International Physics Olympiad
«Formula of Unity» / «The Third Millennium»
Year 2023/2024. Qualifying round

Solutions to problems for grade R8

8.1. (6 points) A drop of oil with a volume of 0.003 mm^3 are spread over the surface of water in a thin layer and cover an area of 300 cm^2.

[1] Determine the average diameter of the oil molecule.

(*G.N.Stepanova*)

Answer: 10^{-10}.

Solution. 1. Since the spreading of oil on the surface of water produces a layer of 1 molecule thickness, and the total volume of oil does not change, then

$$V = Sd$$

2. Transform and substitute the numerical values

$$d = \frac{V}{S} = \frac{3 * 10^{-12}}{3 * 10^{-2}} = 10^{-10}\ (m)$$

8.2. (5 points) The first astronaut of the Earth Yu.A. Gagarin flew around the Earth in 108 min.

[2] Neglecting the height of the ship's orbit compared to the Earth's radius, find the average velocity of the Vostok spacecraft on orbit. Consider the orbit to be circular. The average radius of the Earth is 6400 km. Give the answer in km/h rounded to integers.

(*G.N.Stepanova*)

Answer: 22329.

Solution. 1. Determine the time of the 1 turn in hours

$$t = \frac{108}{60} = 1{,}8(h)$$

2. Determine the value of the Vostok spacecraft velocity in the orbit

$$v = \frac{2\pi R}{t}$$

3. Substitute the numerical values

$$v = \frac{2 * 3{,}14 * 6400}{1{,}8} = 22328{,}8\ (\frac{km}{h})$$

4. Round to integers:

8.3. (7 points) The pool with an area of 100 m^2 is filled with water up to a level of 1.2 m and divided in halves by a partition. The partition is slowly moved so that it divides the pool in a 1:3 ratio.

[3] What work must be done if the water does not penetrate the partition?

(*A.G.Areshkin, O.S. Komarova, V.G. Mozgovaya, D.L. Fedorov*)

Answer: 240000.

Solution. 1. Find the volume of water in the pool

$$V = Sh = 100 * 1{,}2 = 120\ (m^3)$$

2. Determine the mass of water on one side of the baffle (see. Fig. 1)

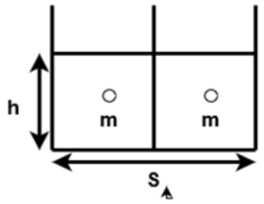

Fig. 1

$$m = \frac{\rho V}{2} = \frac{1000 * 120}{2} = 60000\ (kg)$$

3. Since the volumes of both parts were initially equal, then

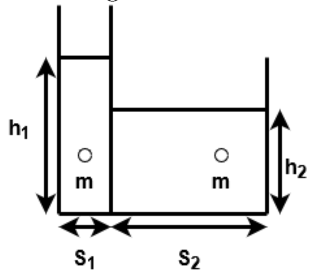

Fig. 2

$$S_1 h_1 = S_2 h_2 \qquad (1)$$

See Fig. 2 for the designations
4. Transform (1) and get

$$\frac{S_1}{S_2} = \frac{h_2}{h_1} = \frac{1}{3}; h_1 = 3h_2; S_2 = 3S_1$$

5. Make a system of equations to determine the areas

$$\begin{cases} S_1 + S_2 = 100 \\ S_2 = 3S_1 \end{cases}$$

6. Solving the system, we get

$$4S_1 = 100; S_1 = 25\ (m^2); S_2 = 75\ (m^2)$$

7. Find new water levels in both parts of the pool

$$\frac{S}{2}h = S_1 h_1 = S_2 h_2; h_1 = \frac{Sh}{2S_1} = \frac{120}{50} = 2{,}4\ (m)$$

$$h_2 = \frac{Sh}{2S_2} = \frac{120}{150} = 0{,}8\ (m)$$

8. Work is equal to the change in potential energy of the water in the pool

$$A = \frac{mgh_1}{2} + \frac{mgh_2}{2} - 2\frac{mgh}{2} = \frac{mg}{2}(h_1 + h_2 - 2h) = \frac{60000 * 10}{2}(3{,}2 - 2{,}4) = 240 * 10^3\ (J) = 240\ (kJ)$$

8.4. (7 points) An aluminum cylinder has a height of 10 cm.
[4] What is the height of the iron cylinder of the same diameter if it exerts the same pressure on the table? Give the answer in cm rounded to the tenth.

(G.N.Stepanova)

Answer: 3,4.
Solution. 1. The pressure is created by the weight of the cylinder. Then by definition

$$p = \frac{mg}{S}$$

2. Since the areas and masses are the same

$$S_1 = S_2; m_1 = m_2$$

then

$$\rho_1 V_1 = \rho_2 V_2; \rho_{Al} h_{Al} = \rho_{Fe} h_{Fe}$$

3. Transform and substitute the numerical values

$$h_{Fe} = \frac{\rho_{Al} h_{Al}}{\rho_{Fe}} = \frac{2688 * 0{,}1}{7874} = 0{,}034\ (m) = 3{,}4\ (cm)$$

8.5. (5 points) A diesel locomotive is pulling the train at a speed of 72 km/h, developing a power of 880 kW.

[5] How large is the traction force in this case?

(*G.N.Stepanova*)

Answer: 44000.
Solution. 1. Express the velocity in meters per second

$$v = 20\ (\frac{m}{s})$$

2. Using the formula for power, express the force

$$F_{\text{тяги}} = \frac{P}{v} = \frac{8{,}8 * 10^5}{20} = 44\ (kN)$$

8.6. (6 points) The driver of a resting electric locomotive notices that a platform with a bunch of organic fertilizer, the distance to which is equal to L=100 m, is rolling directly towards him with the velocity of v0=20 m/s. This very threatening situation is not quite hopeless because the wagon, which is experiencing friction forces, is moving with the constant acceleration of -0.1 m/s^2.

[6] What constant acceleration A must the electric locomotive begin to move with in order for a soft coupling to occur between it and the platform, as a result of which the railroad property would not have to be washed?

(*A.S. Chirtsov*)

Answer: 1,9.
Solution. 1. Denote the accelerations of the platform and the electric locomotive as a_1 and a_2, respectively. We start counting time from the moment of detection of the platform.
2. Write down the equations for the speeds of the platform and the electric locomotive

$$v_1(t) = v_0 - a_1 t \qquad (1)$$

$$v_2(t) = a_2 t \qquad (2)$$

3. Determine the moment of time $t_{\text{в}}$, when the velocities of the bodies are equal

$$v_1(t_{\text{в}}) = v_2(t_{\text{в}}) \qquad (3)$$

Substitute equations (1) and (2) into (3):

$$t_{\text{в}} = \frac{v_0}{a_1 + a_2} \qquad (4)$$

4. We will count the distances from the point of the initial position of the platform. Denote the initial distance between the platform and the electric locomotive as L.
5. Write down the equations for the coordinates of the platform and the electric locomotive

$$L_1(t) = v_0 t - \frac{a_1 t^2}{2} \qquad (5)$$

$$L_2(t) = L + \frac{a_2 t^2}{2} \qquad (6)$$

6. At the time $t_в$ the coordinates of both bodies must coincide

$$L_1(t_в) = L_2(t_в)$$

Substitute (4) - (6):

$$\frac{v_0^2}{(a_1+a_2)} - \frac{a_1 v_0^2}{2(a_1+a_2)^2} = L + \frac{a_2 v_0^2}{2(a_1+a_2)^2} \qquad (7)$$

7. Transform (7):

$$2L(a_1+a_2) = v_0^2 \qquad (0)$$

8. From (8) we obtain the expression for the acceleration of the electric locomotive

$$a_2 = \frac{v_0^2}{2L} - a_1 \qquad (9)$$

9. Substituting the numerical values into (9), we get the answer

$$a_2 = \frac{400}{200} - 0{,}1 = 1{,}9\ (\frac{m}{s^2})$$

8.7. (7 points) To determine the specific heat capacity of a substance, a steel cylinder of a 156 g mass, preheated in boiling water, was placed in an aluminum calorimeter with water. The mass of the calorimeter is 45 g, the mass of the water is 100 g, the initial temperature of the water is 17 °C. After some time the temperature in the calorimeter became 29 °C.

[7] Find the specific heat capacity of steel. Give the answer rounded to integers.

(*G.N.Stepanova*)

Answer: 499 (*allowed* 500).

Solution. 1. When the cylinder cools down, the amount of heat it will lose is equal to the amount of heat gained by the calorimeter and the water. Let us express this in the form of the equation

$$c_{st} m_{st}(t_1 - t_{cal}) = c_{Al} m_{Al}(t_{cal} - t_W) + c_W m_W (t_{cal} - t_W) \qquad (1)$$

2. Here $c_{Al} = 900\ (\frac{J}{kg*K})$, $c_W = 4200\ (\frac{J}{kg*K})$.

3. Express from (1) the heat capacity of iron and substitute the numerical values

$$c_{st} = \frac{(c_{st} m_{st} + c_W m_W)(t_{cal} - t_W)}{m_{st}(t_1 - t_{cal})} = \frac{(900 * 0{,}045 + 4200 * 0{,}1) * 12}{0{,}156 * 71} = 499\ (\frac{J}{kg * K})$$

8.8. (7 points) Two spirals of resistance 100 ohms and 200 ohms were used to make an electric stove, designed for a voltage of 210 V. The power of the stove is changed by switching the spirals.

[8] Find the minimum possible power of the stove.

(*Yu. V. Maksimachev*)

Answer: 147.

Solution. 1. The power of the circuit section is equal to

$$P = \frac{U^2}{R}$$

2. For minimum power, the maximum resistance is needed and it is equal to the sum of resistances

$$P_{min} = \frac{U^2}{R_{max}} = \frac{U^2}{R_1 + R_2}$$

3. Substitute the numerical values

$$P_{min} = \frac{210 * 210}{300} = 7 * 21 = 147\ (W)$$

8.9. (5 points) Two isolated conductors of 10 cm length are tied together and are located perpendicular to the lines of a magnetic field with an induction of 0.2 Tesla.

[9] Find the modulus of the Ampere force, if the currents of 7 A and 9 A flow towards each other in the conductors.

(*Yu. V. Maksimachev*)

Answer: 0,04.

Solution. 1. If the conductor with current and induction of the magnetic field are perpendicular, the Ampere force is equal to

$$F_A = IlB$$

2. Since the conductors are connected and the currents in them are opposite, the resultant force is equal to the difference of forces acting on the conductors

$$F_{Ares} = (I_2 - I_1)lB$$

3. Substitute the numerical values

$$F_{Ares} = 2 * 0{,}1 * 0{,}2 = 0{,}04\ (N)$$

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Solutions to problems for grade R9

9.1. (7 points) Water is pumped out of a large tank by a pump. The power of the pump is 4 kW, the efficiency of the setting is 12.5%.

[1] What is the modulus of the speed the water, having density of ρ=1000 kg/m^3 , flows out from a smooth hose, having cross-sectional area of 10 sm^2, the tip of which is at the same level with the surface of water in the tank?

(*A.G.Areshkin, O.S. Komarova, V.G. Mozgovaya, D.L. Fedorov*)

Answer: 10.

Solution. 1. According to the definition of useful power and the fact that the work goes on acquisition of kinetic energy by some mass of water (there is no change of potential energy, because the tank is large and the tip is at the same level with the surface of water in the tank), we get:

$$P_{useful} = \frac{A}{t} = \frac{mv^2}{2t} \qquad (1)$$

2. From the definition of efficiency we have

$$\nu = \frac{P_{useful}}{P_{full}}; P_{useful} = \nu P_{full} \qquad (2)$$

3. Combining (1) and (2), we get

$$\nu P_{full} = \frac{mv^2}{2t} = \frac{\rho S l v^2}{2t}$$

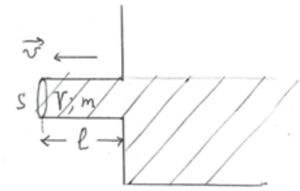


Fig. 3

Here ρ - water density, S - cross-sectional area of the pipe, l - length of the pipe section from which water will flow out during the time t (see figure 3)

4. Since the ratio of l and t is equal to velocity, then

$$\nu P_{full} = \frac{\rho S v^3}{2}$$

5. Transforming (3), substitute the numerical values:

$$v = \sqrt[3]{\frac{2\nu P_{full}}{\rho S}} = \sqrt[3]{\frac{2 * 12{,}5 * 10^{-2} * 4 * 10^3}{10^3 * 10^{-2}}} = 10 \ (\frac{m}{s})$$

9.2. (7 points) A body of a 2 kg mass moves on a horizontal surface under the action of a force equal to 20 N in modulus and directed at an angle of 30° to the horizon.

[2] Determine the modulus of the interaction force between a body and a surface, if the sliding friction coefficient is equal to 1.

(*Yu.V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 14,1.

Solution. 1. Represent all the forces acting on the body in the figure 4

2. The total force acting on the body from the surface is the combination of the support reaction N и F_{fr}, therefore

$$F_{tot} = \sqrt{N^2 + F_{fr}^2} = \sqrt{N^2 + \mu^2 N^2} = N\sqrt{1+\mu^2} = N\sqrt{2} \qquad (1)$$

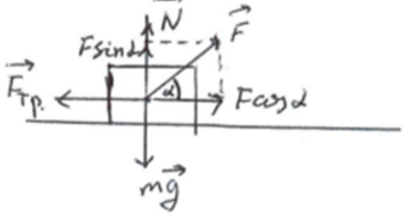


Fig. 4

3. Write the equation for the force components with respect to the vertical axis

$$F\sin\alpha + N = mg \qquad (2)$$

4. Transform (2) and substitute the numerical values

$$N = mg - F\sin\alpha = 20 - 20 * \frac{1}{2} = 10\ (N) \qquad (3)$$

5. Combining (1) and (3), we obtain

$$F_{tot} = 1{,}41 * 10 = 14{,}1\ (N)$$

9.3. (6 points) Ball is 0.2 kg mass and 7 liters in volume

[3] Find the minimum work required to immerse the ball into the water of 1 g/cm^3 density from a depth of 1 m to a depth of 21 m. Neglect the force of water resistance.

(*Yu. V. Maksimachev*)

Answer: 1360.

Solution. 1. Represent all the forces acting on the body in the figure 5

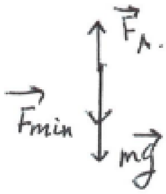


Fig. 5

2. The minimum work will be at an infinitely slow process, when the velocity of the body movement is infinitely small and the resultant of all forces is zero

$$\vec{v} = const; F_{res} = 0$$

3. Then

$$F_{min} + mg = F_A$$

and

$$F_{min} = F_A - mg = \rho g V - mg = g(\rho V - m)$$

4. By definition of the work of a constant force

$$A_{min} = F_{min}(h_2 - h_1) = g(\rho V - m)(h_2 - h_1) \qquad (1)$$

5. Substituting the numerical values into (1), we get

$$A_{min} = 10(10^3 * 7 * 10^{-3} - 0{,}2)20 = 6{,}8 * 200 = 68 * 20 = 1360\ (J)$$

9.4. (5 points) A thin homogeneous rod of 60 grams mass made of wood is hung on a thread at one end and the other end is half submerged in water.

[4] Find the magnitude of Archimedes' force applied to the rod.

(*D.L. Fedorov, V.A. Zhivulin*)

Answer: 0,4.

Solution. 1. Represent all the forces acting on the rod in the figure 6

2. Write down the equation of equilibrium (equality of moments of forces with respect to the point of attachment to the thread)

$$mg\frac{l}{2}\cos\alpha = F_A\frac{3}{4}l\cos\alpha \qquad (1)$$

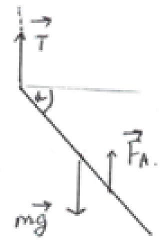


Fig. 6

3. Transform (1)

$$mg = \frac{3}{2}F_A; F_A = \frac{2}{3}mg$$

4. Substitute the numerical values

$$F_A = \frac{2}{3} * 0{,}06 * 10 = 0{,}4\ (N)$$

9.5. (6 points) A hopper was filled with some mass of sand and twice the mass of cement to make a concrete mixture.

[5] Determine the specific heat capacity of the mixture after mixing. The specific heat capacity of sand is 960 J/(kg*K) and that of cement is 810 J/(kg*K).

(*Yu. V. Maksimachev*)

Answer: 860.

Solution. 1. By definition, the specific heat capacity of a mixture is equal to the ratio of the amount of heat transferred to the mixture to the product of the mass of the mixture and the change in its temperature

$$c_{mix} = \frac{Q_{mix}}{m_{mix}\Delta T}$$

2. Taking into account that the total amount of heat is distributed between sand and cement, and the sum of their masses is equal to the mass of the mixture, we get

$$c_{mix} = \frac{Q_{mix}}{m_{mix}\Delta T} = \frac{Q_c + Q_s}{(m_c + m_s)\Delta T} = \frac{c_c m_c \Delta T + c_s m_s \Delta T}{(m_c + m_s)\Delta T} = \frac{c_c m_c + c_s m_s}{m_c + m_s} \qquad (1)$$

3. Take into account the mass ratio of the components and transform (1)

$$c_{mix} = \frac{c_c m_c + c_s m_s}{m_c + m_s} = \frac{c_c * 2m_s + c_s m_s}{2m_s + m_s} \qquad (2)$$

4. Substitute the numerical values into (2) and reduce

$$c_{mix} = \frac{c_c * 2 + c_s}{2 + 1} = \frac{1620 + 960}{3} = 540 + 320 = 860\ \left(\frac{J}{kg * K}\right)$$

9.6. (6 points) There are 120 g of gas in an open vessel of 0.45 m^3 volume. The temperature of the gas is increased from 300 K to 450 K at a constant pressure of 166 kPa.

[6] How many moles of gas will come out of the vessel?

(*Yu. V. Maksimachev*)

Answer: 10.

Solution. 1. Write the Clapeyron-Mendeleev equation for the gas inside the vessel at initial temperature

$$pV = \frac{m_1}{M}RT_1 \qquad (1)$$

2. Denote by Δv the number of moles of gas that will escape from the vessel

3. Write the Clapeyron-Mendeleev equation for the gas inside the vessel at the new temperature

$$pV = \left(\frac{m_1}{M} - \Delta v\right) RT_2 \qquad (2)$$

4. Combine (1) and (2)

$$\frac{m_1}{M} RT_1 = \left(\frac{m_1}{M} - \Delta v\right) RT_2 \qquad (3)$$

5. Substitute numerical values into (3) and transform:

$$300\frac{m_1}{M} = 450\left(\frac{m_1}{M} - \Delta v\right); \frac{m_1}{M} = 1{,}5\left(\frac{m_1}{M} - \Delta v\right); \frac{m_1}{M} = 1{,}5\frac{m_1}{M} - 1{,}5\Delta v; 1{,}5\Delta v = 0{,}5\frac{m_1}{M}$$

6. Then, using (1), we have

$$\Delta v = \frac{1}{3}\frac{m_1}{M} = \frac{1}{3}\frac{pV}{RT_1} = \frac{1}{3}\frac{166 * 10^3 * 0{,}45}{8{,}3 * 300} = \frac{9 * 10^3}{9 * 10^2} = 10\ (moles)$$

9.7. (7 points) Two identical parallel connected capacitors without dielectric were charged to a voltage of 40 V and then were disconnected from the circuit.

[7] Determine the potential difference on the air capacitor, if the space between the linings of the other capacitor was filled with a substance with dielectric constant of $\varepsilon = 7$.

(*Yu. V. Maksimachev*)

Answer: 10.

Solution. 1. Represent in the figure 7 the initial and final situation with the distribution of voltages on capacitors

2. Since the capacitors are disconnected from the circuit after charging, then

$$q_{full} = const$$

3. By definition of capacitance

$$C = \frac{q}{U}$$

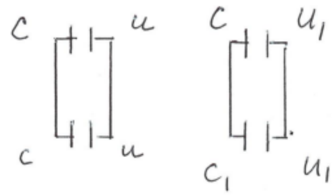


Fig. 7

4. For the first case the charges on both capacitors are the same, so

$$q_{full} = 2CU$$

5. For the second case, the charges on the capacitors are different, but the voltages are equal, therefore

$$q_{full} = (C + C_1)U_1$$

6. Then

$$2CU = (C + C_1)U_1 \qquad (1)$$

7. Using the formula for the capacitance of a flat capacitor

$$C = \frac{\varepsilon_0 \varepsilon_{air} S}{d}; C_1 = \frac{\varepsilon_0 \varepsilon S}{d}; \varepsilon_{air} = 1; C_1 = \varepsilon C \qquad (2)$$

8. From (1) and (2) we get

$$2CU = C(\varepsilon + 1)U_1 \qquad (3)$$

9. Transforming (3) and substituting the numerical values, we have

$$U_1 = \frac{2U}{\varepsilon + 1} = \frac{80}{8} = 10\ (V)$$

9.8. (6 points) A coil of a copper wire has a mass of 1.78 kg and a resistance of 3.4 Ohms.

The resistivity of copper is 1.7*10^{-8} Ohm*m, and the density of copper is 8.9*10^3 kg/m^3 .

[8] Determine the area of the cross section of the wire in mm^2

(*Yu.V. Maksimachev*)

Answer: 1.

Solution. 1. By definition of resistivity

$$R = \frac{\rho_{sp} * l}{S} \qquad (1)$$

2. By definition of density

$$\rho_r = \frac{m}{V} = \frac{m}{Sl} \qquad (2)$$

3. Multiply (1) and (2), then

$$R * \rho_r = \frac{\rho_{sp} * l}{S} * \frac{m}{Sl} = \frac{\rho_{sp} m}{S^2 l}$$

4. Transform (3) and substitute the numerical values

$$S^2 = \frac{\rho_{sp} m}{R * \rho_r} = \frac{1{,}7 * 10^{-8} * 1{,}78}{3{,}4 * 8{,}9 * 10^3} = \frac{1{,}78 * 10^{-8}}{17{,}8 * 10^3} = 10^{-12}$$

5. Find the cross-sectional area

$$S = \sqrt{10^{-12}} = 10^{-6}\ (m^2) = 1\ (mm^2)$$

9.9. (7 points) A small ball is held stationary over a very long and flat inclined plane forming an angle $\alpha = 30°$ with the horizon. The initial distance from the ball to the inclined surface is h = 1m. The ball is released without initial velocity.

[9] Find the distance between the first and the 2022d impacts of the ball on the inclined surface. surface. Neglect the influence of air and consider all impacts to be absolutely elastic. Give the answer rounded to 1 sm.

(*A.S. Chirtsov*)

Answer: 8172924.

Solution. 1. From the law of conservation of mechanical energy we determine the velocity v_0, with which the ball first hits the surface

$$\frac{mv_0^2}{2} = mgh$$

Then $v_0 = \sqrt{2gh}$

2. Put the xOy coordinate system to describe the motion of the ball after the first bounce from the surface. The point O is the place of the first collision, the x-axis is along the inclined plane, the y-axis is perpendicular to the plane and directed upward.

3. Denote as v_{nx} и v_{ny} the components of the ball velocity along the axes at the moment of the n-th bounce, where n takes values from 1 to 2021, since we are not interested in the motion after 2022 bounce.

4. After the first bounce

$$v_{1x} = v_0 \sin\alpha$$

$$v_{1y} = v_0 \cos\alpha$$

5. In the chosen coordinate system the accelerations along the axes

$$g_x = g \sin\alpha$$

$$g_y = g \cos\alpha$$

Where acceleration g_y is directed opposite to the y-axis direction.

6. Write expressions for velocity components after the n-th bounce

$$v_y(t) = v_{ny} - g_y t \qquad (1)$$

$$v_x(t) = v_{nx} + g_x t \qquad (2)$$

It follows from the formula (2) that the velocity component along the x-axis will increase after each bounce.

7. Write the equations for the coordinates of the ball after the n-th bounce (here time is counted from the moment of the n-th bounce)

$$y(t) = v_{ny} t - \frac{g_y t^2}{2} \qquad (3)$$

$$x(t) = L_n + v_{nx} t + \frac{g_x t^2}{2} \qquad (4)$$

Here L_n - distance along the x-axis from the starting point to the n-th rebound point.

8. Determine the time of motion $t_{n\text{п}}$ between the n-th и (n+1)-th bounces. For this purpose equate $y(t_{n\text{п}})$ to zero. Then

$$v_{ny} t_{n\text{п}} - \frac{g_y t_{n\text{п}}^2}{2} = 0$$

Therefore

$$t_{n\text{п}} = \frac{2v_{ny}}{g_y} \qquad (5)$$

9. Substitute (5) into (1) and get the y component of velocity before the next impact

$$v_{n\text{п}} = v_{ny} - g_y \frac{2v_{ny}}{g_y} = -v_{ny}$$

So we get that absolute values of $v_{n\text{п}}$ and v_{ny} are equal. Therefore, all v_{ny} are the same and equal to v_{1y}. Hence, all times between successive impacts are also equal

$$t_{n\text{п}} = \frac{2v_{1y}}{g_y} = \frac{2v_0}{g} \qquad (6)$$

10. Find the path travelled along the x-axis between two consecutive impacts. For this purpose put (6) in (4)

$$\Delta x_n = x(t_{n\text{п}}) - x(0) = v_{nx} t_{n\text{п}} + \frac{g_x t_{n\text{п}}^2}{2} = (v_{1x} + g_x(n-1)t_{n\text{п}})t_{n\text{п}} + \frac{g_x t_{n\text{п}}^2}{2} =$$

$$= (v_0 \sin\alpha + g \sin\alpha (n-1)\frac{2v_0}{g})\frac{2v_0}{g} + \frac{g \sin\alpha}{2}\frac{4v_0^2}{g^2} = 4\frac{v_0^2}{g} n \sin\alpha \qquad (7)$$

11. To determine the distance between the 1st and 2022nd impacts, add up all Δx_n from the 1st to 2021st

$$L_{2021} = \sum_{n=1}^{2021} \Delta x_n = 4\frac{v_0^2}{g} \sin\alpha \sum_{n=1}^{2021} n \qquad (8)$$

12. The sum can be found as the sum of the arithmetic progression

$$\sum_{n=1}^{2021} n = \frac{2022 * 2021}{2}$$

13. Using the expression for v_0, we get

$$L_{2021} = 4\frac{2gh}{g}\sin\alpha * 2021 * 1011 = 8h\sin 30^\circ * 2021 * 1011 = 8172924\ (m)$$

International Physics Olympiad
«Formula of Unity» / «The Third Millennium»
Year 2023/2024. Qualifying round

Solutions to problems for grade R10

10.1. (7 points) There is a column of air trapped by a drop of mercury in a tube sealed on one side. The length of the column of air when the tube is placed vertically with the open end up is 10 cm, and when the tube is deflected 60° from the vertical - 12 cm.

[1] Determine in centimeters the length of the column of air if the tube is placed with the open end downwards.

(*A.G.Areshkin, O.S. Komarova, V.G. Mozgovaya, D.L. Fedorov*)

Answer: 30.

Solution. 1. Represent the tube positions in Figure 8 and mark lengths and pressures on it
2. Write the equations of equilibrium for the first two cases

$$P_1 = P_{at} + \frac{mg}{S}; P_2 = P_{at} + \frac{mgcos\alpha}{S} \qquad (1)$$

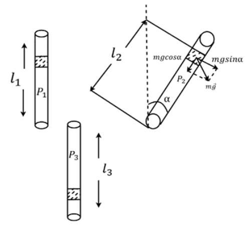


Fig. 8

3. Express the volumes of air through the cross-sectional area of the tube and the length of the air column

$$V_1 = l_1 S; V_2 = l_2 S$$

4. Since the temperature is constant, then

$$PV = const \qquad (2)$$

5. Transform (2) and substitute expressions for volumes

$$P_1 V_1 = P_2 V_2;$$

$$P_1 l_1 S = P_2 l_2 S$$

6. Reduce by S:

$$\frac{P_1}{P_2} = \frac{l_1}{l_2} = \frac{0.12}{0.1} = 1.2$$

Then

$$P_1 = 1.2 P_2 \qquad (3)$$

7. Substitute expressions (1) into (3)

$$P_{at} + \frac{mg}{S} = 1.2\left(P_{at} + \frac{mgcos\alpha}{S}\right)$$

Transform and get

$$P_{at} + \frac{mg}{S} = 1.2P_{at} + 1.2\frac{mg * 0.5}{S}$$

$$P_{at} + \frac{mg}{S} = 1.2P_{at} + 0.6\frac{mg}{S}$$

$$P_{at} = 2\frac{mg}{S} \qquad (4)$$

8. Write the equations of equilibrium for the third case

$$P_3 + \frac{mg}{S} = P_{at} \qquad (5)$$

9. Substitute the relation (4) into (5) and (1), then

$$P_3 = \frac{mg}{S}; P_1 = \frac{3mg}{S} \qquad (6)$$

10. Since the temperature is constant, by analogy with (5) and (6)

$$P_1 V_1 = P_3 V_3$$

$$P_1 l_1 S = P_3 l_3 S; l_3 = \frac{P_1 l_1}{P_3} = \frac{P_1}{P_3} l_1$$

11. Transform, using (6), and substitute the numerical values

$$l_3 = \frac{P_1}{P_3} l_1 = 3 l_1 = 3 * 0.1 = 0.3\ (m) = 30\ (cm)$$

10.2. (7 points) The gas in a cylindrical vessel is divided into two parts by an easily movable piston, which has a mass of 40 kg and an area of 10 cm^2. At horizontal position of the cylinder the gas pressure in the vessel on both sides of the piston is equal to 300 kPa.

[2] Determine in kPa the pressure of the gas above the piston when it is placed vertically. The temperature of the gas on both sides of the piston is the same.

(*Problem Bank in Physics for Applicants of the BSTU «Voenmech» named after D.F. Ustinov*)

Answer: 200.

Solution. 1. Represent the two positions of the cylinder in the figure 9 with notations of the parameters

2 By definition of pressure

$$P = \frac{F}{S}$$

3. Since the piston is in equilibrium at the vertical position of the cylinder, we can write the equation of equilibrium for the piston as follows

$$P_1 S = P_2 S + m_n g \qquad (1)$$

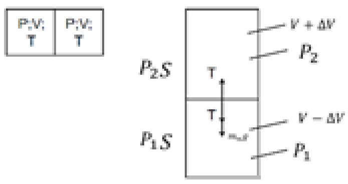


Fig. 9

4. Since the process is isothermal and the total combined volume of the two parts of the cylinder remains constant, we can write the following equation

$$P_2(V + \Delta V) = P_1(V - \Delta V) = PV \qquad (2)$$

5. Transform (1)

$$P_1 = P_2 + \frac{m_n g}{S} \qquad (3)$$

6. Let us introduce the notation

$$\frac{V}{\Delta V} = x$$

7. So we can rewrite (2) in the form:

$$P_2(x+1)\Delta V = P_1(x-1)\Delta V = Px\Delta V \qquad (4)$$

8. Transform (4) and substitute it into (3)

$$P_1 = \frac{Px}{x-1}; P_2 = \frac{Px}{x+1} \qquad (5)$$

$$\frac{Px}{x-1} = \frac{Px}{x+1} + \frac{m_n g}{S} \qquad (6)$$

9. Substitute numerical values into (6), transform and get

$$\frac{3*10^5x}{x-1}=\frac{3*10^5x}{x+1}+\frac{400}{10^{-3}};\frac{3*10^5x}{x-1}=\frac{3*10^5x}{x+1}+4*10^5$$

$$\frac{3x}{x-1}=\frac{3x}{x+1}+4$$

$$3x(x+1)=3x(x-1)+4(x-1)(x+1)$$

$$3x^2+3x=3x^2-3x+4\left(x^2-1\right)$$

$$6x=4x^2-4;3x=2x^2-2;2x^2-3x-2=0$$

$$x_{1,2}=\frac{3\pm\sqrt{9+16}}{4}=\frac{3\pm5}{4}$$

$$x=2 \qquad (7)$$

since the solution can only be a positive root.
10. Substituting the root of (7) into the relation (5), we get

$$P_2=\frac{P_x}{x+1}=\frac{3*10^5*2}{3}=2*10^5=200\ (kPa)$$

10.3. (7 points) Two equally charged balls, suspended on strings of equal length, have separated by some angle.
[3] What should be the material density of the balls so that when they are immersed in kerosene, the angle between them does not change? The density of kerosene is 0.8 g/cm^3, the dielectric constant is equal to 2.
(*Problem Bank in Physics for Applicants of the BSTU «Voenmech» named after D.F. Ustinov*)
Answer: 1600.
Solution. 1. Represent on the figure 10 the forces acting on a charged ball in the air
2. Write down the equation of equilibrium with respect to the vertical axis

$$T_1\cos\frac{\alpha}{2}=mg \qquad (1)$$

3. Write the equation of equilibrium with respect to the horizontal axis

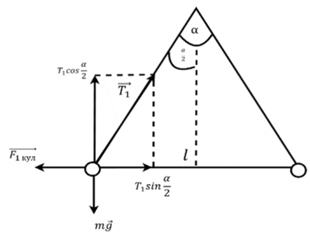


Fig. 10

$$T_1\sin\frac{\alpha}{2}=9*10^9\frac{q^2}{l^2} \qquad (2)$$

4. Divide (2) by (1) and get

$$tg\frac{\alpha}{2}=9*10^9\frac{q^2}{l^2mg} \qquad (3)$$

5. Represent on the figure 11 the forces acting on a charged ball in kerosene in the figure
6. Write the equation of equilibrium with respect to the vertical axis

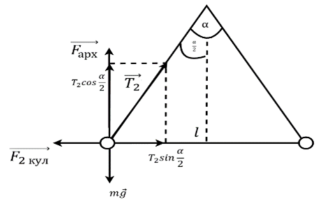


Fig. 11

$$F_{\text{арх}}+T_2\cos\frac{\alpha}{2}=mg$$

then

$$T_2\cos\frac{\alpha}{2}=mg-F_{\text{арх}} \qquad (4)$$

7. Let us write the equation of equilibrium along the horizontal axis

$$T_2\sin\frac{\alpha}{2}=9*10^9\frac{q^2}{\varepsilon l^2} \qquad (5)$$

8. Divide (5) by (4) and get

$$tg\frac{\alpha}{2} = 9 * 10^9 \frac{q^2}{\varepsilon l^2(mg - F_{\text{арх}})} \qquad (6)$$

9. Since the angle has not changed, from (3) and (6) we get

$$\frac{1}{mg} = \frac{1}{\varepsilon\,(mg - F_{\text{арх}})} \qquad (7)$$

transform (7)

$$mg = \varepsilon(mg - \rho_{ker} g V) \qquad (8)$$

10. Express the mass of the ball through density and volume and substitute into (8)

$$V\rho_{ball} g = \ \varepsilon\,(V\rho_{ball} g - \rho_{ker} g V)$$

then

$$\rho_{ball} g = \varepsilon g\,(\rho_{ball} - \rho_{ker})$$

Reducing by g and substituting numerical value of ε, we get

$$\rho_{ball} = 2\rho_{ball} - 2\rho_{ker} \qquad (9)$$

11. Solve equation (9) with respect to the density of the ball and substitute the numerical values. Then

$$\rho_{ball} = 2\rho_{ker} = 2 * 800 = 1600\ (\frac{kg}{m^3})$$

10.4. (7 points) External circuit consisting of two identical resistors

[4] Find the power released in the external circuit if it is known that the same power is released on the resistors both when they are connected in series and in parallel. The source is an element with EMF of 12 V and internal resistance of 2 Ohms.

(*Problem Bank in Physics for Applicants of the BSTU «Voenmech» named after D.F. Ustinov*)

Answer: 16.

Solution. 1 Represent the circuit in the figure 12 when conductors are connected in series

2. Since $R_{seq} = 2R$, the current in series connection is equal to

$$I_1 = \frac{\varepsilon}{2R + r}$$

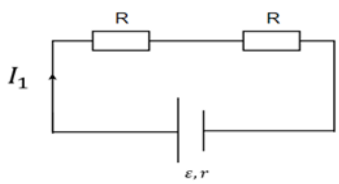


Fig. 12

3. then the power realesed in this case

$$P_1 = 2I_1^2 R = 2\frac{\varepsilon^2 R}{(2R + 2)^2} \qquad (1)$$

4. Represent the circuit in the figure 13 for parallel connection of conductors

5. Since $R_{par} = \frac{R^2}{2R} = \frac{R}{2}$, the current in parallel connection is equal to

$$I_2 = \frac{\varepsilon}{\frac{R}{2} + r}$$

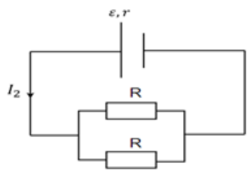


Fig. 13

6. Then the power released in this case is

$$P_2 = I_2^2 R_{\text{пар}} = \frac{\varepsilon^2 R}{(\frac{R}{2} + r)^2 * 2} \qquad (2)$$

7. Since the powers in both cases are equal, from (1) and (2) we get

$$\frac{2\varepsilon^2 R}{(2R+r)^2} = \frac{\varepsilon^2 R}{(\frac{R}{2}+r)^2 2}$$

or

$$4(\frac{R}{2}+r)^2 = (2R+r)^2 \qquad (3)$$

8. Open brackets in (3), transform and get

$$2\left(\frac{R}{2}+r\right) = 2R+r; R+2r = 2R+r; R = r$$

9. Then the power is equal

$$P = \frac{\varepsilon^2 R}{(\frac{R}{2}+r)^2 2} = \frac{\varepsilon^2 r}{2 * 2.25 r^2} = \frac{\varepsilon^2}{4.5r} \qquad (4)$$

10. Substituting numerical values into (4), we get

$$P = \frac{144}{9} = 16\ (W)$$

10.5. (7 points) A Nano-car rolls on a perfectly smooth horizontal road surface, on which there is a rectangular pit, having depth of H and width of L. The dimensions of the pit are essentially larger than those of the Nano-car, which allows us to consider the Nano-car as a material point.

[5] What speed must the Nano-car have in order for it to continue moving along the road surface on the other side of the hole? Assume that all impacts of the Nano-car on the bottom and the walls of the pit are absolutely elastic. Find all possible solutions and give a single concise and elegant form of mathematical notation for them.

(*A.S. Chirtsov*)

Answer: L*(2k+1)/n*(g/8H)^0,5 (allowed L/n*(g/8H)^0,5).

Solution. 1. Since there are no losses at impact with the bottom, the possibility of Nano-car motion continuation exists only when the upper point of the trajectory, where the Nano-car has only a horizontal velocity component, coincides with the pit boundary. Therefore, the width of the pit must be equal to twice the length of the fall (the distance along the horizontal axis that the body travels from the beginning of the fall to the first impact with the bottom) multiplied by any integer n.

2. Since the fall length is $vt_п$, where $t_п$ - is the fall time, the required relationship between velocity and pit width is as follows

$$L = 2nvt_п \qquad (1)$$

3. Determine the time of fall from the formula for equal-accelerated motion

$$H = \frac{gt_п^2}{2}$$

Then

$$t_п = \sqrt{\frac{2H}{g}} \qquad (2)$$

4. Substitute (2) in (1):

$$L = 2nv\sqrt{\frac{2H}{g}} \qquad (3)$$

5. Transforming (3) we get

$$v = \frac{L}{n}\sqrt{\frac{g}{8H}}$$

6. If we take into account reflections from vertical walls, then the answer must be multiplied by any odd integer

$$v = \frac{L(2k+1)}{n}\sqrt{\frac{g}{8H}}$$

10.6. (6 points) Two stones are thrown horizontally from a tower in opposite directions with the velocities of 8 m/s and 2 m/s.

[6] What is the time taken for the velocity vectors to become mutually perpendicular? Neglect the air resistance.

(*Problem Bank in Physics for Applicants of the BSTU «Voenmech» named after D.F. Ustinov*)

Answer: 0,4.

Solution. 1. Represent in the figure 14 the trajectories of motion of both stones and velocity vectors at an arbitrary moment of time

2. The condition of perpendicularity of the vectors is the equality to zero of their scalar product

$$\left(\vec{V_1},\vec{V_2}\right) = V_1 * V_2 \cos\left(\widehat{\vec{V_1},\vec{V_2}}\right) = V_{1x}V_{2x} + V_{1y}V_{2y} = 0 \qquad (1)$$

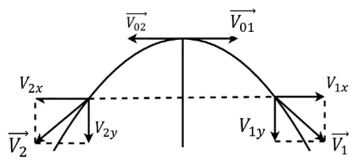


Fig. 14

3. Transform (1) and, based on the equality of the vertical components of both stones, get

$$V_{1x}\left|V_{2x}\right| = V_{1y}V_{2y}$$

$$V_{1y} = V_{2y} = \sqrt{V_{1x}\left|V_{2x}\right|} = \sqrt{V_{01}V_{02}} \qquad (2)$$

4. Substitute the numerical values into (2)

$$V_{1y} = V_{2y} = \sqrt{V_{01}V_{02}} = \sqrt{8*2} = 4\ \left(\frac{m}{s}\right)$$

5. Since

$$V_y = gt$$

find the time and substitute the numerical values

$$t = \frac{V_y}{g} = \frac{4}{10} = 0{,}4\ (s)$$

10.7. (6 points) Ball is 0.2 kg mass and 7 liters in volume

[7] Find the minimum work required to immerse the ball into the water of 1 g/cm^3 density from a depth of 1 m to a depth of 21 m. Neglect the force of water resistance.

(*Yu. V. Maksimachev*)

Answer: 1360.

Solution. 1. Represent the forces acting on the body in the figure 15

2. The minimum work will be when the process is infinitely slow so the velocity of the body movement is infinitesimal and the resultant of all forces is zero

$$F_{res} = 0$$

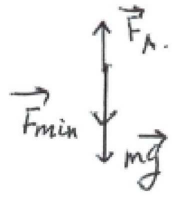


Fig. 15

3. Then

$$F_{\min} + mg = F_{\text{A}}$$

and

$$F_{\min} = F_{\text{A}} - mg = \rho g V - mg = g(\rho V - m);$$

4. By the definition of work of constant force

$$A_{\min} = F_{\min}\,(h_2 - h_1) = g(\rho V - m)\,(h_2 - h_1) \qquad (1)$$

5. Substituting into (1) the numerical values, we get

$$A_{\min} = 10\left(10^3 * 7 * 10^{-3} - 0{,}2\right) 20 = 6{,}8 * 200 = 68 * 20 = 1360\ (J)$$

10.8. (5 points) A voltmeter designed to measure voltages up to 30 V has an internal resistance of 3 kOhm.

[8] What must be the additional resistance connected to the voltmeter to measure voltages up to 300V? Give the answer in kiloohms.

(*Yu. V. Maksimachev*)

Answer: 27.

Solution. 1. Denote by n the ratio of maximum possible voltages measured by the voltmeter in two cases

$$\frac{U}{U_v} = n$$

2. Represent the voltage on the voltmeter in the second case as the sum of the main and additional voltages (voltage drops on the main and additional resistance)

$$U = U_v + U_{add}$$

then

$$U_{add} = U - U_v = U_v(n-1) \qquad (1)$$

3. Let equal currents flow through the voltmeter in both cases

$$I_v = I_{add}$$

then, from (1) and Ohm's law we get

$$\frac{U_v}{R_v} = \frac{U_v(n-1)}{R_{add}} \qquad (2)$$

3. Reduce (2) and substitute the numerical values

$$R_{add} = R_v(n-1) = 3 * 10^3(10-1) = 27 * 10^3\ (Ohm) = 27\ (kOhm)$$

10.9. (7 points) A thermally insulated cylindrical horizontal vessel with the volume of 5V is divided by very thin heat-conducting pistons into 5 identical compartments. The pistons can move freely in the vessel. The cross-sectional area of the vessel is S. Initially the pistons are fixed, the compartments are filled with ideal gas, and in each compartment the pressure of Np_0 and the temperature of NT_0 are maintained, where N is the number of the compartment (N = 1, 2, 3, 4, 5). The pistons are released and after some time the system comes to equilibrium.

[9] How many times will the distance between the second and the third pistons change as a result?

(*A.S. Chirtsov*)

Answer: 1.

Solution. 1. For an ideal gas there is the equation

$$pV = nRT$$

where n is the number of moles.

2. Then it follows from the problem statement that the number of moles in each compartment is the same.
3. After equilibrium establishing (thermal and mechanical), the temperatures and pressures in all compartments should become the same.
4. Since the temperatures, pressures and number of moles in all compartments are the same, therefore the volumes are the same. That is, the volume will not change, so the distance will not change.

International Physics Olympiad
«Formula of Unity» / «The Third Millennium»
Year 2023/2024. Qualifying round

Solutions to problems for grade R11

11.1. (7 points) Stone is thrown from the Earth's surface.

[1] What is the minimum modulus of initial velocity for the stone to fly over a wall 5.2 m thick? The height of the wall is equal to its thickness. The point of throwing the stone is at a distance of 5.2 m from the wall. The trajectory of the stone is symmetrical with respect to the wall. Neglect the air resistance.

(*A.G.Areshkin, O.S. Komarova, V.G. Mozgovaya, D.L. Fedorov*)

Answer: 13 (*allowed* 13,0 *with decrease in score to* 6).

Solution. 1. In the figure 16 there are the wall and the optimal trajectory of the stone as it flies over the wall

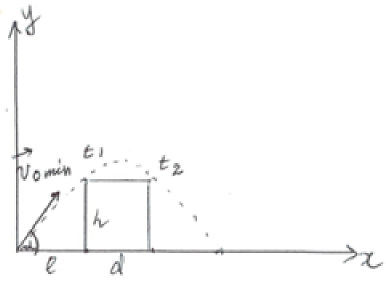


Fig. 16

2. At an arbitrary moment of time the velocity of the stone is equal to

$$\vec{v} = \vec{v_0} + \vec{a}t \qquad (1)$$

3. Project (1) for the optimal trajectory onto the x-axis

$$v_x = v_{0min}\cos\alpha$$

4. The optimal trajectory should pass through both corner points of the wall, at moments t_1 and t_2 respectively, then

$$l = v_x t_1 = v_{0min}\cos\alpha * t_1 \qquad (2)$$

5. Since l=d, then

$$t_2 = 2t_1$$

and

$$d + l = 2l = v_x t_2 = v_{0min}\cos\alpha * t_2$$

6. For uniformly accelerated motion

$$\vec{r} = \vec{v_0} + \frac{\vec{a}t^2}{2} \qquad (3)$$

7. Project (3) for the optimal trajectory onto the y-axis

$$h = v_{0min}\sin\alpha * t - \frac{gt^2}{2}$$

then

$$\frac{gt^2}{2} - v_{0min}\sin\alpha * t + h = 0 \qquad (4)$$

8. Substitute in (4) the numerical values

$$5t^2 - v_{0min}\sin\alpha * t + 5.2 = 0 \quad (5)$$

9. Denote $v_{0min}\sin\alpha = x$, then

$$5t^2 - xt + 5.2 = 0 \qquad (6)$$

10. Equation (6) solutions:

$$t_{1,2} = \frac{x \pm \sqrt{x^2 - 104}}{10} \qquad (7)$$

11. Sinse

$$t_2 = 2t_1 \qquad (8)$$

, substituting into (8) the solutions of (7), we get:

$$x = 3\sqrt{x^2 - 104} \qquad (9)$$

12. Square both sides of equation (9)

$$x^2 = 9x^2 - -936$$

then

$$x^2 = \frac{936}{8} = 117\ (\frac{\text{м}^2}{\text{с}^2})$$

13. Take the square root, then

$$v_{0min} \sin\alpha = \sqrt{117} = 3\sqrt{13} \qquad (10)$$

14. Since

$$sin^2\alpha + cos^2\alpha = 1 \qquad (11)$$

15. Multiply (11) by v_{0min}^2, then

$$v_{0min}^2 \sin^2\alpha + v_{0min}^2 \cos^2\alpha = v_{0min}^2 \qquad (12)$$

16. From (2) and (7) we get

$$v_{0min} \cos\alpha = \frac{l}{t_1} = \frac{l}{\frac{x - \sqrt{x^2-104}}{10}} = \frac{10l}{x - \sqrt{x^2 - 104}} = \frac{52}{3\sqrt{13} - \sqrt{117 - 104}} = \frac{26}{\sqrt{13}} = 2\sqrt{13} \qquad (13)$$

then

$$v_{0min}^2 \cos^2\alpha = 52 \qquad (14)$$

17. Substitute (10) and (14) into (12), then

$$v_{0min} = \sqrt{117 + 52} = 13\ (\frac{m}{s})$$

11.2. (7 points) On the one of the islands of the Bermuda Triangle, called Cosogravia for some reason, the acceleration of free fall is equal to g $= 9.8\ m/s^2$ in magnitude as everywhere else, but directed at an angle $\alpha = 15°$ to the vertical, that is to say it is slightly oblique to the north. A very short aborigine makes a bow shot at an angle $\beta = 60°$ to the surface of the island, giving the arrow a known initial velocity of $v_0 = 3$ m/s.

[2] At what distance from the aborigine will the arrow fall on the surface of the island if the shot was made in a northerly direction? In a southerly direction? In a westerly direction?

Замечание. Give the answer rounded to 1 sm. Indicate your answers separated by semicolons.

(*A.S. Chirtsov*)

Answer: 1,21; 0,44; 0,92 (*allowed* 1,21; 0,44; 0,91 *with decrease in score to* 6).

Solution. 1. Put a coordinate system with the origin at the shot point, the z-axis pointing vertically upward, the x-axis pointing northward, and the y-axis pointing westward.

2. Then the components of acceleration along the axes will have the following values

$$g_x = g\sin\alpha$$

$$g_z = -g\cos\alpha$$

$$g_y = 0$$

3. Since the aborigene is short, it can be assumed that the shot is fired from the ground.
4. Consider the first case. The problem is two-dimensional.
4.1. Components of the initial velocity

$$v_{0z} = v_0 \sin\beta \qquad (1)$$

$$v_{0x} = v_0 \cos\beta \qquad (2)$$

4.2. Write the equations for the coordinates of the arrow at an arbitrary moment of time

$$x(t) = v_{0x}t + \frac{g_x t^2}{2} \qquad (3)$$

$$z(t) = v_{0z}t + \frac{g_z t^2}{2} \qquad (4)$$

4.3. Determine the flight time t_f, by equating the altitude to zero

$$v_{0z}t_f + \frac{g_z t_f^2}{2} = 0$$

Then

$$t_f = \frac{2v_{0z}}{-g_z} \qquad (5)$$

4.4 Substitute (5) in (3)

$$x\left(t_f\right) = v_{0x}t_f + \frac{g_x {t_f}^2}{2} = v_{0x}\frac{2v_{0z}}{-g_z} + \frac{g_x}{2}\left(\frac{2v_{0z}}{-g_z}\right)^2 \qquad (6)$$

4.5. Transforming (6), we get

$$L_{\text{север}} = \frac{2v_0^2}{g}\left(\frac{\cos\beta\sin\beta}{\cos\alpha} + \frac{\sin\alpha\left(\sin\beta\right)^2}{\left(\cos\alpha\right)^2}\right)$$

4.6. Substituting the numerical values, we get

$$L_{\text{север}} = 1{,}21\ (m)$$

5. Consider the second case. The problem is two-dimensional.
5.1. Components of the initial velocity

$$v_{0z} = v_0 \sin\beta \qquad (7)$$

$$v_{0x} = -v_0 \cos\beta \qquad (8)$$

5.2. Write the equations for the coordinates of the arrow at an arbitrary moment of time

$$x(t) = v_{0x}t + \frac{g_x t^2}{2} \qquad (9)$$

$$z(t) = v_{0z}t + \frac{g_z t^2}{2} \qquad (10)$$

5.3. Determine the flight time t_f by equating the altitude to zero

$$v_{0z}t_f + \frac{g_z t_f^2}{2} = 0$$

Then

$$t_f = \frac{2v_{0z}}{-g_z} \qquad (11)$$

5.4 Substituting (11) in (9), we get

$$x\left(t_f\right) = v_{0x}t_f + \frac{g_x {t_f}^2}{2} = v_{0x}\frac{2v_{0z}}{-g_z} + \frac{g_x}{2}\left(\frac{2v_{0z}}{-g_z}\right)^2 \qquad (12)$$

5.5. Transform (12):

$$L_{\text{юг}} = \frac{2v_0^2}{g}\left(\frac{\cos\beta\sin\beta}{\cos\alpha} - \frac{\sin\alpha\left(\sin\beta\right)^2}{\left(\cos\alpha\right)^2}\right)$$

5.6. Substituting the numerical values, we get

$$L_{\text{юг}} = 0{,}44\ (m)$$

6. Consider the third case. The problem is three-dimensional.
6.1. Components of the initial velocity

$$v_{0z} = v_0\sin\beta \qquad (13)$$

$$v_{0y} = v_0\cos\beta \qquad (14)$$

$$v_{0x} = 0 \qquad (15)$$

6.2. Write the equations for the coordinates of the arrow at an arbitrary moment of time

$$x(t) = \frac{g_x t^2}{2} \qquad (16)$$

$$z(t) = v_{0z}t + \frac{g_z t^2}{2} \qquad (17)$$

$$y(t) = v_{0y}t \qquad (18)$$

6.3. Determine the flight time t_f by equating the altitude to zero

$$v_{0z}t_f + \frac{g_z t_f^2}{2} = 0$$

Then

$$t_f = \frac{2v_{0z}}{-g_z} \qquad (19)$$

6.4. Substituting (19) in (16) and (18), we get

$$x\left(t_f\right) = \frac{g_x {t_f}^2}{2} = \frac{g_x}{2}\left(\frac{2v_{0z}}{-g_z}\right)^2 \qquad (20)$$

$$y\left(t_f\right) = v_{0y}t_f = v_{0y}\frac{2v_{0z}}{-g_z} \qquad (21)$$

6.5. In this case

$$L_{\text{запад}} = \sqrt{\left(x\left(t_f\right)\right)^2 + \left(y\left(t_f\right)\right)^2} \qquad (22)$$

6.6. Transform (20) - (22)

$$L_{\text{запад}} = \frac{2v_0^2}{g}\frac{\sin\beta}{\cos\alpha}\sqrt{\left(\sin\beta\tan\alpha\right)^2 + \left(\cos\beta\right)^2}$$

6.7. Substituting the numerical values, we get

$$L_{\text{запад}} = 0{,}92 \ (m)$$

11.3. (5 points) A body of 10 g mass uniformly sinks in water.

[3] Assuming that 50% of the heat released during the motion is used to heat the body, determine by how many degrees the temperature of the body will increase when it sinks to depth of 10 m. The heat capacity of the body is 0.4 J/K. The density of the body is much greater than the density of water.

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 1,3 (*allowed* 1,25 *with decrease in score to* 2).

Solution. 1. Represent in the figure 17 the forces acting on the body

2 Since the velocity is $v = const$, then

$$\vec{F}_{result} = 0 \qquad (1)$$

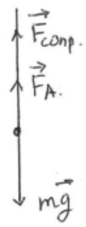


Fig. 17

3. Project (1) onto the vertical axis

$$F_{resist} + F_{\text{A}} - mg = 0 \qquad (2)$$

4. From the formulas for Archimedes' force

$$F_{\text{A}} = \rho_{\text{в}} g V$$

5. Since the density of the body

$$\rho = m/V$$

we get

$$mg = \rho_{\text{т}} g V$$

6. Since $\rho_b >> \rho_{\text{в}}$, then $mg >> F_{\text{A}}$ and therefore Archimedes' force can be neglected and

$$F_{resist} = mg \qquad (3)$$

7. Total heat losses are equal to the power of force F_{resist}. Then, using (3), we obtain

$$Q_{full} = F_{resist} h = mgh \qquad (4)$$

8. By definition of heat capacity

$$Q_{body} = c\Delta T$$

9. Since 50% of the heat released during motion is used to heat the body:

$$c\Delta T = 0.5mgh \qquad (5)$$

10. Transform (5) and substitute the numerical values

$$\Delta T = \frac{0.5mgh}{c} = \frac{0.5 * 0.01 * 10 * 10}{0.4} = 1.25 \ (K)$$

11. Rounding

11.4. (6 points) An electric kettle has two windings. If only the first winding is switched on the water boils in 40 minutes, if only the second one - in 60 minutes.

[4] How many minutes will it take for water to boil if both windings are switched on in parallel at the same time?

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 24.

Solution. 1. By Joule-Lenz law

$$Q = \frac{U^2 t}{R}$$

2. Since a certain (and the same in all cases) amount of heat is needed for water to boil

$$\frac{U^2 t_1}{R_1} = \frac{U^2 t_2}{R_2} = \frac{U^2 t_{par}}{R_{par}} \qquad (1)$$

3. Resistance in parallel connection

$$R_{par} = \frac{R_1 R_2}{R_1 + R_2}$$

4. Converting (1), find the ratio of tile winding resistances

$$\frac{U^2 t_1}{R_1} = \frac{U^2 t_2}{R_2}$$

then

$$\frac{t_1}{R_1} = \frac{t_2}{R_2} \qquad (2)$$

5. Substitute the numerical values in (2)

$$\frac{R_2}{R_1} = \frac{t_2}{t_1} = 1.5$$

then

$$R_2 = 1.5 R_1 \qquad (3)$$

6. Using (3), determine the resistance in parallel connection

$$R_{par} = \frac{R_1 R_2}{R_1 + R_2} = \frac{R_1 * 1.5 R_1}{R_1 + 1.5 R_1} = \frac{1.5 R_1^2}{2.5 R_1} = 0.6 R_1$$

7. From (1) we get

$$\frac{t_1}{R_1} = \frac{t_{par}}{R_{par}} \qquad (4)$$

8. Transform (4) and substitute the numerical values

$$t_{par} = \frac{t_1 R_{par}}{R_1} = 0.6 * 40 = 24 \ (min)$$

11.5. (7 points) From the top of a long inclined plane, which forms an angle of 60° with the horizon, a body with an initial velocity of 10 m/s is thrown downward at an angle of 30° to the inclined plane.

[5] At what distance from the point of throwing is the point of falling of the body on the inclined plane? Neglect the air resistance.

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 34,6.

Solution. 1. Represent the inclined plane, the introduced coordinate system and the velocity and acceleration components in the figure 18

2. The position of the body at an arbitrary moment of time in the case of uniformly accelerated motion is given by the equation

$$\vec{r} = \vec{v_0} + \frac{\vec{a}t^2}{2} \qquad (1)$$

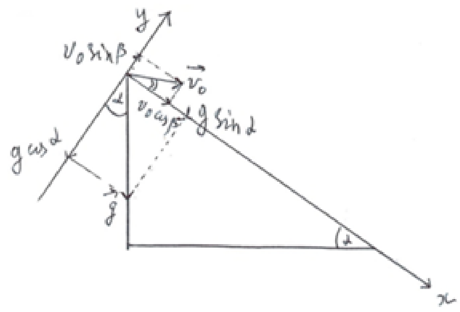


Fig. 18

3. Project (1) onto the x-axis

$$l = v_0 \cos\beta * t + \frac{g\sin\alpha * t^2}{2} \qquad (2)$$

4. Determine the moment of time when $v_y = 0$. For this purpose, let us write the equation for the velocity at an arbitrary moment of time in the case uniformly accelerated motion

$$\vec{v} = \vec{v_0} + \vec{a}t \qquad (3)$$

5. Project (3) onto the y-axis and equate the result to zero

$$v_y = v_0 \sin\beta - g\cos\alpha * t = 0 \qquad (4)$$

6. Solving (4) and substituting the numerical values, we get

$$t = \frac{v_0 \sin\beta}{g\cos\alpha} = 1 \text{ (c)}$$

7. Since $t_{full} = t_{ascent} + t_{descent} = 2t_{ascent} = 2s$, then substituting the numerical values into (2), we have

$$l = 20\frac{\sqrt{3}}{2} + 20\frac{\sqrt{3}}{2} = 20\sqrt{3} = 34.6\ (m)$$

11.6. (7 points) A body of 0.4 kg mass starts sliding with an initial velocity of 12 m/s up an inclined plane, which forms an angle of 30° with the horizon.

[6] Determine the work of friction forces in first 3.6 s of motion, if the coefficient of friction is 6 times less than the square root of 3.

(*Yu.V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: −14,6.

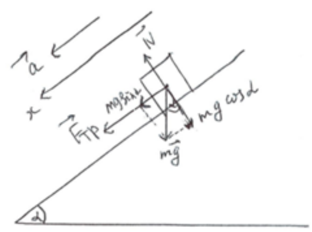
Solution. 1. Divide the process of movement into 2 stages: ascent and descent

2. Consider the first stage - ascent

3. Represent the inclined plane, the body, the direction of the axis and the forces acting on the body in the figure 19

4. By definition of the work of a constant force

$$A_{F_{fr}} = -F_{fr}S \qquad (1)$$


Ашп. 19

5. Since $F_{fr} = \mu N$ и $N = mg\cos\alpha$, substituting the numerical values, we get

$$F_{fr} = \mu mgcos\alpha = 1\ (N) \qquad (2)$$

6. Project the equation of Newton's second law $F_{res} = ma$ onto the x-axis

$$\mu mg\cos\alpha + mg\sin\alpha = ma \qquad (3)$$

7. Solving (3) and substituting the numerical values, we get

$$a = g(\mu\cos\alpha + \sin\alpha) = 10\left(\frac{\sqrt{3}}{6}\frac{\sqrt{3}}{2} + \frac{1}{2}\right) = 7.5\ (\frac{m}{s^2}) \qquad (4)$$

8. For an arbitrary moment of time at an equally accelerated motion the following relation is true

$$v_0^2 - v^2 = 2aS_{rise} \qquad (5)$$

then

$$S_{rise} = \frac{v_0^2}{2\text{a}} = 9.6\ (m) \qquad (6)$$

9. According to (1)

$$A_{F_{fr}rise} = -F_{fr}S_{rise} = -9.6\ (J) \qquad (7)$$

10. Determine the ascent time. For this purpose we equate to zero the velocity of motion along the plane

$$v = v_0 - at_{rise} = 0$$

then

$$t_{rise} = \frac{v_0}{a} = 1.6\ (s) \qquad (8)$$

11. Consider the second stage - descent
12. Represent the inclined plane, the body, the direction of the axis and the forces acting on the body in the figure 20
13. Considering (8), determine the descent time

$$t_{descent} = t - t_{rise} = 3{,}6 - 1{,}6 = 2\ (s) \qquad (9)$$

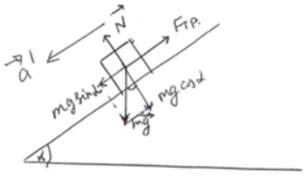


Fig. 20

14. Determine the descent distance

$$S_{descent} = \frac{a' t_{descent}^2}{2} \qquad (10)$$

15. Project the equation of Newton's second law $F_{res} = ma$ onto the x-axis

$$mg\sin\alpha - \mu mg\cos\alpha = ma' \qquad (11)$$

then

$$a' = g(\sin\alpha - \mu\cos\alpha) = g\left(\frac{1}{2} - \frac{\sqrt{3}}{6}\frac{\sqrt{3}}{2}\right) = 2.5\ (\frac{m}{s^2}) \qquad (12)$$

16. Substituting (12) into (10), we get

$$S_{descent} = 5\ (m)$$

17. According to (1)

$$A_{F_{fr}descent} = -F_{fr}S_{descent} = -5\ (J) \qquad (13)$$

18. According to (1)

$$A_{F_{fr}} = A_{F_{fr}rise} + A_{F_{fr}descent} = -9.6 - 5 = -14.6\ (J)$$

11.7. (5 points) 2 moles of ideal gas were given 249 J of heat during the isothermal expansion. The gas was then returned to its initial state by isobaric compression and isochoric heating. The work of the gas during the cycle is 83 J.

[7] Determine the difference between the maximum and minimum temperature of the gas in the cycle.

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 10.

Solution. 1. Represent the graph 21 of the considered cycle

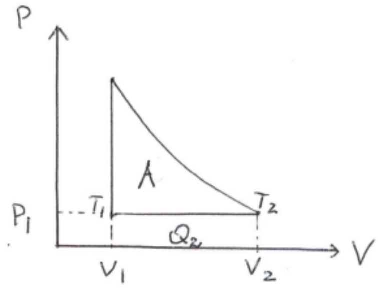


Fig. 21

2. Since a cycle is considered, the initial and final states coincide, hence, the total change in internal energy is zero. Therefore, from the first law of thermodynamics

$$A = Q_1 - Q_2 \qquad (1)$$

3. Determine the amount of heat lost during isobaric compression and isochoric heating

$$Q_2 = Q_1 - A = 249 - 83 = 166\ (J) \qquad (2)$$

4. If we consider the whole process consisting of isobaric compression and isochoric heating, then at the initial and final points the temperature of the gas is the same and, therefore, the change in internal energy is zero. Therefore, the amount of heat in the whole process is equal to the work in isobaric compression (since in isochoric heating the work is zero).

$$Q_2 = P_1(V_2 - V_1) = P_1 \Delta V \qquad (3)$$

5. Then from the Michelson-Mendeleev equation

$$Q_2 = P_1 \Delta V = \nu R \Delta T \qquad (4)$$

6. Transform (4) and substitute the numerical values

$$\Delta T = \frac{Q_2}{\nu R} = \frac{166}{16.6} = 10\ (\text{K}) \qquad (5)$$

7. It's obvious that $T_1 = T_{min}$ since $P_1 = P_{min}$ and $V_1 = V_{min}$ (see fig. 21)
8. Let's prove, that $T_2 = T_{max.}$ Choose and fix V, such that V1 < V < V2; it corresponds to 2 pressure values p' and p, and p' > p. Since V = const, then T' > T, hence the isotherm temperature is higher than the isobaric temperature at any $V_1 < V < V_2$. since T_2 - isotherm temperature, then $T_2 = T_{max}$.
9. Therefore

$$T_{max} - T_{min} = 10\ (K)$$

11.8. (6 points) A bead can slide freely along a hoop of 4.5 m radius, which rotates with an angular velocity of 2 rad/s around a vertical axis passing through its center.

[8] What is the maximum height, relative to bead's initial position, the bead will rise to? The axis lies in the plane of the hoop.

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 2 (*allowed* 2,0).

Solution. 1. Represent the hoop and the forces acting on the bead at some angle of deviation from the vertical on the figure 22

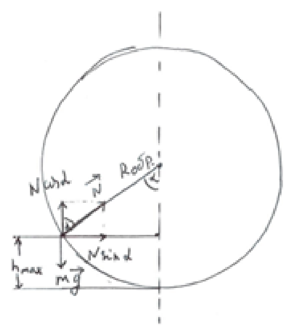


Fig. 22

2. Project the equation of Newton's second law $\vec{F_{res}} = m\vec{a}$ onto the vertical and horizontal axes

$$N \cos\alpha - mg = 0 \qquad (1)$$

(since $a_y = 0$)

$$N \sin\alpha = ma_{cent} = m\omega^2 R_{hoop} \sin\alpha \qquad (2)$$

3. Transform (1) and (2) and write them as a system of equations

$$\begin{cases} N\cos\alpha = mg \\ N = m\omega^2 R_{hoop} \end{cases} \qquad (3)$$

4. From (3) we get

$$\cos\alpha = \frac{g}{\omega^2 R_{hoop}} \qquad (4)$$

5. Using the notations in the figure and formula (4) ,substitute the numerical values and get

$$h_{max} = R_{hoop} - R_{hoop}\cos\alpha = R_{hoop} - R_{hoop}\frac{g}{\omega^2 R_{hoop}} = 4.5 - 4.5 * \frac{5}{9} = 2\ (m)$$

11.9. (6 points) Two balls have masses of 0.2 g and 0.8 g and charges of 0.3 μCl and 0.2 μCl. Balls are connected by a thin thread of 20 cm length and move along the force line of a homogeneous electric field with intensity of 10 V/m, directed vertically downwards.
[9] Determine in millinewtons the modulus of the thread tension force.

(*Yu. V. Maksimachev, T.N. Strelkova, B.K. Galyakevich*)

Answer: 11,5 or 15,5.
Solution. 1. Represent the thread, the directions of the axis and the electric field strength vector and the forces acting on both balls in the figure 23

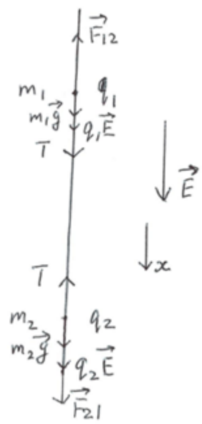


Fig. 23

2. The problem is one-dimensional, all forces are directed along the x-axis. Write the equation of Newton's second law for each of the balls

$$\vec{F_{res}} = m\vec{a} \qquad (1)$$

3. According to Newton's 3rd law: $F_{12} = F_{21}$
4. Project equation (1) for each of the balls onto the x-axis.

$$T + q_1E + m_1g - F_{12} = m_1a \qquad (2)$$

$$F_{21} + q_2E + m_2g - T = m_2a \qquad (3)$$

Since the balls are moving together, they have the same accelerations.
5. Divide (3) by (2) and substitute the numerical values

$$\frac{F_{21} + q_2E + m_2g - T}{T + q_1E + m_1g - F_{12}} = \frac{m_2a}{m_1a} = \frac{8 * 10^{-4}}{2 * 10^{-4}} = 4 \qquad (4)$$

therefore

$$F_{21} + q_2E + m_2g - T = 4T + 4q_1E + 4m_1g - 4F_{12} \qquad (5)$$

6. Since $\frac{m_2}{m_1} = 4$, from (5) we get

$$T = F_{21} + \frac{E(q_2 - 4q_1)}{5} = 9*10^9 * \frac{q_1q_2}{c^2} - \frac{E(4q_1 - q_2)}{5} = 9*10^9\frac{6*10^{-14}}{0.04} + \frac{10\,(8*10^{-4} - 8*10^{-4}) - 10^4(12*10^{-7}}{5}$$

$$= \frac{9*6*10^{-3}}{4} - \frac{10*10^{-3}}{5} = 11.5*10^{-3}N = 11.5\ (mN)$$

Possible answer: 15,5 (if balls have different location)

Table of constants

Gravitational acceleration	g = 10 m/s^2
Speed of light	c = 3·10^8 m/s
Universal gas constant	R = 8,3 J/(mol·K)
Elementary charge	e = 1,6·10^{-19} C
Avogadro's number	NA = 6·10^{23} mol^{-1}
Coulomb's constant	k= 9·10^9 Nm^2/Кл^2
Molar mass of hydrogen	M_{H2} = 2·10^{-3} kg/mol
Planck's constant	h = 6,6·10^{-34} J·s
Molar mass of helium	M_{He} = 4·10^{-3} kg/mol
Electronvolt	1 eV = 1,6·10^{-19} J
π = 3,14	π^2 = 10
$\sqrt{2}$ = 1,41	$\sqrt{3}$ = 1,73