of rescuers is searching for a lost child in the forest at night, forming a chain with a distance of 20.0 meters between each rescuer. At regular time intervals the chain stops, sends out a signal, and listens for the child's response.

[7] Determine the maximum allowable time between stops if the speed of the rescuers is 2.00 m/s and the distance at which the child's response can be clearly heard is 30.0 meters. *(Yakovlev A.B.)*

Answer: 28,3 s.

Solution. 1) Let us translate the problem into geometric language. The area in which a lost child can be well heard is a circle of radius $R = 30m$. The chain of rescuers is a straight line with points placed at equal distances $2l = 20m$. The maximum allowable time between stops is achieved when the child is exactly between the two rescuers, and the rescuers at both stops are on the boundary of hearing area, where child can be heard. Thus, the problem reduces to finding the distance between two parallel chords of length 2l in a circle of radius R.

2) Thus, we draw radii at the ends of one of the chords to form an isosceles triangle. Let's draw a height in it, which is half of the required distance between the chords (due to the symmetry of the problem). We find the height by the Pythagorean Theorem, taking into account that the drawn height divides the chord in half. Thus, the required distance is:

$$S = 2h = 2\sqrt{R^2 - l^2}$$

3) Now we can determine the time between the stops:

$$T = \frac{S}{v} = \frac{2\sqrt{R^2 - l^2}}{v} = 28{,}3s$$

Solutions to problems for grade R9

9.1. (8 points) A cube with an edge length of a = 5.00 cm and mass m = 200.0 g floats in a rectangular vessel (length 20.0 cm, width 10.0 cm, height 20.0 cm) filled with a liquid of density $\rho = 2.00 g/cm^3$.

[1] How much water (density 1.00 g/cm^3) must be added to the vessel for the body to start sinking? Give the answer in liters.

Comment. The initial amount of liquid is 2.45 liters. Assume that the acceleration of free fall is 10.0 m/s^2 (*Yakovlev A.B.*)

Answer: a) If the liquids are miscible, the problem has no solution. б) If the liquids are immiscible, 0.175 l of water must be added.

Solution. 1) Two cases are possible: the liquids are either miscible or immiscible. Let's consider both cases and perform some preliminary calculations:
Density of the cube: $\rho = \frac{m}{a^3} = \frac{0.200}{(0.0500)^3} = 1.60 g/cm^3$.
Volume of the vessel: $V_v = 20{,}0 \times 10{,}0 \times 20{,}0 = 4000 cm^3 = 4{,}00 l$
Area of the base of the vessel: $S = 20{,}0 \times 10{,}0 = 200 cm^2$
2) Miscible liquids. The cube will sink, if the average density of the liquid mixture is less than the cube's density:

$$\frac{\rho_1 V_0 + \rho_2 V}{V_0 + V} < \rho.$$

where ρ_1,ρ_2 - densities of liquid and water respectively, V_0,V - initial volume occupied by liquid and volume of water to be added to it. Solving this inequality for V:

$$V > \frac{\rho_1 - \rho}{\rho - \rho_2} V_0 = 1{,}63 l.$$

Note that in this case the total volume of liquids is greater than the volume of the vessel. Therefore this case is not possible.
3) Immiscible liquids. The cube will sink if enough water is added so that the liquid level reaches the top face of the cube.
From Archimedes' principle we get:

$$\rho_1 g V_3 = mg$$

From where we obtain the volume of the immersed part of the cube:

$$V_3 = \frac{m}{\rho_1}$$

So the immersion depth is equal to:

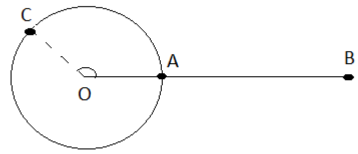
$$h = \frac{V}{a^2} = \frac{m}{\rho_1 a^2}$$

Height of protruding part:

$$H = a - h = a - \frac{m}{\rho_1 a^2}$$

Then the required volume of water, taking into account that the cube occupies part of the volume:
$V = SH - a^2 H = (S - a^2)(a - \frac{m}{\rho_1 a^2}) = 175\ cm^3 = 0{,}175$ l

9.2. (5 points) Kolobok starts moving along a circular path of radius R = 10.0 m from point A with a speed of 0.500 m/s. At the same moment, a fox starts chasing him from point B with a speed of 0.670 m/s.



[2] Will the fox catch Kolobok before he reaches point C?

Comment. The distance between the center of the circle O and point B is L = 25.0 m. The angle AOC is 150 degrees. (*Yakovlev A.B.*)

Answer: Fox manages to catch up with the kolobok.

Solution. 1) The fox will be able to catch up with the kolobok if its travelling time to point C is shorter than the kolobok's travelling time to this point. It is favourable for the fox to move directly from point B to C. Let's calculate this distance using the cosine theorem from triangle OBC:

$$BC = \sqrt{OB^2 + OC^2 - 2OB * OC\cos(\alpha)} = \sqrt{L^2 + R^2 - 2LR\cos(\alpha)},$$

where $\alpha = 150^o$

Then the travelling time of the fox is:

$$t_1 = \frac{BC}{v_1} = \frac{\sqrt{L^2 + R^2 - 2LR\cos\alpha}}{v_1} \approx 50.8s.$$

2) The kolobok must travel along the arc AC, the length of which is calculated as:

$$S = R\alpha,$$

where the angle α is expressed in radians:

$$\theta = \frac{\alpha \times \pi}{180^o} = \frac{5\pi}{6} rad.$$

Then the time of motion of the kolobok:

$$t_2 = \frac{S}{v_2} = \frac{26.2}{0.500} = 52.4s.$$

6) Since $t_2 > t_1$, the fox manages to catch up with the kolobok before it reaches the point C.

9.3. (9 points) In a deep cavity two immiscible liquids are poured (both layers have the same thickness of 20.0 m), with refractive indices of 1.50 (lower layer) and 1.25 (upper layer). A small boat starts moving along the surface of the upper layer at a constant speed of 20.0 m/s. After 1.20 s, from a point vertically aligned with the boat's starting position, a narrow beam of sound is emitted at an angle of 30.0^o to the vertical.

[3] After how many seconds will the sound catch up with the boat?

Comment. The speed of sound in the lower liquid layer is 300.0 m/s. Assume that sound follows the same laws of reflection and refraction as light. (*Yakovlev A.B., Minarsky A.M.*)

Answer: The sound will not catch up with the boat..

Solution. 1) Let's introduce the notations:

$h = 20.0$ m — thicknesses of liquid layers

$n_1 = 1.50$ — refractive index of the lower liquid.

$n_2 = 1.25$ — refractive index of the upper liquid.

$v_K = 20.0$ m/s — speed of the boat.

$t_0 = 1.20$ s — delay before the beam release.

1) Let's calculate the time it takes for the sound to travel from the bottom to the surface. The path is divided into two segments — movement through the first liquid and the second liquid, where sound travels at different speeds. Thus, we get:

$$t = t_1 + t_2 = \frac{s_1}{v_1} + \frac{s_2}{v_2}$$

where s_1 and s_2 - are the distances traveled by the sound in the lower and upper liquids, with speeds v_1 and v_2 respectively

2) By definition, the refractive index of a medium is the ratio of the speed of sound in the air to the speed of sound in that medium:

$$n = \frac{c}{v}$$

So, we get:

$$n_1 v_1 = n_2 v_2$$

Thus, the speed of sound in the upper liquid is:

$$v_2 = \frac{v_1 n_1}{n_2}$$

3) When sound moves from the lower liquid to the upper one, its direction changes according to Snell's law:

$$n_1 \sin\theta_1 = n_2 \sin\theta_2.$$

where - θ_1,θ_2 - are the angles of incidence and refraction at the boundary between the two liquids. From geometrical considerations, it is clear that $\theta_1 = 30^o$
then we have

$$\sin\theta_2 = \sin\theta_1 \frac{n_1}{n_2}$$

4) From geometric considerations we find the path travelled by the sound in each layer:

$$s_1 = \frac{h}{\cos\theta_1} = \frac{h}{\sqrt{1-(\sin\theta_1)^2}}$$

$$s_2 = \frac{h}{\cos\theta_2} = \frac{h}{\sqrt{1-(\sin\theta_2)^2}} = \frac{h}{\sqrt{1-(\sin\theta_1\frac{n_1}{n_2})^2}}$$

The horizontal distance covered by the sound:

$$L = l_1 + l_2 = s_1\sin\theta_1 + s_2\sin\theta_2 = h\sin\theta_1\left(\frac{1}{\sqrt{1-(\sin\theta_1)^2}} + \frac{1}{\sqrt{1-(\sin\theta_1\frac{n_1}{n_2})^2}}\frac{n_1}{n_2}\right) = 26.55m$$

The total time for sound to reach the surface:

$$t = \frac{s_1}{v_1} + \frac{s_2}{v_2} = \frac{h}{v_1}\left(\frac{1}{\sqrt{1-(\sin\theta_1)^2}} + \frac{1}{\sqrt{1-(\sin\theta_1\frac{n_1}{n_2})^2}}\frac{n_1}{n_2}\right) = 0.15\text{c}$$

5) Since the sound started traveling later than the boat, by the time the sound reaches the surface, the boat will have moved:

$$l = (t_0 + t)v_K = 27m$$

6) Since $l > L$, the sound will not catch up with the boat.

9.4. (5 points) Nastya attends a school physics club. During one of the lessons, she assembled an electrical circuit consisting of three identical light bulbs with a resistance of R = 5 Ohms. She connected two bulbs in parallel and the remaining one in series with them.

[4] Determine the maximum power dissipated in one of the bulbs if the bulbs are connected to a 3 V battery.

Comment. Neglect the internal resistance of the battery. (*Cherenkov A.A.*)

Answer: 0.8 W.

Solution. 1) To calculate the power generated by the bulbs, let us find the current flowing through them. The total resistance of the bulbs connected in parallel is:

$$R_{12} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R^2}{2R} = \frac{R}{2}$$

These bulbs are connected in series with the third bulb. Then the total resistance of the circuit is:

$$R_0 = R_3 + R_{12} = \frac{3}{2}R$$

2) According to Ohm's law for the circuit section:

$$I = \frac{U}{R_0} = \frac{2U}{3R}$$

3) The current flowing through the bulbs connected in parallel is divided in half due to symmetry. Thus, the powers released on these bulbs are:

$$P_1 = P_2 = P_{12} = \left(\frac{I}{2}\right)^2 R = \frac{U^2}{9R}$$

4) The power released on the third bulb:

$$P_3 = I^2 R_3 = \frac{4U^2}{9R} > P_{12}$$

Substituting the numerical values, we find:

$$P_3 = 0.8W$$

9.5. (7 points) A group of light sport aircraft must take off from point A at the same time, drop a pennant at point B, and return back. The distance from A to B is 450 km. Each aircraft has a fuel tank capacity of 240 liters, and 1 liter of fuel allows it to travel 2.5 km at a cruising (most fuel-efficient) speed of 300 km/h.

[5] What is the minimum number of aircraft in the group required for all of them to complete the mission and return to A, assuming instantaneous fuel transfer between aircraft is possible?

[6] What is the total flight time of all aircraft?

(*Gamov G.A., Yakovlev A.B.*)

Answer: 3.

Answer: 6 h.

Solution. 1) Let's define the variables:

Fuel tank capacity: V = 240 l

Fuel consumption: $\alpha = \frac{1}{2{,}5}$ l/km

Aircraft speed: v = 300 km/h

Distance from A to B and back: S = 900 km

Let's solve the problem iteratively, first considering one aircraft, then two, and so on.

2) Let one plane fly. The maximum distance it can cover:

$$S_0 = \frac{V}{\alpha} = 600km$$

Таким образом, самолет не долетит $\Delta S = S - S_0 = 300km$ more, for what he needs $\Delta V = \Delta S\alpha = 120l$ of additional fuel, i.e. half a tank.
3) Let two aircraft fly. It is clear that one of the aircraft must fly ΔS, transfer half a tank of fuel to the other aircraft and turn around. Otherwise, if it flies less or more than ΔS, it will not be able to transfer the necessary amount of fuel to the second aircraft to complete the flight to A. However, by this moment it will have already used up half a tank of fuel, so if it gives the second half of the tank, it will not be able to return to A itself.
4) Let three aircraft fly. Consider the following strategy. Let the aircraft fly $\Delta S/2$, then they use up a quarter of their tank each. At this moment, one of the planes pumps its fuel to the other planes up to a full tank. Then it will have exactly a quarter of a tank left, and this is enough to return back. Next, the remaining planes fly another $\Delta S/2$, again using a quarter of a tank each. One of the planes pumps its fuel to the second plane until it has a full tank. Then it will have exactly half a tank left, which is just enough for it to return back to A. The second aircraft will have a full tank, on which it will fly the remaining distance.
5) Let's calculate the total number of hours the aircraft will be in the air. Total distance travelled:

$$L = 2 * (\Delta S/2 + \Delta S + S/2) = 3\Delta S + S$$

Which means the total time in the air:

$$T = \frac{L}{v} = \frac{3\Delta S + S}{v} = 6h$$

Solutions to problems for grade R10

10.1. (8 points) It is known that when 1 mole of gas A reacts with 3 moles of gas B at a temperature of T = 298.15 K, 2 moles of substance C are formed, and the enthalpy change at constant pressure of 1 atmosphere is -40.32 kJ/mol.

[1] What amount of energy will be released if 1 mole of gas A reacts with gas B at a constant volume and temperature T?

Comment. By definition, enthalpy is equal to the sum of internal energy and the product of pressure and volume: H = U + pV. The universal gas constant is 8.31441 J/(K*mol).

(*Prigogine I., Yakovlev A.B.*)

Answer: -35.36 kJ.

Solution. Let us write down the first law of thermodynamics:

$$\Delta U = A + Q$$

Since by the problem statement the volume is constant, no work is done:

$$\Delta U = Q$$

2) Let's write the expression for enthalpy change:

$$\Delta H = \Delta U + \Delta pV$$

Then the internal energy:

$$\Delta U = Q = \Delta H - \Delta PV$$

3) It follows from the Mendeleev-Clapeyron equation that

$$\Delta pV = \Delta \mu RT = \Delta \mu RT$$

it follows from the problem statement that

$$\Delta \mu = 2 - 1 + 3 = -2$$

4) Thus, the final expression for the released energy is

$$Q = \Delta H - \Delta \mu RT = -35.36kJ$$

10.2. (5 points) A flat capacitor consists of three aluminum foil plates separated by two layers of quartz with a thickness of d = 0.20 mm. The outer plates are connected together.

[2] What is the capacitance of this capacitor? Give the answer in microfarads.

Comment. The area of each plate is s = 10 cm^2, and the dielectric constant of quartz is $\varepsilon = 4.3$.

(*Cherenkov A.A.*)

Answer: 0,00038 μF.

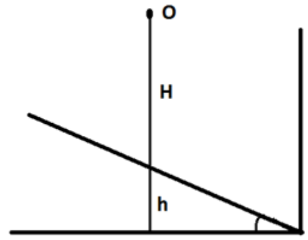
Solution. 1) Let's charge the outer plates with charge q. This induces a charge of q of the opposite sign on each side of the middle plate. Thus, the setup can be considered as two identical capacitors connected in parallel. Let's calculate the capacitance of these capacitors:

$$C = \frac{\varepsilon\varepsilon_0 S}{d}$$

The total capacitance of two such capacitors connected in series is:

$$C_0 = 2\mathrm{C} = 2\frac{\varepsilon\varepsilon_0 S}{d} = 0{,}00038\mu F$$

10.3. (9 points) A small ball with a radius of 5 mm starts falling from point O with no initial velocity in a region enclosed by two perfectly elastic planes (see the figure).



[3] For what ratio H/h will the ball be able to return to its initial position?

Comment. The distance from point O to the vertical wall l is greater than the radius of the ball. Assume that the acceleration of free fall is 10.0 m/s^2. The inclination angle of the plane is α. (*Yakovlev A.B.*)

Answer: $\frac{H}{h} = \frac{\text{ctg}(\alpha)}{\sin(4\alpha)}$, $\alpha \in 0^o, 45^o$.

Solution. 1) Using the law of conservation of total mechanical energy, let's find the ball's speed before hitting the inclined plane:

$$\frac{mv_0^2}{2} = mgH$$

From this, we get:

$$v_0^2 = 2gH$$

2) Let's assume that after the first collision with the inclined plane, the ball bounces off at an angle β to the horizontal. The equation of motion for the ball along the horizontal direction is

$$x(t) = v_0 \cos(\beta) t$$

Let the angle of the inclined plane be α. Then the horizontal distance the ball must travel before hitting the second wall is:

$$l = h\,\text{ctg}(\alpha)$$

Let's find the relationship between anglesα and β. For an elastic collision against the surface, the magnitude of the speed remains unchanged — only the direction changes. From geometric considerations, we find that the ball reflects off the surface at an angle:

$$\beta = 90 - 2\alpha$$

Let's determine the time when the ball hits the second wall:

$$l = x(t_0)$$

$$l = v_0 \cos(\beta) t_0$$

From this, we get:

$$t_0 = \frac{l}{v_0 \cos(\beta)}$$

3) The ball can return to the starting point if the collision with the vertical wall happens perpendicular to its surface. This means the ball's velocity must be horizontal at time t_0, meaning the vertical component of the velocity must be zero:

$$0 = v_y(t_0) = v_0 \sin(\beta) - gt_0$$

From this, we get

$$t_0 = \frac{v_0 \sin(\beta)}{g}$$

4) Let's equate the two expressions for t_0

$$\frac{v_0 \sin(\beta)}{g} = \frac{l}{v_0 \cos(\beta)}$$

This gives:

$$v_0^2 \sin(\beta) \cos(\beta) = lg$$

Substitute the expressions for speed and horizontal distance:

$$2gH \sin(\beta) \cos(\beta) = h \operatorname{ctg}(\alpha) g$$

Let's express the ratio of heights:

$$\frac{H}{h} = \frac{\operatorname{ctg}(\alpha)}{\sin(2\beta)} = \frac{\operatorname{ctg}(\alpha)}{\sin(2 * (90^o - 2\alpha))} = \frac{\operatorname{ctg}(\alpha)}{\sin(4\alpha)}$$

5) It's important to note that the solution holds only when the ball's vertical velocity component is positive after bouncing off the first plane, meaning

$$\sin\beta > 0$$

Otherwise, if

$$\sin\beta \leqslant 0$$

Then the ball will bounce downward and will not hit the vertical wall perpendicularly. This can also be seen from the formula for t_0, which becomes negative when $\sin\beta < 0$. Thus, we have:

$$\sin\beta > 0$$

$$\sin(90^o - 2\alpha) = \cos(2\alpha) > 0$$

This implies

$$2\alpha \in (-90^o; 90^o)$$

, Since the angle must be positive, the final constraint on angle α:

$$\alpha \in (0^o, 45^o)$$

10.4. (8 points) Petya decided to replicate Fedor Konyukhov's round-the-world flight in a hot air balloon with a volume of V = 1000.0 m^3 and a mass of m = 1000.0 kg. The required gas temperature for the balloon to stay in flight is T = 330.0^o K. At a distance of 5.000 km from the shore over the Atlantic Ocean, Petya realized that the temperature inside the balloon had dropped to T_1 = 290.0^o K, and the burners had just enough fuel to maintain this temperature.

[4] How much ballast must be dropped for the balloon to reach the shore, if the balloon is at a height of h = 300.0 m and moving at a speed of v = 15.00 km/h

Comment. Assume that the mass of air inside the balloon remains constant and density of the air is 1.275 kg/m^3. Take the acceleration of free fall as 9.80000 m/s^2 and assume the surrounding air density is independent of altitude. (*Yakovlev A.B.*)

Answer: 154,6 kg.

Solution. 1) Let's write the Mendeleev-Clapeyron equation for the gas inside the balloon at temperatures T and T_1

$$\begin{cases} pV = \mu RT \\ p_1 V_1 = \mu R T_1 \end{cases}$$

Since the pressure in both cases equals the external atmospheric pressure, we find the balloon's volume when the gas cools to temperature T_1:

$$V_1 = V\frac{T_1}{T}$$

2) Let's apply Newton's Second Law to the balloon when the gas temperature is T. According to the problem, this is the necessary temperature for the balloon to fly and hence Archimedes' force is balanced by the gravitational force of the balloon with gas. Let the total mass of the ball and the gas in it be M, then:

$$F_{arh} = Mg$$

The air density is ρ, then

$$\rho g V = Mg$$

wherefrom

$$M = \rho V$$

3) When the gas cools, the balloon's volume decreases. Consequently, the buoyant force also decreases, causing the balloon to accelerate downward. Let's assume ballast of mass Δm is dropped to give the balloon needed acceleration to reach the shore. Now, the balloon's total mass (balloon + gas) becomes $M_1 = M - \Delta m$. We apply Newton's Second Law again

$$M_1 g - F_{arh1} = M_1 a$$

so

$$M_1 g - \rho g V_1 = M_1 a$$

, therefore

$$a = g\left(1 - \frac{\rho V_1}{M_1}\right) = g\left(1 - \frac{\rho V_1}{M - \Delta m}\right) = g\left(1 - \frac{\rho V_1}{\rho V - \Delta m}\right)$$

4) The balloon must travel a horizontal distance of $l = 5{,}000$ km at a speed v . The time required for this trip is:

$$T = \frac{l}{v}$$

During this same time, the balloon must rise vertically by distance h with acceleration a. The vertical displacement is:

$$h = y(T) = \frac{aT^2}{2} = \frac{al^2}{2v^2}$$

So

$$a = \frac{2v^2 h}{l^2} = g\left(1 - \frac{\rho V_1}{\rho V - \Delta m}\right)$$

After a bit of algebraic manipulation, we find the ballast that must be dropped

$$\Delta m = \rho V \left(1 - \frac{T_1}{T(1 - \frac{2hv^2}{l^2 g})}\right) = 154{,}6kg$$

10.5. (5 points) At the vertices of a regular hexagon with a side length of 10.0 cm, there are charges of q = 0.000100 C.

[5] What charge must be placed at the center of the hexagon for the system to be in equilibrium? Will this equilibrium be stable?

(*Yakovlev A.B.*)

Answer: $0{,}000183C$.
Answer: unstable.
Solution. 1) Due to the central symmetry of the charge system, it is sufficient to consider the equilibrium of only one of the charges at the vertices of the hexagon. For this purpose, let us number the other charges. Let the two nearest charges in the vertices be with numbers 1 and 2, the charge in the opposite vertex with number 5, and the two remaining charges with numbers 3 and 4, and the charge Q in the centre with number 6. It still follows from the symmetry of the system that the resulting force F_{12} from charges 1 and 2 will be directed along the diagonal where the selected charge is located. Similarly, the resulting force F_{34} from charges 3 and 4 will also align along this diagonal. The forces from charges 5 and 6 are also along this diagonal since all three charges lie on it.
2) Let us calculate the distances between the charge in question and the others. Obviously, the distances to charges 1,2, 6 are equal to side a of the hexagon, and to charge 5 - 2a. The distances to charges 3 and 4 are equal to the small diagonal of the hexagon, which is $a\sqrt{3}$.
3) Let's start calculating the forces using Coulomb's law. Force from charges 1 and 2:

$$|\vec{F_{12}}| = |\vec{F_1} + \vec{F_2}| = |\vec{F_1}| = \frac{kq^2}{a^2}$$

Force from charges 3 and 4:

$$|\vec{F_{34}}| = |\vec{F_3} + \vec{F_4}| = 2|\vec{F_3}|\cos(30^o) = 2\frac{kq^2}{(a\sqrt{3})^2}\frac{\sqrt{3}}{2} = \frac{kq^2}{\sqrt{3}a^2}$$

Forces from charges 5 and 6:

$$|\vec{F_5}| = \frac{kq^2}{(2a)^2} = \frac{kq^2}{4a^2}, \quad |\vec{F_6}| = \frac{kqQ}{a^2}$$

4) Since $\vec{F_{12}}$,$\vec{F_{34}}$,$\vec{F_5}$ are all repulsive forces, they act in the same direction. Let's write the equilibrium equation for the selected charge projected onto the hexagon's diagonal:

$$F_{12} + F_{34} + F_5 + F_6 = 0$$

Substitute the expressions for forces:

$$\frac{kq^2}{a^2} + \frac{kq^2}{\sqrt{3}a^2} + \frac{kq^2}{4a^2} + \frac{kqQ}{a^2} = 0$$

From this we get

$$Q = -\frac{15 + 4\sqrt{3}}{12}q = -0{,}000183C$$

Solutions to problems for grade R11

11.1. (9 points) The coach challenged Ivan to hit a table tennis ball with a racket in such a way that it bounces to the same height of 20 cm all the time.

[1] Determine the amplitude and frequency of the racket's motion, considering that the ball loses 10% of its velocity due to air resistance between bounces

Comment. Assume that the acceleration of free fall is 10 m/s^2, and the collisions are perfectly elastic. Also, for simplicity, assume that the force of air resistance remains constant throughout the ball's motion. (*Yakovlev A.B., Cherenkov A.A.*)

Answer: 16 ; 0.0067.

Solution. 1) Let's assume that after bouncing off the racket, the ball has a speed $v_0$which is sufficient for the ball to reach the desired height of h=20cm. According to the problem, the ball loses 10% of its speed between impacts with the racket. Therefore, the ball's speed before the next collision is:

$$v_1 = \alpha v_0$$

где $\alpha = 0{,}9$

2) Let's analyze the ball's motion between two consecutive impacts with the racket. Using the Work-Energy Theorem, the kinetic energy loss of the ball is related to the work done by the friction force F:

$$2Fh = \frac{mv_0^2}{2} - \frac{mv_1^2}{2} = \frac{mv_0^2}{2}(1-\alpha^2)$$

From this, we find:

$$F = \frac{mv_0^2}{4h}(1-\alpha^2)$$

3) Let's consider the ball's motion as it falls from the top of its trajectory. According to the Law of Conservation of Energy, the potential energy converts into kinetic energy, with part of it lost to friction:

$$mgh = \frac{mv_1^2}{2} + Fh$$

Substitute the expression for friction force:

$$mgh = \frac{mv_0^2\alpha^2}{2} + \frac{mv_0^2}{4}(1-\alpha^2)$$

From this, we get:

$$v_0 = 2\sqrt{\frac{gh}{1+\alpha^2}}$$

The friction force formula now becomes

$$F = mg\frac{1-\alpha^2}{1+\alpha^2}$$

4) Let's find the racket's speed before impact with the ball. Since the racket's mass is much larger than the ball's, the collision can be treated as one with a perfectly heavy wall. The collision is perfectly elastic, so in the racket's reference frame, the ball's speed remains the same before and after the collision:

$$v_1' = v_0'$$

In the lab frame, this gives:

$$v_1 + V = v_0 - V$$

where V is the racket's speed. So we get

$$V = \frac{v_0 - v_1}{2} = v_0\frac{1-\alpha}{2} = \sqrt{\frac{gh}{1+\alpha^2}}(1-\alpha)$$

5) Let's calculate the period of the racket's motion, which is equal to the total time the ball travels between impacts. First, let's consider the ball's ascent from the racket to the peak of its trajectory. Using Newton's Second Law:

$$mg + F = ma_1$$

wherefrom

$$a_1 = g + F/m = g\left(1 + \frac{1-\alpha^2}{1+\alpha^2}\right) = g\frac{2}{1+\alpha^2}$$

The time to reach the top is determined by the condition that the ball's speed at the peak is zero:

$$0 = v_0 - a_1 t_1$$

So

$$t_1 = \frac{v_0}{a_1} = \sqrt{\frac{h}{g}(1+\alpha^2)}$$

Similarly, let us consider the motion of the ball from the upper point of the trajectory to the racket. According to Newton's second law:

$$mg - F = ma_2$$

This gives:

$$a_2 = g - F/m = g\left(1 - \frac{1-\alpha^2}{1+\alpha^2}\right) = g\frac{2\alpha^2}{1+\alpha^2}$$

The time to descend is found from the condition that the ball's speed before collision with the racket is v_1:

$$v_1 = a_2 t_2$$

so

$$t_2 = \frac{v_1}{a_2} = \frac{1}{\alpha}\sqrt{\frac{h}{g}(1-\alpha^2)}$$

The total time of the ball's motion between collisions is

$$T = t_1 + t_2 = (1 + \frac{1}{\alpha})\sqrt{\frac{h}{g}(1+\alpha^2)}$$

5) Knowing the racket's period, we can now compute its frequency:

$$\omega = \frac{2\pi}{T} = \frac{2\pi\alpha}{1+\alpha}\sqrt{\frac{g}{h(1+\alpha^2)}} = 15{,}6 rad/s$$

Let's find the amplitude from the expression for maximum speed:

$$V = A\omega$$

so

$$A = \frac{V}{\omega} = \frac{(1-\alpha)(\alpha+1)}{2\pi\alpha}h = 0.0067m$$

11.2. (11 points) To prevent flooding, concrete blocks in the shape of right triangular prisms, 4 meters high, are placed along the riverbank at a height of 1 meter. The side length of the prism's base is 1.5 m. Determine the water level at which one such block

[2] can be overturned,

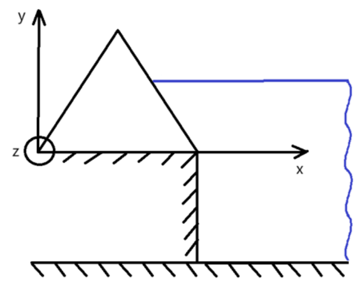
[3] can be shifted, given that the coefficient of friction between concrete and the ground is 0.30.

Comment. Assume that the densities of concrete and water are $2.3*10^3\ kg/m^3$ and $1.0*10^3\ kg/m^3$, respectively. (*Yakovlev A.B.*)

Answer: Water cannot tip the block over.

Answer: 2.3 m.

Solution. 1) Let us introduce a rectangular coordinate system as in the figure, with the origin at the vertex of the base of the prism. The block will tip over if the moment created by the force of water pressure is greater than the moment of the gravitational force with respect to the oz axis.



Let the water level be h meters above the shore. Let us calculate the projections of the water pressure force on the face of the block. Since the pressure distribution doesn't change along the z direction, it's enough to solve a 2D problem for a liquid layer of height h on the edge of the cross-section and multiply the result by the prism's height l. Thus, the elementary force df acting on the edge element da:

$$df = \rho g(h-y)da$$

where ρ - is the density of water. From geometric considerations, we know

$$\sin\alpha da = dy$$

where $\alpha = 60^o$ So we get

$$df = \frac{\rho g(h-y)}{\sin\alpha}dy$$

$$f = \int_0^h \frac{\rho g(h-y)}{\sin\alpha}dy = \frac{\rho g h^2}{2\sin\alpha}$$

Therefore, the total pressure force on the entire face is:

$$F = lf = \frac{\rho g l h^2}{2\sin\alpha}$$

Since the pressure force is perpendicular to the edge, we split it into components

$$\begin{cases} F_x = F\sin\alpha = \frac{\rho g l h^2}{2} \\ F_y = F\cos\alpha = \frac{\rho g l h^2}{2}\operatorname{ctg}\alpha \end{cases}$$

And for the elementary forces we have:

$$\begin{cases} df_x = df \sin\alpha = \rho g(h-y)dy \\ df_y = df \cos\alpha = \rho g(h-y)\operatorname{ctg}\alpha dy \end{cases}$$

2) Let's calculate the moments of forces F_x,F_y relative to the oz axis. Following the same logic as before, we get:

$$M_x = l * \int_0^h y df_x = l\rho g * \int_0^h (hy - y^2)dy = \frac{l\rho g}{6}h^3$$

$$M_y = l * \int_0^h (a - y\operatorname{ctg}\alpha)df_y = l\rho g * \operatorname{ctg}\alpha * \int_0^h (a - y\operatorname{ctg}\alpha)(h-y)dy = l\rho g \operatorname{ctg}\alpha \frac{h^2}{2}(a - \frac{h}{3}\operatorname{ctg}\alpha)$$

Where a is the side of the prism's base.

3) The gravitational force acting on the concrete block is calculated as:

$$mg = \rho_1 V g = \rho_1 l S g = \rho_1 a^2 l g \frac{\sqrt{3}}{4}$$

where ρ_1 is the density of concrete, and $S = a^2\frac{\sqrt{3}}{4}$ - s the area of the prism's base.

We also need the lever arm of the weight relative to the oz axis. From symmetry, the center of gravity aligns with the center of the prism's cross-section, equidistant from its bases. This is the intersection point of the medians of the cross-section. The lever arm is half the base side. Therefore, the moment of the gravitational force is:

$$M_0 = mg\frac{a}{2}$$

4) Force F_x tries to tip the block, while the gravitational force and F_y counteract this tipping. Let's introduce the resulting moment function:

$$M(h) = M_x - M_y - M_0 = l\rho g(1 + \operatorname{ctg}^2\alpha)\frac{h^3}{6} - l\rho g a \operatorname{ctg}\alpha \frac{h^2}{2} - mg\frac{a}{2}$$

If at some value of h the resulting moment is positive, it will mean that the block will tip over. It's clear that at h = 0, the moment is negative. Let's investigate further by introducing an auxiliary function:

$$M_1(h) = \frac{2M(h)}{l\rho g} = (1 + \operatorname{ctg}^2\alpha)\frac{h^3}{3} - \operatorname{ctg}\alpha * ah^2 - \frac{ma}{\rho l}$$

Since the water level can't exceed the block's height, then $h \in \left[0, \frac{a\sqrt{3}}{2}\right]$. Therefore, we investigate $M_1(h)$ on this segment. Let us calculate the derivative:

$$\frac{dM_1}{dh} = \left((1 + \operatorname{ctg}^2\alpha)h - 2a\operatorname{ctg}\alpha\right)h$$

The derivative is a quadratic function, the graph is a parabola, branching upwards. The roots of this parabola are:

$$\begin{cases} h_1 = 0 \\ h_2 = 2a\operatorname{ctg}\alpha\sin^2\alpha = a\sin(2\alpha) = \frac{a\sqrt{3}}{2} \end{cases}$$

Moreover

$$\frac{dM_1}{dh} < 0, h \in \left[0; \frac{a\sqrt{3}}{2}\right]$$

i.e., the moment is decreasing all the time on this segment. But since the momentum was negative

at h = 0, it will remain negative for the whole segment. Thus, there is no way the water can tip the block over.
5) For the water to move the block, the force F_x must exceed the frictional force. Thus, let us write Newton's second law in projection on the vertical axis:

$$F_y + mg = N$$

Where N is the normal reaction force. The friction force is:

$$F_f = \mu N = \mu(F_y + mg)$$

For sliding to occur, we need:

$$F_x > F_f$$

$$\frac{\rho g l h^2}{2} > \mu \rho_1 \alpha^2 l g \frac{\sqrt{3}}{4} g + \mu \frac{\rho g l h^2}{2} \operatorname{ctg} \alpha$$

This gives

$$h^2 > \frac{\sqrt{3} \mu \rho_1 a^2}{2\rho(1 - \mu \operatorname{ctg} \alpha)}$$

So:

$$h > 1.3m$$

Let's take into account that we were measuring the height of water from the shore, then the actual water level:

$$H = 1 + h > 2.3m$$

But this level is higher than the top of the block, so the water will start to flow over the block earlier and will not be able to move it.

11.3. (8 points) Peter, whose mass is m = 70 kg, is preparing for a cycling race with a mountain stage where the maximum road incline reaches 10 degrees. Peter's racing bike has the following parameters: mass - 12 kg, external diameter of wheels - 700 mm, radius of pedals circumference - 180 mm, minimum front and maximum rear sprockets, to which the chain is attached, have 40 and 25 teeth, respectively.
[4] Determine whether he will be able to maintain a constant speed on the steepest section of the course, assuming that Peter applies an average force of 0.85 * mg vertically downward on the pedals
Comment. Neglect friction. Assume that the acceleration of free fall is 9.8 m/s^2. (*Yakovlev A.B.*)
Answer: Peter will not be able to maintain a constant speed. .
Solution. 1) Peter will be able to move at a constant speed on the steepest part of the incline if the work he performs exceeds the work done by gravity. Let's assume Peter makes half a turn of the pedals. Then the work he performs is:
$A_1 = F * 2r = 2\beta g m r = 210$ J
where $\beta = 0{,}85$, r is radius of pedals circumference.
2) Let's calculate how many revolutions the rear wheel makes during this motion:

$$N = \frac{1}{2}\frac{n_1}{n_2} = \frac{1}{2} i$$

where $i = \frac{40}{25}$ is the gear ratio of the sprockets
The distance the bicycle travels along the inclined section is:

$$S = 2\pi NR = \pi iR$$

And the vertical displacement is:

$$h = S\sin\alpha = \pi iR\sin\alpha$$

3) Finally, let's calculate the work done by gravity:
$A_2 = (m+M)gh = (m+M)g\pi iR\sin\alpha =$ 245 J
Since $A_2 > A_1$, Peter will not be able to maintain a constant speed.

11.4. (5 points) In a school laboratory pupils study electrostatics. As part of an experiment, teacher takes a flat capacitor, grounds one of its plates, and applies a voltage of U = 220 V. Then a flat uncharged metal plate is placed in the air gap between the capacitor plates at a distance of l = 2.00 cm from the lower grounded plate.

[5] What will be the potential of the inserted plate?

[6] What will be the electric field strength on both sides (above and below) of the inserted plate, if the distance between the capacitor's plates is d = 3.00 cm? (*Cherenkov A.A.*)

Answer: 147 V.
Answer: 7,33 kV/m.
Solution. 1) When an uncharged plate is inserted, charges are induced on its sides, equal in magnitude but opposite in sign to the charges on the capacitor plates. As a result, the electric field strength E inside the capacitor remains unchanged. We get:

$$E = \frac{U}{d} = 7{,}33 kV/m$$

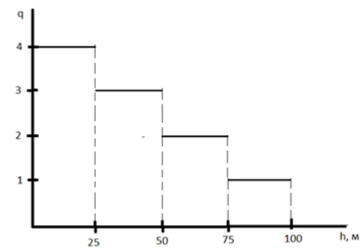
2) Let's denote the desired potential as ϕ. Then we have

$$E = \frac{U}{d} = \frac{\phi}{l}$$

From this, we find:

$$\phi = \frac{Ul}{d} = 147V$$

11.5. (7 points) The ionosphere of the planet Zhelezyaka is structured in such a way that the electric charge of a body changes abruptly at altitudes from 0 to 100 meters, as shown in the figure.



[7] Determine whether oscillatory motion is possible for a body with a mass of m = 4 grams, given that in this altitude range the strength of the upward-directed electric field is constant and equal to 10,000 V/m, and the gravitational field strength is also constant and equal to 5 m/s^2

Comment. The charge of the body is given in microCoulombs. (*Yakovlev A.B.*)

Answer: Тело может пребывать в колебательном движении..
Solution. 1) Let's note that the forces acting on the body are potential. Oscillatory motion of the body is possible if it falls into a potential well. The body's potential energy consists of gravitational and electric components:

$$\Pi(h) = mgh - Eq(h)h = (mg - Eq(h))h.$$

This means the potential energy is a piecewise linear function of height, as the charge changes abruptly with height. Let's now calculate $(mg - Eq(h))$ for each segment:
For $0 \leqslant h \leqslant 25$ m:

$$mg - Eq = (4 \times 10^{-3} * 5) - (10000 \times 4 \times 10^{-6}) = -20 \times 10^{-3} N.$$

Potential energy decreases.
For $25 \leqslant h \leqslant 50$ m:

$$mg - Eq = (4 \times 10^{-3} * 5) - (10000 \times 3 \times 10^{-6}) = -10 \times 10^{-3} N.$$

Potential energy still decreases .
For $50 \leqslant h \leqslant 75$ m:

$$mg - Eq = (4 \times 10^{-3} * 5) - (10000 \times 2 \times 10^{-6}) = 0N.$$

Potential energy remains constant.
For $75 \leqslant h \leqslant 100$ m:

$$mg - Eq = (4 \times 10^{-3} * 5) - (10000 \times 1 \times 10^{-6}) = 10 \times 10^{-3} N.$$

Potential energy increases.
2) Therefore, for $h \in [0; 50]$ Potential energy decreases, for $h \in [50; 75]$ it remains constant, and for the further increase of height $h \in [75; 100]$ the potential energy increases. This means that for the interval $h \in [50; 75]$ the body enters a potential well and oscillatory motion is possible.