International Physics Olympiad
«Formula of Unity» / «The Third Millennium»
Year 2023/2024. Qualifying round

Solutions to problems for grade R8

8.1. (7 points) The first point moves along the Y axis of a rectangular coordinate system with a velocity of 4 m/s, and the second point moves along the X axis. The distance between the points is constant and equal to 5 m.

[1] Determine the modulus of the velocity of the second point at the moment when the first point is at a distance of 3 m from the origin.

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 3 m/s.

Solution. According to the problem statement, the first point moves along the Y axis with constant velocity. The second point moves along the X axis with variable acceleration, which is a consequence of the constant distance between the points during the motion.

If the second point is at the origin of coordinate system at the initial moment, then, after time t, its coordinate is determined by the expression

$$x(t) = [L^2 - (L - v_1 t)^2]^{1/2}$$

Thus, the dependence of x-coordinate of the second point on time is not linear (motion with constant velocity) or quadratic (motion with constant acceleration), i.e. the second point moves with variable acceleration.

A typical technique that simplifies the solution of problems on joint motion is the transition to the analysis of the motion of only one body, but in a moving frame of reference. Let us link the stationary frame of reference with the first body (thus we "stop" the first body).

Then the second body moves with the velocity $\vec{v}_2$ relative to the reference frame associated with the X-axis, which, in turn, moves with the velocity $-\vec{v}_1$ relative to the first body.

The velocity of the second body $\vec{v}_{21}$ relative to the frame of reference in which the first body is at rest is determined by the addition of velocities law:

$$\vec{v}_{21} = \vec{v}_2 - \vec{v}_1$$

The motion trajectory of the second body in the moving frame of reference is a circle, since the distance between the bodies is constant according to the problem statement. It follows that the vector $\vec{v}_{21}$ is perpendicular to the radius of the circle, i.e. the segment L, connecting the points (Fig. 2). From the similarity of velocity and displacement triangles, we have

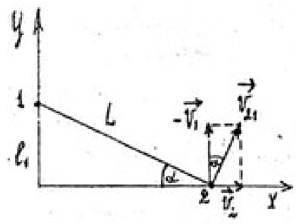

Fig. 2

$$\frac{v_2}{v_1} = \frac{l_1}{\sqrt{L^2 - l_1^2}}$$

hence the expression for the velocity of the second point:

$$v_2 = \frac{v_1 l_1}{\sqrt{L^2 - l_1^2}} = \frac{4 * 3}{\sqrt{25 - 9}} = 3\ m/s$$

8.2. (7 points) Ivan has a measuring cup with graduations and a thermometer. He took a cup of cold water ($T_0 = 10^o$ C), poured 50 cm^3 that water out of it, and then poured in the same amount of hot water of constant (but unknown) temperature from the boiler. As a result, the temperature of the water in the glass was $T_1 = 37^o$ C. He then poured 50 cm^3 out of the cup and added the same amount of water from the boiler. Then he measured the temperature and got $T_2 = 53^o$ C.

[2] Determine the volume of water in the cup and the temperature of the water in the boiler.

(*Minarsky A.M.*)

Answer: $V = 150\ cm^3$, $T = 91^o\ C$.

Solution. Let the heat capacity of water be c, unknown mass of water in the cup - M, amount of water poured out and poured in - $m = 50\ g$, unknown temperature of water in the boiler - T.
Let's write the heat balance equation for single pouring and refilling:

$$c(M - m)(T_1 - T_0) = cm(T - T_1)$$

or

$$cM(T_1 - T_0) = cm(T - T_0)$$

wherefrom

$$T_1 = T_0 + \frac{m}{M}(T - T_0) \qquad (1)$$

Similarly, for the second mixing:

$$T_2 = T_1 + \frac{m}{M}(T - T_1) \qquad (2)$$

Subtracting equation (1) from equation (2), we obtain:

$$T_2 - T_1 = T_1 - T_0 + \frac{m}{M}(T_0 - T_1) \qquad (3)$$

Transform and get:

$$M = m(T_0 - T_1)/(T_0 + T_2 - 2T_1)$$

Substituting the numerical values of temperatures T_0, T_1 и T_2, we obtain: $M = 3m = 150\ g$, so the volume of the water: $V = 150 cm^3$. From equation (1) we get:

$$T = T_0 + \frac{M}{m}(T_1 - T_0)$$

and knowing already that $\frac{M}{m} = 3$, we get: $T = 91^o\ C$.
So, the volume of water in the cup is $V = 150\ cm^3$, and the temperature of water in the boiler is $T = 91^o\ C$.

8.3. (12 points) Ski competitions are held on a circular track. Skiers are divided into two groups: professionals and amateurs. Professionals start at the same time, pass on the track 3 laps and have speeds from 24 to 27 km / hr. Amateurs start at the same time half an hour later, pass 2 laps and have speeds from 12 to 20 km/hour. It is known that each amateur had to give way to each professional during the race, but exactly once.

[3] What can be the length of one lap of the track?

Comment. If possible, give both the minimum and maximum values of the lap length.

(*Minarsky A.M.*)

Answer: 18<L<20 (km).

Solution. Let the length of one lap of the track be equal to L, the speed of a professional - u, the speed of amateur - v, and the time elapsed until they meet each other - t. Therefore, if the amateur started $T = 0.5\ h$ later, then the time he skied is $t - T$, meanwhile the professional has done one more lap:

$$ut = v(t - T) + L \qquad (1)$$

However, the professional caught up with the amateur during the race, which means he completed less than 3 laps:

$$ut < 3L \qquad (2)$$

Carry over vt to the left in the equation (1), divide (2) by transformed (1) and get:

$$\frac{u}{u-v} < \frac{3L}{L-vT} \qquad \text{или} \qquad L(3v-2u) < uvT$$

Substitute here T = 0.5h, the highest speed of an amateur v=20km/h and the lowest speed of a professional u=24km/h, to get the lowest upper limit of the circle length:

$$L < 20(km). \qquad (3)$$

Let's get the condition that the professional could not overtake the amateur 2 times, i.e. for 2 laps. If on the contrary it happened, then

$$ut = v(t-T) + 2L \qquad (4)$$

and it is necessary by the problem statement that at the moment of such possible overtaking he has already passed more than 3 laps of the race:

$$ut > 3L \qquad (5)$$

Carry over vt to the left in the equation (4), divide (5) by transformed (1) and get:

$$\frac{u}{u-v} > \frac{3L}{2L-vT} \qquad \text{или} \qquad L(3v-u) > uvT$$

Substitute here T = 0,5h, the lowest speed of an amateur v=12km/h and the highest speed of a professional u=27km/h to get the highest lower bound of the circle length:

$$L > 18(km). \qquad (6)$$

Combining the inequalities (3) and (6) we get the final answer:

$$18 < L < 20(km)$$

8.4. (10 points) A ball with the density 2 times less than the density of water is immersed in a vertical cylindrical vessel of 10 cm radius ,which is partially filled with water.

[4] How many millimeters will the water level rise after the ball is immersed if the radius of the ball is 3.0 cm?

Comment. Consider that the volume of the ball is equal to $\frac{4}{3}\pi R_b^3$, the area of the circle is equal to πR_c^2. (*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 1,8 mm.

Solution. The density of the ball, according to the problem statement, is less than the density of water, i.e. the ball does not sink in water, and the equilibrium condition of the ball has the form

$$\vec{F}_A + m\vec{g} = 0 \qquad (1)$$

or

$$\rho_{\text{в}} g V_{i.p.} = \rho g V \qquad (2)$$

where $\vec{F}_A$ – Archimedes' force acting on the ball; V – ball's volume and $V_{i.p.}$ – volume of the part of the ball immersed in water at equilibrium.

Using the condition $\rho = \frac{1}{2}\rho_w$, it is easy to find, that $V_{i.p.} = \frac{1}{2}V$.

The further reasoning is of a purely geometric nature.

The volume of water displaced by the ball is equal to

$$V' = V_{i.p.} - V_1 \qquad (3)$$

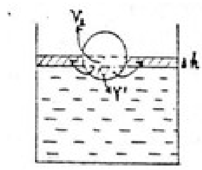

Fig. 3

where V_1 – volume of the ball part, located between the planes corresponding to the initial and final position of the water level in the cylinder and separated from each other by the desired value Δh (fig. 3).
On the other hand, this volume is equal to the volume of liquid that is above the water level in the cylinder before the balloon is lowered:

$$V' = S\Delta h - V_1 = \pi R^2 \Delta h - V_1 \qquad (4)$$

where R – radius of the cylinder. Equating the right parts of expressions (3) and (4), we obtain

$$\pi R^2 \Delta h = \frac{1}{2}V = \frac{2}{3}\pi R^3 \qquad (5)$$

wherefrom

$$\Delta h = \frac{2r^3}{3R^2} = \frac{2 * 27 * 10^{-6}}{3 * 0.010} = 1{,}8 * 10^{-4}\ m = 1{,}8\ mm$$

8.5. (4 points) A weightlifter lifted a 200 kg barbell from shoulder height (170 cm above the floor) to a height of 210 cm above the floor.
[5] How much has the potential energy of the barbell changed?

Comment. Assume that the acceleration of free fall is 10 m/s^2 (*G.N.Stepanova*)
Answer: $\Delta E_{pot} = 800\ J$.
Solution. Let's use the formula for work in the field of gravity and substitute the numerical values

$$\Delta E_{pot} = mg\Delta h = 200 * 10{,}0 * 0{,}400 = 0{,}800 * 10^3 = 800J$$

Solutions to problems for grade R9

9.1. (7 points) A 76 cm long tube with both ends opened is half immersed in mercury. The atmospheric pressure is 76 cm Hg.

[1] Determine in centimeters the length of the column of mercury in the tube if the tube is removed from the mercury after tightly closing the upper end.

Comment. The temperature is constant. (*Yu.V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 22 cm.

Solution. When a tube with a tightly closed upper end is pulled out, some mercury will pour out of the tube and some will remain. This is due to the fact that when the mercury is poured out, the pressure of the air isolated between the upper closed end of the tube and the surface of the remaining mercury decreases as the volume increases, and the difference between the pressure forces of the atmosphere and the air in the tube balances the force of gravity acting on the column of mercury remaining in the tube. The equilibrium condition of the mercury column is as follows:

$$\vec{F_0} + m\vec{g} + \vec{F_1} = 0 \qquad (1)$$

where $\vec{F_0}$ – atmospheric pressure force, $\vec{F_1}$ – air pressure force in the tube and $m\vec{g}$ – the force of gravity acting on the mercury remaining in the tube (fig. 4).

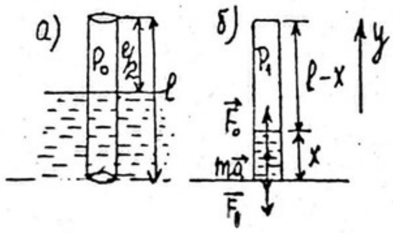

Fig. 4

Let us project the equation of equilibrium (1) on the vertical axis

$$p_0S - mg - p_1S = 0 \qquad (2)$$

Where p_0 – atmospheric pressure, p_1 – air pressure in the tube, S – is the cross-sectional area of the tube. When the upper end is tightly closed, the mass of air in the tube remains constant, i.e. the process of air expansion is isothermal. Based on the Boyle-Marriott law:

$$p_0\frac{l}{2}S = p_1(l - x)S$$

wherefrom $p_1 = \frac{p_0 l}{2(l-x)}$ (3)

where x – is the length of the mercury column remaining in the tube. The mass of the mercury column in the tube is expressed through the density of mercury and its volume:

$$m = \rho_{hg}xS \qquad (4)$$

The atmospheric pressure is given in the problem statement in cm. Hg, i.e. it corresponds to the hydrostatic pressure of a column of mercury of $l_0 = 76\ cm$ height ($l_0 = l$),

$$p_0 = \rho_{hg}gl_0 = \rho_{hg}gl \qquad (5)$$

Substituting the obtained expressions (3), (4), (5) for p_1, m, p_0 into the equilibrium condition (2) of the mercury column, we obtain

$$l - x - \frac{l^2}{2(l-x)} = 0 \qquad (6)$$

This equation reduces to a quadratic equation with respect to the unknown height of the mercury column in the tube:

$$2x^2 - 4lx + l^2 = 0 \qquad (7)$$

Solving the quadratic equation (7), we obtain

$$x_1 = \frac{l(2-\sqrt{2})}{2}; x_2 = \frac{l(2+\sqrt{2})}{2} \qquad (8)$$

The second root of equation (8) has no physical meaning, since $x_2 > l$. As a result, for the required length of the mercury column remaining in the tube, we have

$$x = \frac{l(2-\sqrt{2})}{2} = \frac{0.76*(2-\sqrt{2})}{2} = 22\ cm$$

9.2. (7 points) A body is thrown at a velocity of 10 m/s at an angle of 45^o to a long inclined plane forming 60^o with the horizon.
[2] Determine the maximum distance of the body from the inclined plane.

Comment. Neglect the air resistance. Assume that the acceleration of free fall is 10 m/s^2

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 5 m.
Solution. Let us choose a reference frame as follows: origin O is at the throwing point, X-axis is along the inclined plane and Y-axis is perpendicular to the plane.

In the absence of air resistance, the body moves with free fall acceleration $\vec{g}$, and by the time t the displacement vector $\Delta\vec{r}$ is defined by the expression:

$$\Delta\vec{r} = \vec{v}_0 t + \vec{g}\frac{t^2}{2}$$

The situation corresponding to the problem is illustrated in the figure below.

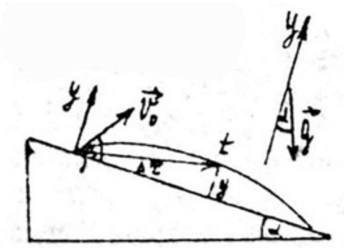

Fig. 5

Projecting the displacement vector $\Delta\vec{r}$ onto the y-axis, we obtain the expression for the distance of the body from the inclined plane by the moment of time t:

$$y = v_0 \sin\beta t - \frac{g\cos\alpha t^2}{2} \qquad (1)$$

Thus, the Task is reduced to determining the maximum value of the function y, which is a quadratic function of time. Let us plot the dependence of y on time t (fig. 6). The roots of the function are $t_1 = 0$ and $t_2 = \frac{2v_0\sin\beta}{g\cos\alpha}$.

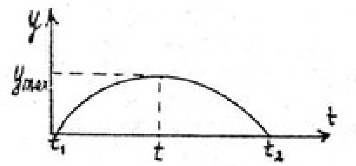

Fig. 6

The maximum value of y is reached at the moment of time $t = \frac{v_0\sin\beta}{g\cos\alpha}$ (since the curve is a parabola). Substituting the obtained time value into the expression (1) for the distance of the body from the inclined plane, we obtain y_{max}:

$$y_{max} = \frac{v_0^2\sin^2\beta}{2g\cos\alpha} = \frac{100*\frac{1}{2}}{20*\frac{1}{2}} = 5\ m$$

9.3. (10 points) The chain, which is made up of small perfectly smooth links, is held so that its lower end touches the surface of the table. The chain is released. The mass of the chain is 50 g and its length is 50 cm.
[3] Determine the modulus of the pressure force of the chain on the table after 0.2 s.

Comment. Assume that the acceleration of free fall is 10 m/s^2.

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 0,6 N.
Solution. Suppose that the links of the chain are very small (the size of the ring is much smaller than the length of the chain). Therefore, the chain can be considered as a continuous flexible ribbon. Let's estimate the time after which the whole chain will be on the table. The links of the chain are absolutely smooth according to the problem condition, i.e. neither friction nor elasticity forces arise

during their motion. Each link of the chain before falling on the table moves with the acceleration of free fall, and the time of movement of the last link of the chain to the table is determined by the relation

$$t_1 = \sqrt{\frac{2l}{g}} = \frac{1}{\sqrt{10}} > 0{,}3\ s$$

This estimate implies that not all elements of the chain are on the table by the time t=0.2 s. According to Newton's 2nd law, the modulus of the chain pressure force on the table is numerically equal to the modulus of the normal reaction force of the table. We can distinguish two components of the normal reaction force of the table:

1) Component N_1, which compensates the pressure on the table of those links of the chain, which are already on the table at time t,

$$N_1 = M_1 * g = \frac{M}{l} * \frac{gt^2}{2} * g = \frac{Mg^2t^2}{2l}$$

2) Component N_2, which decreases the momentum of the chain link, which at time t comes into contact with the table.

$$N_2 = \frac{\Delta p}{\Delta t} = \frac{\Delta mv}{\Delta t} = \frac{(M/l)v * \Delta t * v}{\Delta t} = \frac{M * v^2}{l}$$

where v – velocity of the moving links of the chain at time t. Taking into account that by this moment the velocity of the chain v=g*t, we obtain

$$N_2 = \frac{Mg^2t^2}{l}$$

Let's note that $N_2 > N_1$.

Summing up both components, we obtain the final expression for the normal reaction of the table, numerically equal to the force of the chain pressure on the table:

$$N = N_1 + N_2 = \frac{3Mg^2t^2}{2l} = \frac{3 * 0.050 * 100 * 0.04}{2 * 0.50} = 0.6\ N$$

Note that by the time t =0,2 s pressure of the chain on the table is greater than in the case when the whole chain rests on the table (Mg = 0.5 N).

9.4. (10 points) Resistors of 1.0 Ohm and 4.0 Ohm resistance are connected in parallel to the plus side of a battery with EMF of 16.8 V and resistance of 2.1 Ohm. Resistors of 2.0 Ohm and 3.0 Ohm resistance are connected to the minus side of the battery.

[4] Find the modulus of potential difference between the connection points of resistors of 1.0 Ohm and 2.0 Ohm and resistors of 4.0 Ohm and 3.0 Ohm.

(A.G.Areshkin, O.S. Komarova, V.G. Mozgovaya, D.L. Fedorov)

Answer: 2,0 V.

Solution. Let's depict the circuit diagram (fig.7).

where I – total current flowing through the circuit. Since the resistors R_1 and R_3 are connected in series, their total resistance:

$$R_{13} = R_1 + R_3 = 1 + 2 = 3\ Ohm$$

Resistors R_2 and R_4 are also connected in series:

$$R_{24} = R_2 + R_4 = 3 + 4 = 7\ Ohm$$

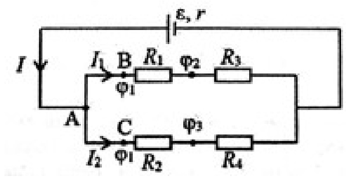

Fig. 7

Cascades R_{13} and R_{24} are connected in parallel, so

$$\frac{1}{R_{total}} = \frac{1}{R_{13}} + \frac{1}{R_{24}} = \frac{1}{3} + \frac{1}{7} = \frac{10}{21}$$

$$R_{total} = \frac{21}{10} = 2{,}1\ Ohm$$

According to Ohm's law for a complete circuit

$$I = \frac{\varepsilon}{R_{total} + r} = \frac{16.8}{2.1 + 2.1} = 4{,}0\ \text{A}$$

At point A the total current is branched. According to the charge conservation law,

$$I = I_1 + I_2$$

Since the cascades R_{13} и R_{24} are connected in parallel, then their voltage is the same: $U_{13} = U_{24}$; by Ohm's law for the circuit section, U=IR;

$$I_1(R_1 + R_3) = I_2(R_2 + R_4)$$

numerically:

$$\begin{cases} 3I_1 = 7I_2 \\ I_1 + I_2 = 4 \end{cases}$$

Solving this system, we obtain

$$I_1 = 2{,}8\ A;\ \ I_2 = 1{,}2\ A$$

. In points B and C the potentials are the same, since these points are short-circuited, let's denote them by ϕ_1. Desired potential difference: $|\Delta\phi| = |\phi_2 - \phi_3| =$

$$= |(\phi_2 - \phi_1) - (\phi_3 - \phi_1)| = |U_1 - U_2| = |I_1R_1 - I_2R_2| = |2.8 * 1{,}0 - 1.2 * 4{,}0| = 2{,}0\ V$$

9.5. (4 points) From the field side the force of 1 µN acts on a charged particle flying into a homogeneous magnetic field with an induction of 0.1 Tesla with a velocity of 10 m/s perpendicular to the field lines.

[5] Determine the charge of the particle in microcoulombs.

(*Yu. V. Maksimachev*)

Answer: $q = 1\ (\mu C)$.

Solution. Let us write the expression for the magnitude of the Lorentz force

$$F_{Lor} = |q| * v * B * \sin\alpha$$

Since $\alpha = 90^o$, then

$$F_{Lor} = qvB \qquad (1)$$

Let's express from (1) the charge value and substitute the numerical values

$$q = \frac{F_{Lor}}{vB} = \frac{10^{-6}}{(10 * 0{,}1} = 10^{-6}\ (C) = 1\ (\mu C)$$

International Physics Olympiad
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Year 2023/2024. Qualifying round

Solutions to problems for grade R10

10.1. (7 points) A cup of 300 cm^3 volume and 100 g mass is slowly immersed upside down in water of 1000 kg/m^3 density. The atmospheric pressure is 100 kPa, the temperature is constant and the same for air and water.

[1] At what minimum depth will the cup begin to sink without the help of an external force?

Comment. Count the depth from the level of water in the cup. Assume that the acceleration of free fall is 10 m/s^2. (*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 20 m.

Solution. First, the cup was in the atmosphere; the air parameters in the cup are shown in figure 8.

When the cup was deepened, the water partially went under the edges of the cup, compressing the air in it to a volume of V_1 and pressure p_1.

At the depth h (see Fig.) the cup can begin to sink without the action of an external force. The condition of the beginning of such motion: $\vec{F}_{total} = 0$.

In projection on the vertical axis we have:

$$F_A - m_{cup} g = 0 \qquad (1)$$

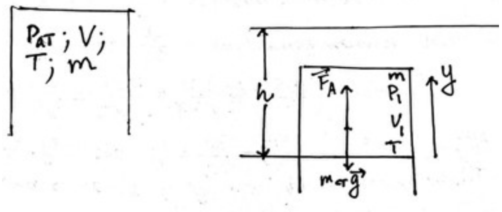

Fig. 8

Since the gravitational force of air in the volume V_1 can be neglected.

Let us transform (1)

$$F_A = \rho g V_1; \quad \rho g V_1 = m_{cup} g; \quad V_1 = \frac{m_{cup}}{\rho};$$

According to problem statement T = const, m = const, since compressed to the volume V_1 air is not escaping from the cup.

According to Boyle-Marriott's law: pV = const. For the case under consideration:

$$p_{atm} V = p_1 V_1$$

where p_1- total pressure at depth h. Since $p_1 = p_{atm} + \rho g h$, then

$$p_{atm} V = (p_{atm} + \rho g h) * \frac{m_{cup}}{\rho}; \quad p_{atm} + \rho g h = \frac{p_{atm} * \rho V}{m_{cup}};$$

$$\rho g h = p_{atm} (\frac{\rho V}{m_{cup}} - 1); \quad h = p_{atm} (\frac{\rho \frac{V}{m_{cup}} - 1)}{\rho g} = \frac{1{,}00 * 10^5 (\frac{3{,}00 * 10^{-4} * 10^3}{0{,}100} - 1)}{10^4} = 20\ m$$

10.2. (7 points) A mole of helium performs a cycle consisting of two isochores and two isobars. The maximum pressure in the cycle is 2 times larger, than the minimum pressure, and the maximum volume is 1.5 times larger, than the minimum volume.

[2] Determine the efficiency of the cycle in percents.

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 12,5%.

Solution. Let's depict the cycle on a p-V diagram.

By definition, $\eta = \frac{A}{Q}$, where A - gas work per cycle, Q - heat received by the gas during the cycle.

The work of the gas is numerically equal to the area under the graph of p(V) bounded by the corresponding values of p and V (see Fig. 9). Then

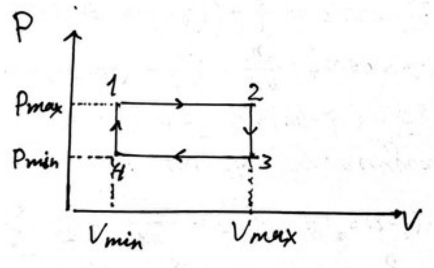

Fig. 9

$$A = (p_{max} - p_{min})(V_{max} - V_{min}) = p_{max}(V_{max} - V_{min}) - p_{min}(V_{max} - V_{min})$$

From the Clapeyron - Mendeleev equation: $pV = \nu RT$. When p=const we have $p\Delta V = \nu R\Delta T$. Then

$$p_{max}(V_{max} - V_{min}) = \nu R(T_2 - T_1); p_{min}(V_{max} - V_{min}) = \nu R(T_3 - T_4)$$

Hence,

$$A = \nu R(T_2 - T_1 - T_3 + T_4) \qquad (1)$$

where T_1, T_2, T_3, T_4 – gas temperatures at points 1, 2, etc. (see Fig. 9)

The gas receives heat from the heater in processes 1-2 and 4-1;

In the others it gives heat to the cooler.

According to the first principle of thermodynamics $Q = \Delta U + A$. Then, taking into account that the gas is one-atomic, we obtain

$$Q_{12} = \Delta U_{12} + A_{12} = \frac{3}{2}\nu R(T_2 - T_1) + \nu R(T_2 - T_1) = \frac{5}{2}\nu R(T_2 - T_1)$$

Since the 1-4 transition is isochoric

$$Q_{41} = \Delta U_{41}; \quad Q_{41} = \frac{3}{2}\nu R(T_1 - T_4)$$

Total heat input in the cycle is equal to

$$Q = Q_{12} + Q_{41} = \frac{5}{2}\nu R(T_2 - T_1) + \frac{3}{2}\nu R(T_1 - T_4) \qquad (2)$$

Express T_2, T_3 and T_4 in terms of T_1;

In process 1-2:

$$p = const; \quad \frac{V}{T} = const; \quad \frac{V_1}{T_1} = \frac{V_2}{T_2}; \quad \frac{T_2}{T_1} = \frac{V_2}{V_1} = \frac{V_{max}}{V_{min}} = 1{,}5; \quad T_2 = 1{,}5T_1$$

In process 2-3:

$$V = const; \quad \frac{p}{T} = const; \frac{p_2}{T_2} = \frac{p_3}{T_3}; \quad \frac{T_2}{T_3} = \frac{p_2}{p_3}; \quad \frac{p_{max}}{p_{min}} = 2$$

Then

$$T_3 = \frac{T_2}{2} = 0{,}75T_1$$

In process 4-1:

$$V = const; \quad \frac{p_1}{T_1} = \frac{p_4}{T_4}; \quad \frac{T_1}{T_4} = \frac{p_1}{p_4} = \frac{p_{max}}{p_{min}} = 2; \quad T_4 = 0{,}5T_1$$

Let's calculate the work by the formula (1)

$$A = \nu RT_1(1{,}5 - 1 - 0{,}75 + 0{,}5) = 0{,}25\nu RT_1$$

Total heat input in the cycle according to the formula (2)

$$Q = \frac{5}{2}\nu RT_1(1{,}5 - 1) + \frac{3}{2}\nu RT_1(1 - 0{,}5) = 2\nu RT_1$$

Then

$$\eta = \frac{A}{Q} = \frac{0{,}25\nu RT_1}{2\nu RT_1} = 0{,}125 = 12{,}5\%$$

10.3. (10 points) A uniformly loaded sled moving on ice at a speed of 5 m/s drives onto a road covered with sand.

[3] Determine the distance traveled by the sled on the road, if the length of the skids is 1 m and the coefficient of sliding friction on the road surface is 0.5.

Comment. Neglect the friction on ice. Assume that the acceleration of free fall is 10 m/s^2.

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 3 m.
Solution. Let us depict in Figure 10 the initial stage of the sled's transition from ice (where there is no friction) to sand (where the friction force is proportional to the weight of the part of the sled that is on the sand).

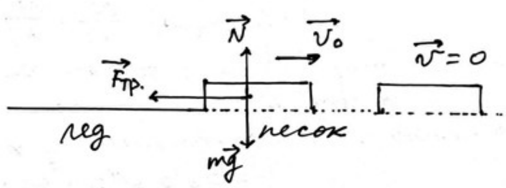

Fig. 10

Since the frictional force is proportional to the mass of the load above the road, and therefore proportional to the length of the skid, as the sled enters the road $F_{fr} \propto S$. Let us denote by l the length of the skid.

Let's draw a graph $F_{fr}(S)$.

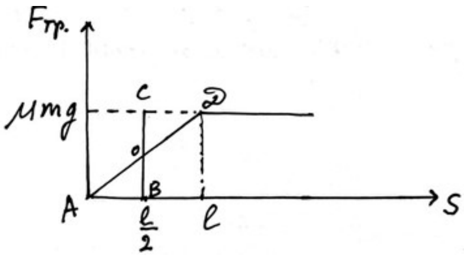

Fig. 11

The work of the sled to overcome the friction force as it enters the road is numerically equal to the area under the graph $F_{fr}(S)$, bounded by the value l (see Fig.11), and is equal to $A_1 = \mu mgl/2$. When the sled is fully on the road the work is equal to $A_2 = \mu mgS_1$. According to the energy-work relation theorem, the work of the resultant force is equal to the change in kinetic energy. Since the terminal velocity is zero, then

$$\frac{mv^2}{2} = A_1 + A_2$$

Substituting the expressions for the works A_1 and A_2 and transforming the expression, we obtain

$$S_1 = \frac{v^2}{2\mu g} - \frac{l}{2}$$

Let's substitute the numerical values taking into account that the total length is sum of S_1 and l. Then

$$S = S_1 + l = 2 + 1 = 3\ m$$

10.4. (10 points) A thin hoop of 0.5 kg mass rolls on a horizontal surface without slipping. The velocity of the center of the hoop relative to the Earth is 2 m/s.

[4] Determine the kinetic energy of the hoop in the frame of reference associated with the Earth.

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 2 J.
Solution.

In the frame of reference associated with the center of the hoop, all points of the hoop move uniformly along the circle, and their velocities are tangent to the circle (hoop). Since the center of the hoop is moving relative to the Earth with velocity $\vec{v}$ (fig. 12), then in the coordinate system associated with the center of the hoop, the hoop point O', touching the Earth is moving at a velocity - $\vec{v}$. So, in this frame of reference, all points of the hoop have the same modulus of velocity which is equal to $v' = v$ and tangent to the circle.

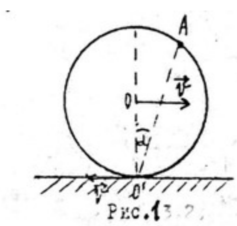

Fig. 12

In the coordinate system associated with the Earth (Fig. 13), the velocity of point A is determined by the law of addition of velocities:

$$\vec{v}_1 = \vec{v}' + \vec{v}$$

where $\vec{v}_1$ the velocity of point A of the hoop relative to the Earth; $\vec{v}'$ - velocity of the same point A in the coordinate system associated with the hoop center; $\vec{v}$ – velocity of the hoop center relative to the Earth, given by the problem statement. Thus $\vec{v}_1$ – is directed along the diagonal of a rhombus with sides being equal to v.

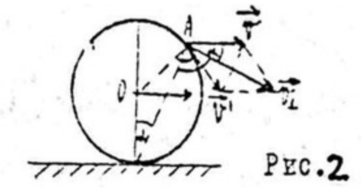

Fig. 13

The velocity of the point O' in this coordinate system will be equal to zero $|\vec{v}_1| = 0$, since, according to the problem statement, the hoop moves without slipping. From this fact we have $v' = v$. So, the velocity of point A of the hoop relative to the Earth is equal to $|\vec{v}_1| = 2v\cos\alpha$.

Let us now proceed to calculate the kinetic energy of the hoop in the reference frame associated with the Earth. For this purpose, it is necessary to take into account that the points of the hoop have different modulus of velocities in this coordinate system. Let us divide the whole hoop into N elementary masses Δm ($\Delta m = m/N$) and calculate the kinetic energy of a pair of hoop points located at the ends of an arbitrary hoop diameter (fig. 14).

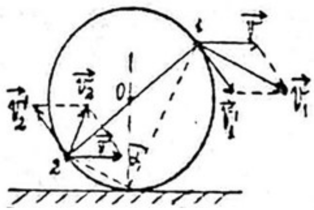

Fig. 14

The velocity of a small section of the hoop of mass Δm at point 1 (on Fig.14 – point A) is equal to $2v\cos\alpha$ and the velocity of the same section at point 2 is equal to $2v\cos(90^o - \alpha) = 2v\sin\alpha$

The kinetic energy of the sections at the pair of points 1 and 2 will be equal to

$$E_k' = \frac{\Delta m(2v\cos\alpha)^2}{2} + \frac{\Delta m(2v\sin\alpha)^2}{2} = \frac{2mv^2}{N}$$

Note, that $E_к'$ is independent of α, i.e., will be the same for all similar pairs of points located at the ends of an arbitrary diameter of the hoop. Then the kinetic energy of the hoop will be equal to the sum of kinetic energies of these pairs of points. The number of pairs is equal to $N/2$.

$$E_k = \sum E_k' = E_k' * \frac{N}{2} = \frac{2mv^2}{N} * \frac{N}{2} = mv^2 = 0.5 * 4 = 2J$$

10.5. (4 points) Flux of the induction vector of a homogeneous magnetic field goes through the side surface of a cone with an angle at the apex of 60^o and a 1 meter long generatrix. The field induction is 4.0 Tesla. The axis of the cone is parallel to the field lines.

[5] Find the magnitude of the flux of the induction vector.

(*Problem Bank in Physics for Applicants of the BSTU «Voenmech» named after D.F. Ustinov*)

Answer: 3,1 Wb.

Solution. Let us depict in the figure 15 the cone and the direction of the vectors used in solution of the problem.

By definition, the magnitude of magnetic flux is equal to

$$\Phi = BS_{lat}\cos(\vec{B},\vec{n})$$

From the figure determine the angle between the normal and the induction vector

$$\beta = 90^o - \frac{\alpha}{2} = 60^o$$

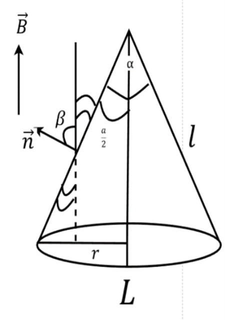

Fig. 15

Find the length of the circle at the base of the cone

$$L = 2\pi r = 2\pi l \sin\frac{\alpha}{2} = \pi l$$

and the lateral surface area of the cone

$$S_{lat} = \frac{\pi l^2}{2}$$

Then

$$\Phi = \frac{B\pi l^2}{2}\cos\beta = \frac{4{,}0 * 3.14}{2 * 2} = 3.1\ (Wb)$$

International Physics Olympiad
«Formula of Unity» / «The Third Millennium»
Year 2023/2024. Qualifying round

Solutions to problems for grade R11

11.1. (5 points) 5 moles of an ideal gas are heated up to 10 K so that the temperature of the gas changes proportionally to the square of the volume of the gas.

[1] What work does the gas do when heated?

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 0,21 kJ.

Solution. According to the Clapeyron-Mendeleev equation $pV = \nu RT$.
Let's use the given dependence for temperature

$$pV = \nu R\alpha V^2$$

then $p = \nu R\alpha V$.
Let's calculate the work of the gas

$$A = \int_{V_1}^{V_2} pdV = \nu R\alpha \int_{V_1}^{V_2} VdV = \nu R\alpha \frac{V^2}{2}\Big|_{V_1}^{V_2} = \frac{\nu R\alpha}{2}(V_2^2 - V_1^2) =$$

$$= \frac{\nu R}{2}\Delta T = \frac{5{,}0 * 8{,}31}{2} * 10 = 0{,}21(kJ)$$

11.2. (7 points) A thin conducting rod of rectangular cross-section slips from rest on a smooth inclined dielectric plane in a vertical homogeneous magnetic field of B = 0.2 Tesla induction (see Fig. 1).

The length of the rod is L = 30 cm, the plane is inclined to the horizon at an angle of $\alpha = 30^o$.The longitudinal axis of the rod keeps the horizontal direction when moving.

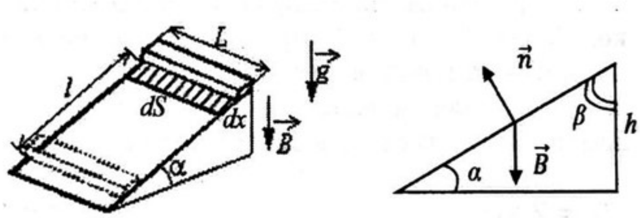

Fig. 1

[2] Calculate the EMF of induction at the ends of the rod at the moment when the rod travels l = 1.5 m distance along the inclined plane.

Comment. Assume that the acceleration of free fall is 10 m/s^2.

(*A.G.Areshkin, O.S. Komarova, V.G. Mozgovaya, D.L. Fedorov*)

Answer: 0,2 V.

Solution. Let us depict the geometry of the problem and the directions of the vectors used in the solution.
According to Faraday's law of electromagnetic induction:

$$\varepsilon_i = -\frac{d\Phi}{dt} \quad (1)$$

where the induction flux $\Phi = BS\cos(\vec{B}, n)$ and $\vec{n}$ – perpendicular to the inclined plane.
By the time dt the rod will move by dx, the area crossed by the magnetic field lines will change by $dS = Ldx$ (see fig.1).
The figure shows that $\beta = 90^o - \alpha = 90^o - 30^o = 60^o$,

then $\angle(\vec{B}, \vec{n}) = 90^o + \beta = 90^o + 60^o = 150^o$. From (1) we get

$$\varepsilon_i = -B\frac{dS}{dt}\cos(\vec{B}, \vec{n}) = -BL\frac{dx}{dt}\cos(\vec{B}, \vec{n}) = -BLv\cos 150^o \qquad (2)$$

According to the conservation of total mechanical energy law

$$mgh = \frac{mv^2}{2}$$

Then $v = \sqrt{2gh}$, where $h = l\sin\alpha$. Substitute in (2):

$$\varepsilon_i = -BL\sqrt{2gl\sin\alpha}\cos 150^o = -BL\sqrt{2gl\sin\alpha}\cos(180^o - 30^o) = BL\sqrt{2gl\sin\alpha}\cos 30^o =$$

$$= 0.2 * 0.3\sqrt{2 * 10 * 1.5 * 0.5} * \frac{\sqrt{3}}{2} = 0.03 * 3\sqrt{5} \approx 0.09 * 2.25 \approx 0.2\ V$$

11.3. (10 points) The density ρ of a rod of 1 m length varies according to the law: $\rho = (1-x)10^3\ \frac{kg}{m}^3$, where x is the distance from the end of the rod in meters. The rod is immersed in water with a density of 1000 kg/m^3.

[3] Determine the length of the submerged part of the rod when it reaches the equilibrium position.

(Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich)

Answer: 0,5 m.

Solution. Let us depict in the figure the y-axis, the rod and the forces acting on it.
Let S be the cross-sectional area of the rod, l – rod's length.
Condition of equilibrium of the rod: $\vec{F}_{total} = 0$.
Project onto the y-axis

$$F_A - mg = 0$$

Transform

$$F_A = \rho_w g V_{i.p.} = \rho_{\text{в}} g S l_{i.p.} \qquad (1)$$

By definition, the mass of the rod is

$$m = \int \rho_r S dx$$

Substitute the expression for the density

$$m = S10^3\int_0^l (1-x)dx = 10^3 S(l - \frac{l^2}{2})$$

Then, from (1):

$$\rho_{\text{в}} g S l_{i.p.} = 10^3 g S(l - \frac{l^2}{2})$$

Transform and substitute the numerical values

$$l_{i.p.} = \frac{10^3(l - \frac{l^2}{2})}{\rho_w} = \frac{10^3(1 - \frac{1^2}{2})}{10^3} = 0{,}5\ m$$

11.4. (10 points) At two different load resistances, the voltage ratio at the terminals of the current source is 5, and the useful power in both cases is 25 W.

[4] Calculate the short-circuit current if the source EMF is 25 V.

(Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich)

Answer: 7,2 A.

Solution. Short-circuit current: $I_{scc} = \frac{\varepsilon}{r}$, where r - internal resistance of the source
Let's find the internal resistance. To do this, first find the relationship between the values of resistances.

From the formula for power we obtain

$$P_1 = \frac{U_1^2}{R_1}; \quad P_2 = \frac{U_2^2}{R_2}; \quad \frac{R_1}{R_2} = \left(\frac{U_1}{U_2}\right)^2 = 5^2 = 25$$

Then

$$R_1 = 25R_2$$

On the other hand: $P = I^2R$;
According to Ohm's law for a complete circuit:

$$I = \frac{\varepsilon}{R + r}$$

Then

$$P_1 = I_1^2R_1 = \frac{\varepsilon^2R_1}{(R_1 + r)^2} = P_2 = \frac{\varepsilon^2R_2}{(R_2 + r)^2} \qquad (1)$$

Substitute the numerical values in (1):

$$25 = \frac{625R_1}{(R_1 + r)^2} = \frac{625R_2}{(R_2 + r)^2}$$

or

$$25R_1 = (R_1 + r)^2; 25R_2 = (R_2 + r)^2$$

We have a system of three equations with three variables:

$$\begin{cases} R_1 = 25R_2 & (2) \\ 25R_1 = (R_1 + r)^2 & (3) \\ 25R_2 = (R_2 + r)^2 & (4) \end{cases}$$

Divide (3) by (4):

$$\frac{R_1}{R_2} = 25 = \frac{(R_1 + r)^2}{(R_2 + r)^2}$$

Extracting the root

$$\frac{R_1 + r}{R_2 + r} = 5$$

Express R_1 in terms of R_2, then

$$\frac{25R_2 + r}{R_2 + r} = 5$$

Expand and get

$$25R_2 + r = 5R_2 + 5r; \quad 20R_2 = 4r; \quad r = 5R_2 = \frac{R_1}{5} \qquad (5)$$

Calculate power

$$P_2 = \frac{\varepsilon^2R_2}{(R_2 + r)^2} = \frac{\varepsilon^2R_2}{36R_2^2} = \frac{\varepsilon^2}{36R_2}$$

Express R_2

$$R_2 = \frac{\varepsilon^2}{36P_2} = \frac{625}{36 * 25} = \frac{25}{36}$$

Using (5), we get

$$I_{scc} = \frac{25 * 36}{125} = \frac{36}{5} = 7.2 \text{ A}$$

11.5. (4 points) A ball of 0.5 kg mass falls on a weightless vertically placed spring with a stiffness coefficient of 1000 N/m.
[5] Determine the maximum compression of the spring if the ball falls from a height of 0.3 m.

Comment. The height is measured from the upper edge of the undeformed spring.

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 6 cm.
Solution. Let's depict in the figure 16 the position of the spring at the initial and final moments of time and the main quantities used to solve the problem

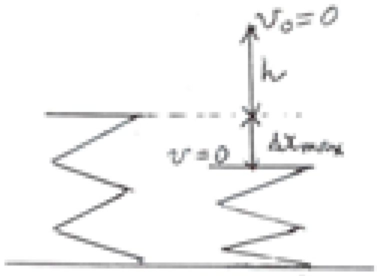

Fig. 16

From the energy conservation law

$$mg(h + \Delta x_{max}) = \frac{k(\Delta x_{max})^2}{2} \qquad (1)$$

Transform (1)

$$\frac{k(\Delta x_{max})^2}{2} - mg\Delta x_{max} - mgh = 0$$

substituting the numerical values and solving the equation, we obtain:

$$500\Delta x_{max}^2 - 5\Delta x_{max} - 1.5 = 0$$

$$100\Delta x_{max}^2 - \Delta x_{max} - 0.3 = 0$$

$$\Delta x_{max} = \frac{1 + \sqrt{1 + 120}}{200} = \frac{12}{200} = 0.06\ m$$

we do not take into account the second root, since it is negative

$$\Delta x_{max} = 0.06\ m$$