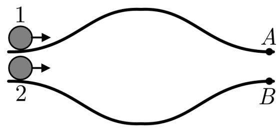
International Physics Olympiad
«Formula of Unity» / «The Third Millennium»
Year 2022/2023. Qualifying round

# Solutions of the problems for R8

**8.1. (2 points)** Two identical circular gingerbread buns («koloboks») roll at the same velocity along parallel horizontal tracks. On the way of the first gingerbread bun there is a hill, on the way of the second one there is a hole (see pic.). The height of the hill and the depth of the hole are the same. Both gingerbread buns overcame their obstacles and ended up at points $A$ and $B$, respectively.




**[1]** Compare the velocities of the gingerbread buns at points $A$ and $B$:
A) $v_1 > v_2$, B) $v_1 = v_2$, C) $v_1 < v_2$.

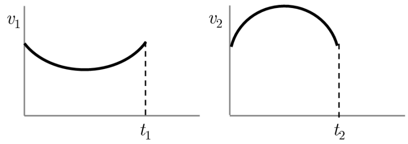
**[2]** Compare the times it took the gingerbread buns to overcome obstacles:
A) $t_1 > t_2$, B) $t_1 = t_2$, C) $t_1 < t_2$.

There is no slippage while the buns are moving. In both questions give only **the letter** of the correct answer from those given. (*S. Starovoytov*)

**Answers: [1]** B. **[2]** A.

**Solution [1].** The velocity of the first gingerbread bun, while it's moving along the hill, decreases first, then the velocity increases and at the point $A$ it equals the initial velocity. The velocity of the second gingerbread bun, when moving along the hole, increases first and later decreases, and then at point $B$ becomes equal to the initial velocity.

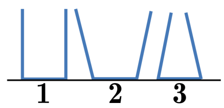
From the law of energy conservation it's evident that $v_1 = v_2$. Qualitative dependences of the gingerbread buns' velocity on time when moving through obstacles are shown in the pictures.




**Solution [2].** The paths traveled by the gingerbread buns are the same, which means that the squares under the curves equal one another. Then $t_1 > t_2$.

**Criteria.** Both questions cost 1 point each.

**8.2. (3 points)** An equal volume of water was poured into three glass cups, the axial sections of which are shown in the picture, in the way so that the water did not spill out of the cups.




**[3]** Choose from the suggested answers the correct one for the hydrostatic pressures of water $(p_1, p_2, p_3)$ on the bottoms of the cups:
A) $p_1 = p_2 = p_3$, B) $p_2 > p_1 > p_3$, C) $p_3 > p_1 > p_2$.

**[4]** Choose from the suggested answers the correct one for the pressure forces $(F_1, F_2, F_3)$ of the cups with water in them on the horizontal surface of the base:
A) $F_1 = F_2 = F_3$, B) $F_2 > F_1 > F_3$, C) $F_3 > F_1 > F_2$.

The height of the cups, the diameter of the base and the thickness of the glass walls are equal. The angle of inclination of the side walls is more than 45°. In both questions give only **the letter** of the correct answer from those given. (*S. Starovoytov*)

**Answers: [3]** C. **[4]** B.

**Solution [3].** The hydrostatic pressure $p = \rho g h$ depends on the height of the liquid level, and it is the largest in the third cup and the smallest in the second.

**Solution [4].** For any angle of inclination, the lateral surface area of the second glass is larger than that of the first and the third. At the indicated angles of inclination of the walls and a small value of the ratio of the radii of the neck and base of the third glass (as in the figure), the side surface area of the third glass is less than that of the first. Since the masses of water in all cups are the same, the weights of cups with water will be bigger for cups with a larger mass of side walls. Therefore, the correct answer is B.

**Remark.** Note that with a small height of the glasses and, respectively, a large value of the ratio of the radii of the neck and base of the third glass, the lateral surface area of the third glass is larger

than that of the first. In this case, the double inequality $F_2 > F_3 > F_1$ would be true. But there is no such inequality in the answers offered for choice. Therefore, the only possible answer is B.
**Criteria.** Question **[3]** costs 1 point. For the question **[4]**: answer «B» costs 2 points, answer «A» costs 1 point.

**8.3.** **(3 points)** Tourist Nikolai Petrovich was 5 minutes late to the departure of his cruise ship, which went down the river. Fortunately, the owner of the speedboat agreed to help Nikolai Petrovich. Having caught up with the cruise ship and left the unlucky tourist on board of it, the speedboat immediately set off back on the return journey.

**[5]** How long did it take from the moment the speedboat started until it returned? Give the answer in minutes.

Assume that the speed of the cruise ship in relation to the water is 3 times bigger than the speed of the river current, and the speed of the boat is 5 times bigger than that of the river. (*S. Starovoytov*)
**Answer:** 25 minutes.
**Solution.** At the very moment when the speedboat catches up with the cruise ship, the cruise ship will have covered the distance down the river:

$$S = S_0 + (v_{\text{CS}} + v_{\text{R}})t_1 = (v_{\text{CS}} + v_{\text{R}})(t_0 + t_1).$$

The speedboat will pass the same way:

$$S = (v_{\text{SB}} + v_{\text{R}})t_1.$$

Here $t_1$ is the time it took the speedboat to get «to» the cruise ship and $t_0$ is how much time the tourist was late.
Equating the distances and considering $v_{\text{SB}}/v_{\text{R}} = 5$, $v_{\text{CS}}/v_{\text{R}} = 3$, we get $t_1 = 2t_0$.
The way back of the speedboat

$$S = (v_{\text{SB}} - v_{\text{R}})t_2,$$

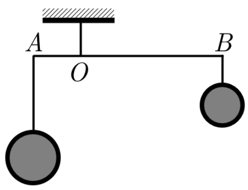
where $t_2$ is the time it took the speedboat to get back.
Equating the distances of the speedboat to and back from we get

$$t_2 = 1.5t_1 = 3t_0.$$

Total time for the speedboat to and from

$$t = t_1 + t_2 = 5t_0 = 25 \text{ min}.$$

**8.4.** **(4 points)** Balls made of different materials are balanced on a lever. The volume of the left ball is 1.25 times bigger than the volume of the ball on the right, and the arm $AO$ is 2.5 times less than the arm $OB$.




**[6]** By how much should the volume of the right sphere be increased so that when the entire system is placed in water, the balance will not get disturbed provided the system density is twice as big as that of water? Give your answer as a percentage, rounded to integers. (*M. Krupina*)

**Answer:** 50%.
**Solution.** Condition of balance in air:

$$\rho_1 V_1 g l_1 = \rho_2 V_2 g l_2,$$

therefore,

$$\rho_1 = 2\rho_2$$

(index «1» refers to the left ball, index «2» refers to the right one).
Condition of balance in water:

$$\rho_1 V_1 g l_1 - \rho_0 V_1 g l_1 = \rho_2 V_2^* g l_2 - \rho_0 V_2^* g l_2.$$

Here $\rho_0$ is the density of water, $V_2^*$ is the new volume of the right ball.
After simple transformations we come to

$$V_2^* = 1.5V_2.$$

Therefore, the increase is by 50%.

**8.5. (2 points)** The boys (triplets) built a raft. When they climbed onto the raft all together, the raft sank completely into the water. But if only two of them ride it, then the raft is immersed in water for $\eta = 0.8$ of its thickness.

**[7]** Find out the density of the wood from which the raft is made. Give your answer in $\mathrm{kg/m^3}$ rounded to integers.

**Remark.** The density of water is $1000\ \mathrm{kg/m^3}$, the thickness of the raft is $h = 20$ cm.

(*T. Vorobyeva, S. Starovoytov*)

**Answer:** 400.

**Solution.** Write down the equilibrium equations first for 3 boys and then for 2 boys:

$$\begin{cases} 3mg + Mg = \rho_0 g S h, \\ 2mg + Mg = \rho_0 g S \Delta h, \end{cases}$$

where $m$ is the mass of each of the boys, $M$ is the mass of the raft, $S$ is the area of the raft, $\Delta h$ is the thickness of the submerged part of the raft in the case of 2 boys on it. Subtract the second equation from the first one:

$$mg = \rho_0 g S (h - \Delta h).$$

Substituting into the first equation, we get

$$Mg = \rho_0 g S (h - 3(h - \Delta h)).$$

By using the definition of the raft mass through its density $\rho$ we get

$$\rho = \rho_0 \left(1 - 3\left(1 - \frac{\Delta h}{h}\right)\right) = \rho_0(1 - 3(1 - \eta)) = 400\ \frac{\mathrm{kg}}{\mathrm{m}^3}.$$

**8.6. (3 points)** Having opened a jar of condensed milk and eaten the half of it, Slavik thought that it would be nice to cook the rest. Having dipped the jar into boiling water, Slavik noticed that the jar was immersed in water for 5/6 of its volume. Time has passed but the milk hasn't turned dark, so Slavik has eaten a quarter more of the milk in the jar. Now the jar started floating being immersed in water on its half only.

**[8]** How much did the full jar weigh provided its volume is 300 ml? Give the answer of the mass in kilos rounded to hundredths.

**Remark.** Consider that the jar is thin-walled. (*T. Andreeva*)

**Answer:** 0.45.

**Solution.** The density of condensed milk is $\rho_{\mathrm{CM}}$, the density of water is $\rho_{\mathrm{W}}$, the empty jar weighs $m$, its volume is $V$. The jar is thin-walled so their inside and outside volumes are almost equal. Considering that when in water, initially the jar was half full and later only quarter full, we write the law of Archimedes:

$$\begin{cases} mg + \dfrac{1}{2} \cdot \rho_{\mathrm{CM}} V g = \dfrac{5}{6} \cdot \rho_{\mathrm{W}} V g, \\ mg + \dfrac{1}{4} \cdot \rho_{\mathrm{CM}} V g = \dfrac{1}{2} \cdot \rho_{\mathrm{W}} V g. \end{cases}$$

Subtract the lower equation from the upper equation:

$$\frac{1}{4} \cdot \rho_{\mathrm{CM}} V g = \frac{1}{3} \cdot \rho_{\mathrm{W}} V g.$$

Therefore, the density of condensed milk is

$$\rho_{\mathrm{CM}} = \frac{4}{3} \cdot \rho_{\mathrm{W}} = 1330\ \frac{\mathrm{kg}}{\mathrm{m}^3}.$$

From the first equation to the law of Archimedes we find out, that

$$m = \frac{5}{6} \cdot \rho_{\mathrm{W}} V - \frac{1}{2} \cdot \rho_{\mathrm{CM}} V = \frac{1}{6} \cdot \rho_{\mathrm{W}} V,$$

and the full jar weighs

$$M = \rho_{\mathrm{CM}} V + \frac{1}{6} \cdot \rho_{\mathrm{W}} V = 0.45\ \mathrm{kg}.$$

**Criteria.** If the given answer is correct but rounded to tenths («0.5»), it costs 1 point.

**8.7.** **(2 points)** Three cylindrical communicating vessels in the form of the cyrillic letter «Ш» are filled with oil of 900 kg/m$^3$ density and are shut with weightless thin pistons. The cross section of each of the vessels makes 60 cm$^2$. A weight of 600 g is being placed over the piston that shuts the middle vessel.

**[9]** What height (in relation to their initial level) will the side pistons rise at after this? Give the answer in millimeters, rounded to integers.

**Remark.** The friction of the pistons over the vessel walls should be neglected. (*S. Starovoytov*)

**Answer:** 37.

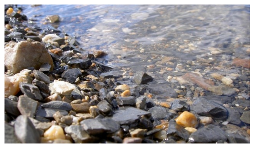
**Solution.** The pressure under the piston in the middle vessel makes $P_{\text{mid}} = mg/s$, where $m$ is the mass of the weight. The pressure of the side pistons at the same height is $P_{\text{edge}} = \rho g(h + x)$, where $h$ is the depth at which the middle piston goes down, $x$ is the height at which the side pistons will rise. From the equality of the pressures we get

$$\frac{mg}{s} = \rho g(h + x).$$

Besides, the liquid squeezed from the middle vessel will go to the side ones. Therefore, $h = 2x$. Substitute it to the equation and getting out $x$, we have

$$x = \frac{m}{3\rho S} = 37 \text{ mm}.$$

**Criteria.** The wrong answers «36» and «74» cost 1 point each.



**8.8.** **(4 points)** Mother took her baby to the river shore. The sun heated the stones by the shore up to 40°C, but the water remained cold (18°C). To bathe the baby, mother got 5 L of water into a small bucket and started heating it by placing the stones in the bucket. Not to spill the water over, she put only one stone in the bucket and then waited until the temperatures of water and a stone equalized. Then she took the stone out and placed in a new one.

**[10]** How much will the temperature of water be after the fourth stone is out of it?

**[11]** Up to what temperature could the water heat up if all the four stones had been possible to be placed in it at once?

Give both answers in degrees Celsius, rounded to integers (but only round up the final results). The mass of each stone is 2.8 kg, the heat capacity of them is 900 $\frac{\text{J}}{\text{kg}\cdot\text{K}}$, and the specific heat capacity of water is 4.2 $\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$. (*T. Andreeva*)

**Answers: [10]** 26°C. **[11]** 25°C.

**Solution [10].** Let's determine the water temperature $t_1$ before removing the first stone:

$$\rho V c(t_1 - t_{\text{W}}) = mc_{\text{F}}(t_{\text{F}} - t_1) \quad \Rightarrow t_1 = \frac{\rho V c t_{\text{W}} + mc_{\text{F}} t_{\text{F}}}{\rho V c + mc_{\text{F}}} = \frac{21 \cdot 18 + 100.8}{23.52} = 20.36°\text{C},$$

where $m$ is the mass of the stone, $V$ is the volume of water, $c$ is the specific heat of water, $c_{\text{F}}$ is the specific heat of the stone, $t_{\text{F}}$ is the temperature of the stone, $\rho$ is the density of water. We can determine the water temperatures before removing other stones the same way:

$$2^{\text{nd}} \text{ stone:} \quad \rho V c(t_2 - t_1) = mc_{\text{F}}(t_{\text{F}} - t_2) \quad \Rightarrow t_2 = \frac{\rho V c t_1 + mc_{\text{F}} t_{\text{F}}}{\rho V c + mc_{\text{F}}} = \frac{21 \cdot 20.36 + 100.8}{23.52} = 22.46°\text{C},$$

$$3^{\text{rd}} \text{ stone:} \quad \rho V c(t_3 - t_2) = mc_{\text{F}}(t_{\text{F}} - t_3) \quad \Rightarrow t_3 = \frac{\rho V c t_2 + mc_{\text{F}} t_{\text{F}}}{\rho V c + mc_{\text{F}}} = \frac{21 \cdot 22.46 + 100.8}{23.52} = 24.34°\text{C},$$

$$4^{\text{th}} \text{ stone:} \quad \rho V c(t_4 - t_3) = mc_{\text{F}}(t_{\text{F}} - t_4) \quad \Rightarrow t_4 = \frac{\rho V c t_3 + mc_{\text{F}} t_{\text{F}}}{\rho V c + mc_{\text{F}}} = \frac{21 \cdot 24.34 + 100.8}{23.52} \approx 26°\text{C}.$$

**Solution [11].** Determine the temperature of the water $t_4^4$ when submerging all 4 stones:

$$\rho V c(t_4^4 - t_{\text{W}}) = 4mc_{\text{F}}(t_{\text{F}} - t_4^4) \quad \Rightarrow \quad t_4^4 = \frac{\rho V c t_{\text{W}} + 4mc_{\text{F}} t_{\text{F}}}{\rho V c + 4mc_{\text{F}}} = \frac{21 \cdot 18 + 4 \cdot 100.8}{31.08} \approx 25°\text{C}.$$

**Criteria.** The right answer in question **[10]** costs 3 points, incorrectly rounded answers «25°C» and «27°C» cost 1 point. Question **[11]** costs 1 point.

**8.9.** **(3 points)** The initial temperature of water of 20°C in the boiler of the steam locomotive is heated up until it boils and evaporates. And then it is heated up to 300°C in the superheater. Superheated vapor goes into the steam engine the efficiency of which makes 30%.

**[12]** How much is the power of the steam engine, provided its water consumption is 7.2 $m^3$/h? Give the answer in MW, rounded to integers.

**Remark.** The specific heat capacity of water is 4.2 $\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$, the specific heat of water vaporization is 2.3 MJ/kg, the average heat capacity of water vapor in the range 100–300°C is 4.2 $\frac{\text{kJ}}{\text{kg}\cdot\text{K}}$.

(*T. Andreeva*)

**Answer:** 2.

**Solution.** According to the definition of efficiency

$$\eta = \frac{P_{\text{Out}}}{P_{\text{In}}}.$$

Calculate the spent power:

$$P_{\text{In}} = \frac{Q_{\text{In}}}{t} = \frac{\rho V c(t_{\text{F}} - t_0) + \rho V r + \rho_{\text{S}} V_{\text{S}} c_{\text{S}}(t_{\text{S}} - t_{\text{F}})}{t}.$$

Here $c$ is the specific heat of water, $c_{\text{S}}$ is the specific heat of steam, $V$ is the volume of water, $\rho$ is the density of water, $t_{\text{F}}$ is the boiling point, $t_{\text{S}}$ is the temperature of the steam, $V_{\text{S}}$ is the volume of the steam, $\rho_{\text{S}}$ is the density of the steam.

Taking into account that $V_{\text{S}}\rho_{\text{S}} = V\rho$, calculate the useful power:

$$P_{\text{Out}} = \frac{\eta Q_{\text{In}}}{t} = \eta\rho \cdot \frac{V}{t} \cdot \big(c(t_{\text{F}} - t_0) + r + c_{\text{S}}(t_{\text{S}} - t_{\text{F}})\big) = 2\ \text{MW}.$$

**Criteria.** If the given answer is correct but rounded to tenths («2.1»), it costs 2 points.

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# Solutions of the problems for R9

**9.1. (3 points)** The train that started from Kings Cross railway station has travelled 1/8 of its way at the speed of $v_1 = 52$ km/h. The average velocity of the train on its whole way has made $\langle v\rangle = 34$ km/h.

**[1]** At what speed has the train made the rest of its way if we know that 1/10 of the whole time it took him to make its way the train has been stopped to let the Hogwarts Express go? Give the answer in km/h, rounded to tenths. (*T. Andreeva*)

**Answer:** 36.4.

**Solution.** The total travel time is the sum of the time of movement at the speed $v_1$ of waiting and movement at speed $v_3$:

$$t = t_1 + \frac{t}{10} + t_3 = \frac{S}{8v_1} + \frac{t}{10} + \frac{7S}{8v_3} = \frac{10S}{72v_1} + \frac{70S}{72v_3}.$$

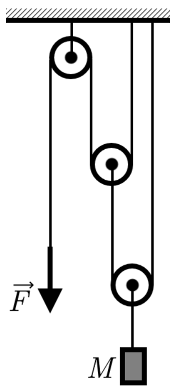
From the definition of average speed:

$$\langle v\rangle = \frac{S}{t} = \frac{S}{\dfrac{10S}{72v_1} + \dfrac{70S}{72v_3}} = \frac{S}{\dfrac{S}{72}\cdot\left(\dfrac{10}{v_1} + \dfrac{13}{v_3}\right)} = \frac{72v_1v_3}{10v_3 + 70v_1},$$

$$v_3 = \frac{70\langle v\rangle v_1}{72v_1 - 10\langle v\rangle} = 36.4\ \frac{\text{km}}{\text{h}}.$$

**Criteria.** The right answer rounded to integers («36») costs 1 point; rounded to hundreds («36.36») costs 3 points. Answers with a tenth of an error («36.3» and «36.5») cost 2 points.

**9.2. (3 points)** The load of $M = 10$ kg in mass has been lifted using the block system shown in the picture. The constant force of 50 N has been applied to the tightrope within 2 sec and then the tightrope has been dropped by accident and it could only get back under control 4 sec later. It afterwards took 5 sec more to get the load to the necessary level with the same force applied.




**[2]** At what level has the load been lifted from its initial position? Give the answer in meters.

**Remark.** Neglect the friction of pulleys and air resistance. Consider the pulleys as weightless, the ropes as light and inextensible, and the parts of ropes that are free of pulleys positioned vertically. Consider the acceleration of gravity to be 10 m/sec$^2$. (*M. Krupina*)

**Answer:** 45.

**Solution.** The equations of motion of the body and blocks, considering that their masses are equal to zero and the tension forces in each thread are constant:

$$\begin{cases} Ma = T_1 - Mg, \\ T_1 = 2T_2, \\ T_2 = 2T_3, \\ T_3 = F \end{cases}$$

(here $T_2$ is the tension force in the right thread, $T_3$ is the tension force in the left thread). Transform and find the acceleration:

$$Ma = 4F - Mg \quad \Rightarrow \quad a = 4 \cdot \frac{F}{M} - g = 4 \cdot \frac{50}{10} - 10 = 10\ \frac{\text{m}}{\text{sec}^2}.$$

The height level and speed at the end of the first two seconds:

$$\begin{cases} x_1 = \dfrac{at_1^2}{2} = \dfrac{10 \cdot 4}{2} = 20\ \text{m}, \\ v_1 = at_1 = 20\ \dfrac{\text{m}}{\text{sec}}. \end{cases}$$

The height level and speed after next 4 seconds:

$$\begin{cases} x_2 = x_1 + v_1 t_2 - \dfrac{gt_2^2}{2} = 20 + 20 \cdot 4 - \dfrac{10 \cdot 16}{2} = 20\ \text{m}, \\ v_2 = v_1 - gt_2 = 20 - 10 \cdot 4 = -20\ \dfrac{\text{m}}{\text{sec}}. \end{cases}$$

therefore, the final height level:

$$x_3 = x_2 + v_2 t_3 + \frac{at_3^2}{2} = 20 - 20 \cdot 5 + \frac{10 \cdot 25}{2} = 45\ \text{m}.$$

**Criteria.** Incorrectly rounded answer «44» costs 2 points.

**9.3.** **(3 points)** The kids are sledging. Andrew is pulling the sledge with Mary by the rope at the angle $\alpha = 30°$ to the horizon while Pete is pushing the same kind of sledge with Daria on it directing the force of $F_2 = 140$ N downwards at the angle $\alpha = 30°$ to the horizon.

**[3]** What force $F_1$ should Adrew apply so that the girls moved with the same acceleration? Give the answer in Newtons, rounded to integers.

The mass of Mary together with her sledge makes $m_1 = 40$ kg, Daria weighs $m_2 = 35$ kg together with her sledge. Consider the acceleration of gravity to be 10 m/sec$^2$. Friction coefficient is $\mu = 0.20$.

(*M. Krupina, S. Starovoytov*)

**Answer:** 127.

**Solution.** The movement of the sledge with Mary. Newton's second law in projections on the horizontal and vertical axes:

$$\begin{cases} F_1 \cos\alpha - F_{\text{fr}1} = m_1 a, \\ F_1 \sin\alpha + N_1 = m_1 g. \end{cases}$$

Besides, $F_{\text{fr}1} = \mu N_1$. Solving the system of equations, we get

$$F_1 \cos\alpha - \mu m_1 g + \mu F_1 \sin\alpha = m_1 a. \qquad (*)$$

The movement of the sledge with Daria:

$$\begin{cases} F_2 \cos\alpha - F_{\text{fr}2} = m_2 a, \\ F_1 \sin\alpha + m_1 g = N_2, \\ F_{\text{fr}2} = \mu N_2. \end{cases}$$

When solving the second system, we express the acceleration:

$$a = \frac{F_2 \cos\alpha - \mu m_2 g - \mu F_2 \sin\alpha}{m_2}.$$

Substitute the acceleration into formula $(*)$ and after transformations we get

$$F_1 = F_2 \cdot \frac{m_1(\cos\alpha - \mu \sin\alpha)}{m_2(\cos\alpha + \mu \sin\alpha)} = 127\ \text{N}.$$

**Criteria.** Incorrect answers from 120 to 135 cost 1 point, except for the answer «126», which costs 2 points (the jury assumes that these errors are caused by incorrect roundings of the trigonometric values).

**9.4. (3 points)** Grandpa Mazai is saving seven hares in the spring floods. Frightened hares are sitting at the stern of the boat, and grandpa Mazai is standing on the bow. The boat is 3.4 m long and weighs 110 kg. The mass of grandpa in his sheepskin coat makes 90 kg.

Not far from the land, the boat stops and Mazai and the hares change their places: grandpa gats to the stern, and the hares get to the bow of the boat. As a result, the boat approaches the land by 1 m.

**[4]** Find out the average weight of a hare. Give the answer in kilos, rounded to tenths.

(*T. Andreeva*)

**Answer:** 3.4.

**Solution.** The center of mass of the system «Boat + Mazai + Hares» does not shift when Mazai and hares move around the boat. Let us find the center of mass of the system in both cases relative to the stern of the boat:

$$\begin{cases} x_1 = \dfrac{M_\mathrm{B}\cdot\dfrac{l}{2} + M_\mathrm{M}l}{M_\mathrm{B} + M_\mathrm{M} + nm}, \\ x_2 = \dfrac{M_\mathrm{B}\cdot\dfrac{l}{2} + nml}{M_\mathrm{B} + M_\mathrm{M} + nm}. \end{cases}$$

Here $m$ is the hare's weight, $M_\mathrm{B}$ is the mass of the boat, $M_\mathrm{M}$ is the Mazai's weight, $l$ is the length of the boat. Then, the shift of the boat makes

$$\Delta x = x_1 - x_2 = \frac{M_\mathrm{M}l - nml}{M_\mathrm{B} + M_\mathrm{M} + nm} \quad\Rightarrow\quad m = \frac{M_\mathrm{M}l - \Delta x(M_\mathrm{M} + M_\mathrm{B})}{n(l + \Delta x)} = 3.4\text{ kg}.$$

**Criteria.** All wrong answers from 3 to 3,8 cost 1 point, between 3,3 and 3,5 — 2 points (the jury assumes that these errors are caused by incorrect roundings).

**9.5. (4 points)** A thin-walled cylindrical glass filled with maple syrup to its quarter (1/4) is floating in a vessel of water, being immersed to its middle. The same glass, but filled with water to its half (1/2), is floating in a vessel with syrup, also being immersed to its middle.


**[5]** What part of the glass can be filled with syrup so that it does not sink in water?

**[6]** And what part of the glass can be filled with syrup so that it does not sink in syrup?

Give answers as decimals, rounded to thousandths. (*T. Andreeva*)

**Answers: [5]** 0.625. **[6]** 0.875.

**Solution [5].** Balance conditions for a glass of mass $m$:

in the 1st case: $$mg + \frac{1}{4}\cdot\rho_\mathrm{S}gV = \frac{1}{2}\cdot\rho gV, \qquad (9.5.1)$$

in the 2nd case: $$mg + \frac{1}{2}\cdot\rho gV = \frac{1}{2}\cdot\rho_\mathrm{S}gV, \qquad (9.5.2)$$

where $V$ is the volume of the glass, $g$ is the free fall acceleration, $\rho$ is the density of water, $\rho_\mathrm{S}$ is the density of the syrup. By subtracting the second from the first equation we get

$$\frac{3}{4}\cdot\rho_\mathrm{S}gV = \rho gV,$$

$$\rho_\mathrm{S} = \frac{4}{3}\cdot\rho, \qquad (9.5.3)$$

$$\rho_\mathrm{S} = \frac{4}{3}\cdot\rho = 1333\ \frac{\text{kg}}{\text{m}^3}.$$

Substituting (9.5.3) in (9.5.1) or (9.5.2), we get

$$m = \frac{1}{6} \cdot \rho V = \frac{1}{8} \cdot \rho_S V.$$

In order the glass of syrup doesn't sink in water there has to be

$$mg + x\rho_S gV \leqslant \rho gV,$$

$$\frac{1}{6} \cdot \rho gV + x \cdot \frac{4}{3} \cdot \rho gV \leqslant \rho gV,$$

$$x \leqslant \frac{5}{8} = 0.625.$$

So no more that 5/8 of the glass volume.

**Solution [6].** In order the glass of syrup doesn't sink in syrup there has to be

$$mg + x\rho_S gV \leqslant \rho_S gV,$$

$$\frac{1}{8} \cdot \rho_S gV + x\rho_S gV \leqslant \rho_S gV,$$

$$x \leqslant \frac{7}{8} = 0.875.$$

So no more that 7/8 of the glass volume.

**Criteria.** Both questions cost 2 points each. The answers, rounded to tenths or hundreds («0.6», «0.62» and «0.63» for the question **[5]** and «0,9», «0.87» and «0.88» for the question **[6]**), cost 1 point.

**9.6. (3 points)** Two satellites move in circular orbits in opposite directions around some planet with linear velocities $v_1 = 5$ km/sec and $v_2 = 8$ km/sec. The radius of the planet equals $R = 17.4$ thousand km and the acceleration of free fall on its surface makes $g = 14$ m/sec$^2$.

**[7]** Find out the time interval within which the satellites periodically approach each other at a minimum distance. Give your answer in hours, rounded to tenths.

(*M. Korobkov, T. Andreeva*)

**Answer:** 11.6.

**Solution.** The correlation between the gravitational force and the force of gravity on the surface of the planet:

$$G \cdot \frac{mM}{R^2} = mg \quad \Rightarrow \quad gR^2 = GM.$$

The equations of motion of satellites in orbits:

$$G \cdot \frac{mM}{r_1^2} = m \cdot \frac{v_1^2}{r_1} \quad \Rightarrow \quad r_1 = \frac{GM}{v_1^2} = \frac{gR^2}{v_1^2},$$

$$G \cdot \frac{mM}{r_2^2} = m \cdot \frac{v_2^2}{r_2} \quad \Rightarrow \quad r_2 = \frac{GM}{v_2^2} = \frac{gR^2}{v_2^2}.$$

At the moment of closest approach (after the previous one), the two satellites together will outline the angle $2\pi$:

$$2\pi = \omega_1 t + \omega_2 t = \frac{v_1}{r_1} \cdot t + \frac{v_2}{r_2} \cdot t,$$

where $\omega_1$ and $\omega_2$ are the satellites' angle velocities.

After the substitution and transformations we get

$$t = \frac{2\pi gR^2}{v_1^3 + v_2^3} = 11.6 \text{ hours}.$$

**Criteria.** The right answer, rounded to integers («12»), costs 2 points.

**9.7.** **(4 points)** Ruth and Mary cooked 2 liters of cranberry juice. There was very little time left before the arrival of the guests, and the fruit drink was still warm (40°C). The girls wanted to cool it down quickly with the help of 20 plastic balls filled with ice (−20°C), but then they argued and divided the fruit drink and the balls equally between themselves. Ruth put all the balls into the fruit drink at once, and Mary poured her fruit drink into two identical jugs and put the balls into one of them, then waited, stirring the fruit drink, until the temperatures of the balls and the fruit drink got equal, and then replaced the balls to the second jug. When the temperatures equalized again, Mary poured the drink into one large jug.

**[8]** Whose drink turned out to be colder in the end? Put either **the letter** R if Ruth's drink is colder, or M if otherwise.

**[9]** By how many degrees that drink turned out to be colder? Round your answer to integers.

The mass of ice in a ball is 20 g.

**Remark.** The specific heat capacity of water (and the fruit drink) is 4.2 $\frac{\text{J}}{\text{kg}\cdot\text{K}}$, the specific heat capacity of ice is 2.0 $\frac{\text{J}}{\text{kg}\cdot\text{K}}$, and the specific heat of fusion of ice is 340 kJ/kg. (*T. Andreeva*)

**Answers: [8]** M. **[9]** 2.

**Solution.** The equations of thermal equilibrium for Ruth's actions:

$$\rho \cdot \frac{V}{2} \cdot c\left(t_1 - t_2^{\text{Ruth}}\right) = \frac{N}{2} \cdot mc_{\text{I}}(0 - t_{\text{I}}) + \frac{N}{2} \cdot m\lambda + \frac{N}{2} \cdot mc\left(t_2^{\text{Ruth}} - 0\right),$$

where $c$ is specific heat capacity of water, $\rho$ is density of water, $V$ is water volume, $m$ is the ball of ice weight, $N$ is the number of balls, $t_{\text{I}}$ is the initial ice temperature, $\lambda$ is the specific heat of ice melting, $t_1$ is the initial fruit juice temperature, $c_{\text{I}}$ is the specific heat capacity of ice.

Then

$$t_2^{Ruth} = \frac{\rho V c t_1 - Nm(c_{\text{I}}(0 - t_{\text{I}}) + \lambda)}{\rho V c + Nmc} = \frac{184}{10.08} \approx 18^\circ\text{C}.$$

Now the equations of thermal equilibrium for Mary's actions. For the first jug:

$$\rho \cdot \frac{V}{4} \cdot c(t_1 - t_2^1) = \frac{N}{2} \cdot mc_{\text{I}}(0 - t_{\text{I}}) + \frac{N}{2} \cdot m\lambda + \frac{N}{2} \cdot mc(t_2^1 - 0),$$

$$t_2^1 = \frac{\rho \cdot \frac{V}{2} \cdot ct_1 - Nm(c_{\text{I}}(0 - t_{\text{I}}) + \lambda)}{\rho \cdot \frac{V}{2} \cdot c + Nmc} = \frac{16}{5.88} = 2.72^\circ\text{C}.$$

For the second jug:

$$\rho \cdot \frac{V}{4} \cdot c(t_1 - t_2^2) = \frac{N}{2} \cdot mc(t_2^2 - t_2^1) \quad \Rightarrow \quad t_2^2 = \frac{\rho \cdot \frac{V}{2} \cdot t_1 + Nmt_2^1}{\rho \cdot \frac{V}{2} + Nm} = \frac{41.09}{1.4} = 29.35^\circ\text{C}.$$

For the third jug:

$$\rho \cdot \frac{V}{4} \cdot c(t_2^2 - t_2^{\text{Mary}}) = \rho \cdot \frac{V}{4} \cdot c(t_2^{\text{Mary}} - t_2^1) \quad \Rightarrow \quad t_2^{\text{Mary}} = \frac{t_2^1 + t_2^2}{2} \approx 16^\circ\text{C}.$$

Therefore, in the end Mary's juice is colder by 18 − 16 = 2°C.

**Criteria.** Both questions cost 2 points each. The right answer for the question **[9]**, not rounded to integers (e.g. «2.1»), costs 2 points.

**9.8.** **(2 points)** The initial temperature of water of 20°C in the boiler of the steam locomotive is heated up until it boils and evaporates. And then it is heated up to 300°C in the superheater. Superheated vapor goes into the steam engine the efficiency of which makes 30%.

**[10]** How much is the power of the steam engine, provided its water consumption is 7.2 m$^3$/h? Give the answer in MW, rounded to integers.

**Remark.** The specific heat capacity of water is $4.2\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$, the specific heat of water vaporization is 2.3 MJ/kg, the average heat capacity of water vapor in the range 100–300°C is $4.2\ \frac{\text{kJ}}{\text{kg}\cdot\text{K}}$.

(*T. Andreeva*)

**Answer:** 2.

**Solution.** According to the definition of efficiency

$$\eta = \frac{P_{\text{Out}}}{P_{\text{In}}}.$$

Calculate the spent power:

$$P_{\text{In}} = \frac{Q_{\text{In}}}{t} = \frac{\rho V c(t_{\text{F}} - t_0) + \rho V r + \rho_{\text{S}} V_{\text{S}} c_{\text{S}}(t_{\text{S}} - t_{\text{F}})}{t}.$$

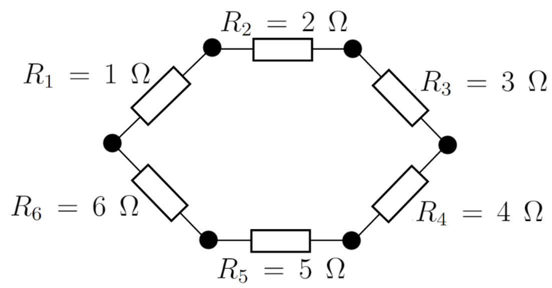
Here $c$ is the specific heat of water, $c_{\text{S}}$ is the specific heat of steam, $V$ is the volume of water, $\rho$ is the density of water, $t_{\text{F}}$ is the boiling point, $t_{\text{S}}$ is the temperature of the steam, $V_{\text{S}}$ is the volume of the steam, $\rho_{\text{S}}$ is the density of the steam.

Taking into account that $V_{\text{S}}\rho_{\text{S}} = V\rho$, calculate the useful power:

$$P_{\text{Out}} = \frac{\eta Q_{\text{In}}}{t} = \eta\rho \cdot \frac{V}{t} \cdot \big(c(t_{\text{F}} - t_0) + r + c_{\text{S}}(t_{\text{S}} - t_{\text{F}})\big) = 2\ \text{MW}.$$

**Criteria.** If the given answer is correct but rounded to tenths («2.1»), it costs 2 points.

**9.9.** **(3 points)** Six resistors $R_1 = 1\ \Omega$, $R_2 = 2\ \Omega$, $R_3 = 3\ \Omega$, $R_4 = 4\ \Omega$, $R_5 = 5\ \Omega$ and $R_6 = 6\ \Omega$ are connected in series one by one and are closed in a ring. A constant voltage source was connected to two contacts of the circuit (black dots in the picture) so that the resistance of the circuit between these contacts is at **maximum**. The source voltage is $U = 36$ V.




**[11]** Find out the power $P_3$ produced in the resistor $R_3$. Give the answer in watts, rounded to integers. (*S. Starovoytov*)

**Answer:** 39.

**Solution.** Between any pair of contacts we have two parallel branches of resistors. The resistance of such a circuit makes

$$R_{\text{total}} = \frac{R_{\text{I}} R_{\text{II}}}{R_{\text{I}} + R_{\text{II}}}.$$

With any connection, the sum of the resistances of both branches is the same (21 Ω). Product is at maximum for the case $10 \cdot 11 = 110$. Therefore, the resistance of the entire circuit is maximum being connected to such a pair of contacts when resistors are in series in one branch: $1\ \Omega + 2\ \Omega + 3\ \Omega + 4\ \Omega = 10\ \Omega$, and in the other branch — resistors $5\ \Omega + 6\ \Omega = 11\ \Omega$.

The current in the first branch:

$$I_{1234} = \frac{U}{R_{1234}} = \frac{36}{10} = 3.6\ \text{A}.$$

Power dissipated in the third resistor:

$$P_3 = I_{1234}^2 R_3 = 3.6^2 \cdot 3 = 38.88 \approx 39\ \text{W}.$$

**Criteria.** The right answer, not rounded to integers (e.g. «38.88»), costs 2 points.

# Solutions of the problems for R10

**10.1. (3 points)** Sherlock Holmes and Professor Moriarty stepped out of the first and last doors of the arrived train of and headed towards each other at a constant speed. After the boarding of passengers was completed, the train began to move with constant acceleration of 1 m/sec$^2$, and in 12 sec the last train carriage passed Professor Moriarty, and then in 2 sec more it passed Sherlock Holmes as well. The length of the train and the platform makes 172 m.


**[1]** How far apart were Sherlock Holmes and Professor Moriarty when the last train carriage left the station? Give your answer in meters, rounded to integers. (*T. Andreeva*)

**Answer:** 15.

**Solution.** If $\tau$ is the time the train didn't move, $L$ is the length of the train, and $v$ is the speed at which Sherlock Holmes and Professor Moriarty went, then the equations describing their meeting to the last train carriage make

$$\begin{cases} \dfrac{at_1^2}{2} = v(t_1+\tau), \\ \dfrac{a(t_1+t_2)^2}{2} = L - v(t_1+t_2+\tau). \end{cases}$$

Solving this system we get $v = 1$ m/sec and $\tau = 60$ sec.
The time at which the last train carriage left the station is

$$t_3 = \sqrt{\frac{2L}{a}} = 18.6 \text{ sec.}$$

By this moment the distance between Sherlock Holmes and Professor Moriarty made

$$S = L - 2v(t_3+\tau) \approx 15 \text{ m.}$$

**10.2. (2 points)** Cliff diver jumped into the water from a 29-meter high cliff. When he had flown the first 10 m, a spectator fell off the edge of the rock into the water.

**[2]** How high is the edge if the athlete and the spectator got to the water at the same time? Give the answer in meters, rounded to integers.

**Remark.** Cliff diving is jumping into the water from natural levels like rocks. Assume that the jumper's initial speed is zero. Consider the acceleration of gravity to be 10 m/sec$^2$. Ignore air resistance. (*T. Andreeva*)

**Answer:** 5.

**Solution.** The height of the cliff is $H_1 = gt_1^2/2$, where $t_1$ is the total time of the diver flying in the air after the jump:

$$t_1 = \sqrt{\frac{2H_1}{g}} = 2.4 \text{ sec.}$$

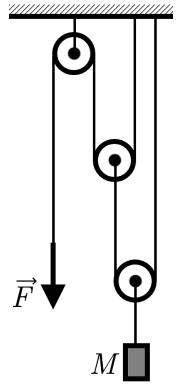
The first 10 meters of the flight after the jump are $\Delta H = g\Delta t^2/2$. In terms of time it took

$$\Delta t = \sqrt{\frac{2\Delta H}{g}} = 1.4 \text{ sec.}$$

The spectator's time of fall is $t_2 = t_1 - \Delta t = 1$ sec, so the height of the edge of the rock:

$$H_2 = \frac{gt_2^2}{2} = 5 \text{ m.}$$

**10.3.** **(3 points)** The load of $M = 10$ kg in mass has been lifted using the block system shown in the picture. The constant force of 50 N has been applied to the tightrope within 2 sec and then the tightrope has been dropped by accident and it could only get back under control 4 sec later. It afterwards took 5 sec more to get the load to the necessary level with the same force applied.




**[3]** At what level has the load been lifted from its initial position? Give the answer in meters.

**Remark.** Neglect the friction of pulleys and air resistance. Consider the pulleys as weightless, the ropes as light and inextensible, and the parts of ropes that are free of pulleys positioned vertically. Consider the acceleration of gravity to be 10 m/sec$^2$.

(*M. Krupina*)

**Answer:** 45.

**Solution.** The equations of motion of the body and blocks, considering that their masses are equal to zero and the tension forces in each thread are constant:

$$\begin{cases} Ma = T_1 - Mg, \\ T_1 = 2T_2, \\ T_2 = 2T_3, \\ T_3 = F \end{cases}$$

(here $T_2$ is the tension force in the right thread, $T_3$ is the tension force in the left thread). Transform and find the acceleration:

$$Ma = 4F - Mg \quad \Rightarrow \quad a = 4 \cdot \frac{F}{M} - g = 4 \cdot \frac{50}{10} - 10 = 10\ \frac{\text{m}}{\text{sec}^2}.$$

The height level and speed at the end of the first two seconds:

$$\begin{cases} x_1 = \dfrac{at_1^2}{2} = \dfrac{10 \cdot 4}{2} = 20\ \text{m}, \\ v_1 = at_1 = 20\ \dfrac{\text{m}}{\text{sec}}. \end{cases}$$

The height level and speed after next 4 seconds:

$$\begin{cases} x_2 = x_1 + v_1 t_2 - \dfrac{gt_2^2}{2} = 20 + 20 \cdot 4 - \dfrac{10 \cdot 16}{2} = 20\ \text{m}, \\ v_2 = v_1 - gt_2 = 20 - 10 \cdot 4 = -20\ \dfrac{\text{m}}{\text{sec}}. \end{cases}$$

therefore, the final height level:

$$x_3 = x_2 + v_2 t_3 + \frac{at_3^2}{2} = 20 - 20 \cdot 5 + \frac{10 \cdot 25}{2} = 45\ \text{m}.$$

**Criteria.** Incorrectly rounded answer «44» costs 2 points.

**10.4.** **(2 points)** The kids are sledging. Andrew is pulling the sledge with Mary by the rope at the angle $\alpha = 30°$ to the horizon while Pete is pushing the same kind of sledge with Daria on it directing the force of $F_2 = 140$ N downwards at the angle $\alpha = 30°$ to the horizon.

**[4]** What force $F_1$ should Adrew apply so that the girls moved with the same acceleration? Give the answer in Newtons, rounded to integers.

The mass of Mary together with her sledge makes $m_1 = 40$ kg, Daria weighs $m_2 = 35$ kg together with her sledge. Consider the acceleration of gravity to be 10 m/sec$^2$. Friction coefficient is $\mu = 0.20$.

(*M. Krupina, S. Starovoytov*)

**Answer:** 127.

**Solution.** The movement of the sledge with Mary. Newton's second law in projections on the horizontal and vertical axes:

$$\begin{cases} F_1 \cos\alpha - F_{\text{fr}1} = m_1 a, \\ F_1 \sin\alpha + N_1 = m_1 g. \end{cases}$$

Besides, $F_{\text{fr}1} = \mu N_1$. Solving the system of equations, we get

$$F_1 \cos\alpha - \mu m_1 g + \mu F_1 \sin\alpha = m_1 a. \qquad (*)$$

The movement of the sledge with Daria:

$$\begin{cases} F_2 \cos\alpha - F_{\text{fr}2} = m_2 a, \\ F_1 \sin\alpha + m_1 g = N_2, \\ F_{\text{fr}2} = \mu N_2. \end{cases}$$

When solving the second system, we express the acceleration:

$$a = \frac{F_2 \cos\alpha - \mu m_2 g - \mu F_2 \sin\alpha}{m_2}.$$

Substitute the acceleration into formula ($*$) and after transformations we get

$$F_1 = F_2 \cdot \frac{m_1(\cos\alpha - \mu \sin\alpha)}{m_2(\cos\alpha + \mu \sin\alpha)} = 127 \text{ N}.$$

**Criteria.** Incorrect answers from 124 to 130 cost 1 point (the jury assumes that these errors are caused by incorrect roundings of the trigonometric values).

**10.5. (3 points)** A thin-walled glass cup is floating in a vessel filled with water. The syrup is being slowly poured into it until the glass is immersed in water exactly to its middle. It appears that a quarter of the glass got filled with syrup. The second glass of the same type, filled up to half with water, is floating in the syrup, immersed to its middle.

**[5]** What part of the second glass will be immersed in the syrup if the contents of the first glass are poured into it? Give your answer as a decimal with an accurancy of hundredths.

(*T. Andreeva*)

**Answer:** 0.75.

**Solution.** The equilibrium equation for the first glass:

$$mg + \rho g \cdot \frac{V}{4} = \rho_0 g \cdot \frac{V}{2}, \qquad (10.5.1)$$

where $m$ is the mass of the empty glass, $V$ is the volume of the glass, $\rho$ is the syrup density, $\rho_0$ is the water density.

The equilibrium equation for the second glass:

$$mg + \rho_0 g \cdot \frac{V}{2} = \rho g \cdot \frac{V}{2}. \qquad (10.5.2)$$

By solving together (10.5.1) и (10.5.2), we get

$$m = \frac{\rho_0 V}{6}, \qquad \rho = \frac{4}{3} \cdot \rho_0.$$

Pour the syrup into the glass filled with water:

$$mg + \rho_0 g \cdot \frac{V}{2} + \rho g \cdot \frac{V}{4} = \rho g x V,$$

where $x$ is the immersed part of the glass. After substituting the mass and density we get $x = 3/4 =$ $= 0.75$.

**Criteria.** The wrong answer «0.73», obtained because of incorrect roundings, costs 1 point.

**10.6. (3 points)** Two satellites move in circular orbits in opposite directions around some planet with linear velocities $v_1 = 5$ km/sec and $v_2 = 8$ km/sec. The radius of the planet equals $R = 17.4$ thousand km and the acceleration of free fall on its surface makes $g = 14$ m/sec$^2$.

**[6]** Find out the time interval within which the satellites periodically approach each other at a minimum distance. Give your answer in hours, rounded to tenths.

(*M. Korobkov, T. Andreeva*)

**Answer:** 11.6.

**Solution.** The correlation between the gravitational force and the force of gravity on the surface of the planet:

$$G \cdot \frac{mM}{R^2} = mg \quad \Rightarrow \quad gR^2 = GM.$$

The equations of motion of satellites in orbits:

$$G \cdot \frac{mM}{r_1^2} = m \cdot \frac{v_1^2}{r_1} \quad \Rightarrow \quad r_1 = \frac{GM}{v_1^2} = \frac{gR^2}{v_1^2},$$

$$G \cdot \frac{mM}{r_2^2} = m \cdot \frac{v_2^2}{r_2} \quad \Rightarrow \quad r_2 = \frac{GM}{v_2^2} = \frac{gR^2}{v_2^2}.$$

At the moment of closest approach (after the previous one), the two satellites together will outline the angle $2\pi$:

$$2\pi = \omega_1 t + \omega_2 t = \frac{v_1}{r_1} \cdot t + \frac{v_2}{r_2} \cdot t,$$

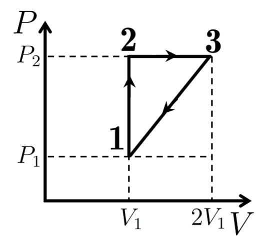
where $\omega_1$ and $\omega_2$ are the satellites' angle velocities.

After the substitution and transformations we get

$$t = \frac{2\pi g R^2}{v_1^3 + v_2^3} = 11.6 \text{ hours}.$$

**Criteria.** The right answer, rounded to integers («12»), costs 2 points.

**10.7. (3 points)** The figure shows a cycle carried out with a monatomic ideal gas. it is known that the efficiency of the Carnot cycle carried out in the same temperature range is 64%, and with isobaric expansion the volume of the gas increases by 2 times.




**[7]** Find out the efficiency of the cycle. Give the answer in percentages with an accuracy of tenths. (*S. Starovoytov*)

**Answer:** 4.8.

**Solution.** From the formula for the efficiency of the Carnot cycle, we determine the temperature ratio:

$$\eta_{\text{Carnot}} = 1 - \frac{T_1}{T_3} \quad \Rightarrow \quad 1 - \eta_{\text{Carnot}} = \frac{T_1}{T_3} = 0.36,$$

where $T_1$ and $T_3$ are the temperatures at points 1 and 3.

The useful work obtained in the cycle is equal to the area inside the cycle, that is, the area of the triangle:

$$W_{\text{cycle}} = \frac{1}{2} \cdot V_1 (P_2 - P_1) = \frac{\nu R}{2} \left( \frac{T_3}{2} - T_1 \right).$$

Here the equations of state at points 1 and 3 and the ratio of volumes at these points are taken into account.

Calculate the work for an isobaric process:

$$W_{23} = P_2 (V_3 - V_1) = P_2 V_1 = \frac{1}{2} \cdot P_3 V_3 = \frac{1}{2} \cdot \nu R T_3.$$

Here the equation of state at point 3 and the ratios of volumes at points 1 and 3 are taken into account.

The total change in the internal energy of a monatomic gas in isochoric and isobaric processes:

$$\Delta U_{13} = \frac{3}{2} \cdot \nu R (T_3 - T_1).$$

Now determine the total amount of heat supplied in isochoric and isobaric processes:

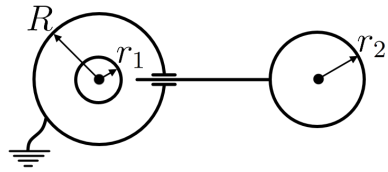
$$Q_{\text{total}} = W_{23} + \Delta U_{13} = \frac{1}{2} \cdot \nu R (4T_3 - 3T_1),$$

and get the answer:

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{total}}} = \frac{\frac{\nu R}{2}\left(\frac{T_3}{2} - T_1\right)}{\frac{\nu R}{2}(4T_3 - 3T_1)} = \frac{\frac{1}{2} - \frac{T_1}{T_3}}{4 - 3 \cdot \frac{T_1}{T_3}} = \frac{0.5 - 0.36}{4 - 1.08} = 0.048.$$

**Criteria.** The wrong answer «4.7», obtained because of incorrect roundings, costs 1 point.

**10.8.** **(4 points)** Charges of $q_1 = 0.6$ $\mu$C and $q_2 = 1.5$ $\mu$C were imparted to two metal balls with radii $r_1 = 10$ cm and $r_2 = 20$ cm respectively, and then they were connected with a thin wire. After that, a ball with radius $r_1$ is placed inside a metal sphere with a radius $R = 3r_1$ and this sphere is grounded.




**[8]** What charge will pass through the connecting wire when the sphere is being grounded? Give the answer in $\mu$C, rounded to tenths.

**Remark.** The distance between the centers of the balls is much greater than their radii.

(*M. Krupina*)

**Answer:** 0.2.

**Solution.** After connecting the balls, the sum of electric charges will not change:

$$q_1 + q_2 = q_1' + q_2'. \qquad (*)$$

The potentials of the connected balls are equal:

$$\frac{kq_1'}{r_1} = \frac{kq_2'}{r_2} \quad \Rightarrow \quad q_1' = q_2' \cdot \frac{r_1}{r_2}.$$

Given the equality of the sums ($*$), we get the magnitude of the charges after the connection:

$$q_1 + q_2 = q_2' \cdot \left(\frac{r_1}{r_2} + 1\right) = q_2' \cdot \frac{r_1 + r_2}{r_2} \quad \Rightarrow \quad \begin{cases} q_2' = (q_1 + q_2) \cdot \dfrac{r_2}{r_1 + r_2}, \\ q_1' = (q_1 + q_2) \cdot \dfrac{r_1}{r_1 + r_2}. \end{cases}$$

After grounding, the surface of the sphere is the surface of zero potential:

$$\frac{kQ}{3r_1} + \frac{kq_1''}{3r_1} = 0 \quad \Rightarrow \quad q_1'' = -Q.$$

In order for the current not to flow through the wire, the potential of the system from the sphere and the first ball and the potential of the second ball must be equal:

$$\frac{kq_1''}{r_1} + \frac{kQ}{3r_1} = \varphi' = \frac{kq_2''}{r_2}.$$

Solving, we get a new connection between the charges of the balls:

$$\frac{kq_1''}{r_1} - \frac{kq_1''}{3r_1} = \frac{kq_2''}{r_2} \quad \Rightarrow \quad q_2'' = \frac{2}{3} \cdot q_1'' \cdot \frac{r_2}{r_1}.$$

Since the amount of charges has not changed (likewise ($*$)), we get the final value of the charge of the first ball:

$$q_1 + q_2 = q_1'' + q_2'' = q_1'' \cdot \left(1 + \frac{2}{3} \cdot \frac{r_2}{r_1}\right) \quad \Rightarrow \quad q_1'' = \frac{q_1 + q_2}{1 + \frac{2}{3} \cdot \frac{r_2}{r_1}}.$$

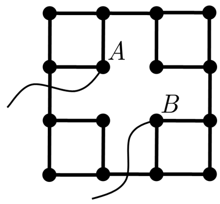
The change in the charge of the first ball, which is equal to the charge that will pass through the connecting wire:

$$\Delta q = |q_1' - q_1''| = \left|\frac{q_1 + q_2}{r_1 + r_2} \cdot r_1 - \frac{q_1 + q_2}{3r_1 + 2r_2} \cdot 3r_1\right| = r_1(q_1 + q_2)\left|\frac{1}{r_1 + r_2} - \frac{3}{3r_1 + 2r_2}\right| =$$

$$= \frac{r_1 r_2 (q_1 + q_2)}{(r_1 + r_2)(3r_1 + 2r_2)} = \frac{0.1 \cdot 0.2 \cdot 2.1 \cdot 10^{-6}}{0.3 \cdot 0.7} = 0.2\ \mu\text{C}.$$

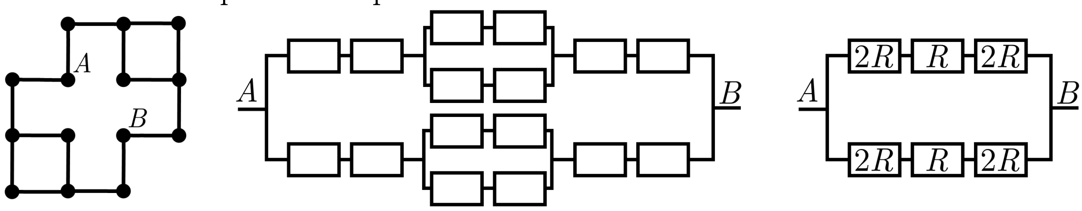
**10.9.** **(3 points)** An electrical circuit consists of identical conductors with a resistance of $R = 10\ \Omega$ forming a grid (see pic). An ohmmeter is connected to the nodes $A$ and $B$.

**[9]** Find out what the ohmmeter reads. Give your answer in $\Omega$ with an accuracy of integers. *(M. Korobkov, T. Andreeva)*




**Answer:** 25.

**Solution.** Construct a sequence of equivalent circuits:




As a result $R_{\text{total}} = 2.5R = 25\ \Omega$.

# Solutions of the problems for R11

**11.1. (3 points)** Sherlock Holmes and Professor Moriarty stepped out of the first and last doors of the arrived train of and headed towards each other at a constant speed. After the boarding of passengers was completed, the train began to move with constant acceleration of 1 m/sec$^2$, and in 12 sec the last train carriage passed Professor Moriarty, and then in 2 sec more it passed Sherlock Holmes as well. The length of the train and the platform makes 172 m.

**[1]** How far apart were Sherlock Holmes and Professor Moriarty when the last train carriage left the station? Give your answer in meters, rounded to integers. (*T. Andreeva*)

**Answer:** 15.

**Solution.** If $\tau$ is the time the train didn't move, $L$ is the length of the train, and $v$ is the speed at which Sherlock Holmes and Professor Moriarty went, then the equations describing their meeting to the last train carriage make

$$\begin{cases} \dfrac{at_1^2}{2} = v(t_1+\tau), \\ \dfrac{a(t_1+t_2)^2}{2} = L - v(t_1+t_2+\tau). \end{cases}$$

Solving this system we get $v = 1$ m/sec and $\tau = 60$ sec.

The time at which the last train carriage left the station is

$$t_3 = \sqrt{\frac{2L}{a}} = 18.6 \text{ sec}.$$

By this moment the distance between Sherlock Holmes and Professor Moriarty made

$$S = L - 2v(t_3+\tau) \approx 15 \text{ m}.$$

**11.2. (3 points)** An athlete — a shot putter, standing on a horizontal surface, pushed the shot at a speed of 12 m/sec at an angle of 30° to the horizon.

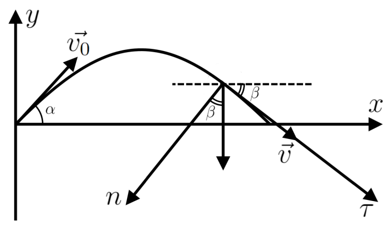
**[2]** What will be the radius of curvature of the shot trajectory in 1 sec after the throw? Give your answer in meters with an accuracy of tenths.

**Remark.** Consider the acceleration of gravity to be 10 m/sec$^2$. Ignore air resistance.

(*M. Krupina*)

**Answer:** 13.3.

**Solution.** Consider the point of the trajectory where the body is at the time $t = 1$ sec (see the fig.). The speed is directed tangentially along the trajectory. The center of curvature is located at the perpendicular to the velocity vector. The angle between the perpendicular and the vertical (the direction of the vector $\vec{g}$) corresponds to $\beta$.




At any point of a curvilinear trajectory, we can write the expression for centripetal acceleration as when moving along a circle with a radius equal to the instant radius of curvature $R$:

$$a_n = \frac{v^2}{R} \quad \Rightarrow \quad R = \frac{v^2}{a_n}. \qquad (*)$$

Project the vector $\vec{g}$ onto the normal to the trajectory:

$$a_n = g\cos\beta.$$

It can be seen from the figure that

$$\cos\beta = \frac{v_x}{v}.$$

The velocities of a body thrown at an angle to the horizon:

$$v_x = v_0\cos\alpha, \qquad v_y = v_0\sin\alpha - gt.$$

Then for any moment of time, the speed modulus is

$$v = \sqrt{v_x^2 + v_y^2} = \sqrt{v_0^2\cos^2\alpha + (v_0\sin\alpha - gt)^2} = \sqrt{v_0^2 - 2gtv_0\sin\alpha + g^2t^2}.$$

Substitute in ($*$) and transform using the expression for the horizontal component of the velocity at the point under consideration and its conservation during the movement process:

$$R = \frac{v^2}{g\cos\beta} = \frac{v^2\cdot v}{gv_x} = \frac{v^3}{gv_0\cos\alpha} = \frac{\left(v_0^2 - 2gtv_0\sin\alpha + g^2t^2\right)^{3/2}}{gv_0\cos\alpha} =$$

$$= \frac{\left(144 - 2\cdot 12\cdot\dfrac{1}{2}\cdot 10\cdot 1 + 100\right)^{3/2}}{10\cdot 12\cdot\dfrac{\sqrt{3}}{2}} = 13.3 \text{ m.}$$

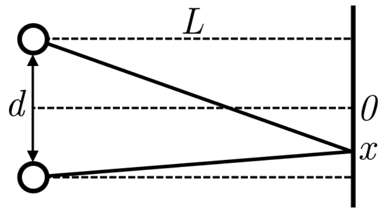
**Criteria.** The wrong answer «13.2», obtained because of incorrect roundings, costs 1 point.

**11.3. (2 points)** Two acoustic systems were installed at the edges of the open-air stage at the distance of 6 m. Due to a mistake by the sound engineer, they «buzzed». The viewer, who was opposite the center of the stage at the distance of 20 m from it, found out that if he moved from his initial position 2 m either to the left or right, then the sound volume turned out to be the smallest.

**[3]** At what frequency did the speakers buzz? Give the answer in Hz, rounded to integers. The speed of sound in air was 345 m/sec. (*T. Andreeva*)

**Answer:** 292.

**Solution.** According to the figure, we determine the distances from the acoustic systems to the viewer in an offset position:




$$S_1 = \sqrt{L^2 + \left(\frac{d}{2} - x\right)^2}, \qquad S_2 = \sqrt{L^2 + \left(\frac{d}{2} + x\right)^2}.$$

Interference minimum condition:

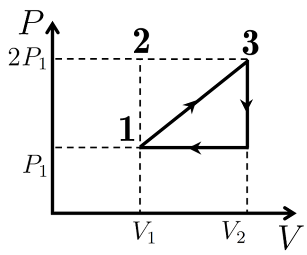
$$\Delta S = \frac{\lambda}{2} = \frac{v}{2f}.$$

Therefore,

$$f = \frac{v}{2\Delta S} = \frac{v}{2\left(\sqrt{L^2 + \left(\dfrac{d}{2} + x\right)^2} - \sqrt{L^2 + \left(\dfrac{d}{2} - x\right)^2}\right)} = 292 \text{ Hz.}$$

**Criteria.** Incorrect answers from 288 to 294 cost 1 point (the jury assumes that these errors are caused by incorrect roundings).

**11.4. (3 points)** The figure shows a cycle carried out with a monatomic ideal gas. it is known that the efficiency of the Carnot cycle carried out in the same temperature range is 64%, and with isobaric expansion the volume of the gas increases by 2 times.




**[4]** Find out the efficiency of the cycle. Give the answer in percentages with an accuracy of tenths. (*S. Starovoytov*)

**Answer:** 6.0.
**Solution.** From the formula for the efficiency of the Carnot cycle, we determine the temperature ratio:

$$\eta_{\text{Carnot}} = 1 - \frac{T_1}{T_2} \quad \Rightarrow \quad 1 - \eta_{\text{Carnot}} = \frac{T_1}{T_2} = 0{,}36,$$

where $T_1$ and $T_2$ are the temperatures at points 1 and 2.
The useful work obtained in the cycle is equal to the area inside the cycle, that is, the area of the triangle:

$$W_{\text{cycle}} = \frac{1}{2} \cdot P_1(V_2 - V_1) = \frac{\nu R}{2}\left(\frac{T_2}{2} - T_1\right).$$

Here the equations of state at points 1 and 2 and the ratio of pressures at these points are taken into account.
The work for the process $1 \to 2$ as the area of the trapezoid and use the equations of state at points 1 and 2:

$$W_{12} = \frac{1}{2} \cdot 3P_1(V_2 - V_1) = \frac{3}{2} \cdot \nu R\left(\frac{T_2}{2} - T_1\right).$$

The change in internal energy in the process $1 \to 2$:

$$\Delta U_{12} = \frac{3}{2} \cdot \nu R(T_2 - T_1).$$

Now determine the total amount of heat in process $1 \to 2$:

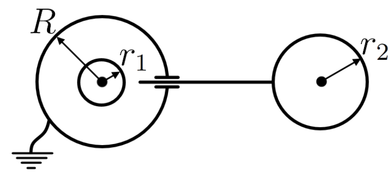
$$Q_{\text{total}} = W_{12} + \Delta U_{12} = \frac{3}{2} \cdot \nu R\left(\frac{3}{2} \cdot T_2 - 2T_1\right),$$

and get the answer:

$$\eta = \frac{W_{\text{cycle}}}{Q_{\text{total}}} = \frac{\dfrac{\nu R}{2}\left(\dfrac{T_2}{2} - T_1\right)}{\dfrac{3}{2} \cdot \nu R\left(\dfrac{3}{2} \cdot T_2 - 2T_1\right)} = \frac{\dfrac{1}{2} - \dfrac{T_1}{T_2}}{3\left(\dfrac{3}{2} - \dfrac{2T_1}{T_2}\right)} = \frac{0.5 - 0.36}{4.5 - 2.16} = 0.060.$$

**Criteria.** The wrong answer «5.9», obtained because of incorrect roundings, costs 1 point.

**11.5. (4 points)** Charges of $q_1 = 0.6\ \mu$C and $q_2 = 1.5\ \mu$C were imparted to two metal balls with radii $r_1 = 10$ cm and $r_2 = 20$ cm respectively, and then they were connected with a thin wire. After that, a ball with radius $r_1$ is placed inside a metal sphere with a radius $R = 3r_1$ and this sphere is grounded.




**[5]** What charge will pass through the connecting wire when the sphere is being grounded? Give the answer in $\mu$C, rounded to tenths.

**Remark.** The distance between the centers of the balls is much greater than their radii.

(*M. Krupina*)

**Answer:** 0.2.
**Solution.** After connecting the balls, the sum of electric charges will not change:

$$q_1 + q_2 = q_1' + q_2'. \tag{$*$}$$

The potentials of the connected balls are equal:

$$\frac{kq_1'}{r_1} = \frac{kq_2'}{r_2} \quad \Rightarrow \quad q_1' = q_2' \cdot \frac{r_1}{r_2}.$$

Given the equality of the sums ($*$), we get the magnitude of the charges after the connection:

$$q_1 + q_2 = q_2' \cdot \left(\frac{r_1}{r_2} + 1\right) = q_2' \cdot \frac{r_1 + r_2}{r_2} \quad \Rightarrow \quad \begin{cases} q_2' = (q_1 + q_2) \cdot \dfrac{r_2}{r_1 + r_2}, \\ q_1' = (q_1 + q_2) \cdot \dfrac{r_1}{r_1 + r_2}. \end{cases}$$

After grounding, the surface of the sphere is the surface of zero potential:

$$\frac{kQ}{3r_1} + \frac{kq_1''}{3r_1} = 0 \quad \Rightarrow \quad q_1'' = -Q.$$

In order for the current not to flow through the wire, the potential of the system from the sphere and the first ball and the potential of the second ball must be equal:

$$\frac{kq_1''}{r_1} + \frac{kQ}{3r_1} = \varphi' = \frac{kq_2''}{r_2}.$$

Solving, we get a new connection between the charges of the balls:

$$\frac{kq_1''}{r_1} - \frac{kq_1''}{3r_1} = \frac{kq_2''}{r_2} \quad \Rightarrow \quad q_2'' = \frac{2}{3} \cdot q_1'' \cdot \frac{r_2}{r_1}.$$

Since the amount of charges has not changed (likewise (*)), we get the final value of the charge of the first ball:

$$q_1 + q_2 = q_1'' + q_2'' = q_1'' \cdot \left(1 + \frac{2}{3} \cdot \frac{r_2}{r_1}\right) \quad \Rightarrow \quad q_1'' = \frac{q_1 + q_2}{1 + \frac{2}{3} \cdot \frac{r_2}{r_1}}.$$

The change in the charge of the first ball, which is equal to the charge that will pass through the connecting wire:

$$\Delta q = |q_1' - q_1''| = \left|\frac{q_1 + q_2}{r_1 + r_2} \cdot r_1 - \frac{q_1 + q_2}{3r_1 + 2r_2} \cdot 3r_1\right| = r_1(q_1 + q_2)\left|\frac{1}{r_1 + r_2} - \frac{3}{3r_1 + 2r_2}\right| =$$

$$= \frac{r_1 r_2 (q_1 + q_2)}{(r_1 + r_2)(3r_1 + 2r_2)} = \frac{0.1 \cdot 0.2 \cdot 2.1 \cdot 10^{-6}}{0.3 \cdot 0.7} = 0.2\ \mu\text{C}.$$

**11.6.** **(3 points)** Having obtained a barrel of potassium dicyanoaurate on a captured ship, Jack the Sparrow decided to make several hundred fake gold coins from copper coins of some ancient forgotten country that had long been lying in an abandoned chest. He carefully measured the coins — they all had a diameter of 22 mm and the thickness of 1.5 mm. As a current source for electroplating coins with gold Jack took 11 giant electric stingrays, who could give the current of as much as 50 mA for 2 hours each.


**[6]** How many coins did Jack manage to gild if the layer of gold applied by him was 4.5 $\mu$m thick? Give the answer as an integer.

**Remark.** The electrochemical equivalent of gold in this process is 2.04 mg/C, the density of gold makes 19.3 g/cm$^3$. (*Folklore*)

**Answer:** 108.

**Solution.** The total mass of the plating is equal to the multiplication product of the electrochemical equivalent of gold and the value of the transferred electric charge $Q$:

$$m = kQ.$$

The value of the transferred electric charge is equal to the multiplication product of the number of stingrays (current sources) by the current strength and the operating time of the each source:

$$Q = nIt.$$

Otherwise, the mass can be expressed as the multiplication product of the number of coins and the density of gold and the volume of the layer applied to each coin:

$$m = N\rho\Delta V.$$

The volume of the layer can be written in the following way:

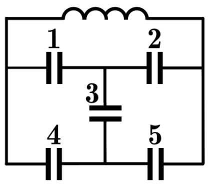
$$\Delta V = \frac{\pi(d + 2x)^2}{4} \cdot (h + 2x) - \frac{\pi d^2}{4} \cdot h \approx \pi dhx + \frac{\pi d^2 x}{2} = \frac{\pi dx}{2} \cdot (d + 2h),$$

where $\Delta V$ is the volume of the gold layer, $d$ is the diameter of the coin, $h$ is the thickness of the coin, $x$ is the thickness of the golden layer. Then the number of coins makes

$$N = \frac{2nkIt}{\rho\pi dx(d+2h)} = 108.$$

**Criteria.** The wrong answers «107» and «109», obtained because of incorrect roundings, cost 1 point.

**11.7. (4 points)** A coil with a number of turns $N = 1000$ and the diameter $d = 1$ cm is placed in a uniform magnetic field parallel to its axis. The field induction increases uniformly with the speed $B_0 = 0.01$ T/sec. The ends of the coil are closed to a battery of five identical capacitors, with a total capacity of $C = 1$ $\mu$F.




**[7]** Find out the charge $Q_5$ on the fifth capacitor. Give your answer in nC with accuracy of hundredths. (*V. Kuzmichev, S. Starovoytov*)

**Answer:** 0.39.

**Solution.** From the law of electro-magnetic induction:

$$|\varepsilon_i| = \frac{\Delta B}{\Delta t} \cdot SN = B_0 \cdot \frac{\pi d^2}{4} \cdot N,$$

where $S$ is the cross-sectional area of the coil.

From the symmetry of the capacitor system, it is clear that the third capacitor is not charged. Then the total capacity:

$$C = \frac{C_0}{2} + \frac{C_0}{2} = C_0,$$

where $C_0$ is capacitance of one item.

The voltages in each of the parallel branches with two capacitors are the same and equal to the EMF value. When two identical capacitors are connected in series, the voltage is divided equally between them. Therefore, the charge on the fifth capacitor :

$$Q_5 = C_0U_5 = C \cdot \frac{\varepsilon}{2} = \frac{CB_0\pi d^2 N}{8} = 0{,}39 \text{ nC}.$$

**11.8. (3 points)** $\alpha$-particle flying at a speed of $V_x = 2 \cdot 10^5$ m/sec along the axis $OX$, enters the region of space where there are uniform electric and magnetic fields, and both the electric field strength and the magnetic field induction are directed along the axis $OY$. $E = 1000$ V/m, $B = 0.5$ T.

**[8]** In how many times will the particle's velocity change when it makes $N = 20$ full turns around the $OY$ axis? Give your answer as a number with an accuracy of tenths. (*S. Starovoytov*)

**Answer:** 1.6.

**Solution.** The particle will move along a helical line with increasing pitch, the helix axis — along $OY$. The value of the velocity projection onto the plane perpendicular to $OY$ is preserved and equal $V_x$.

The movement along the $y$ axis is uniformly accelerated as the electric field strength is constant:

$$V_y = a_y t = \frac{qE}{m}t.$$

The total time of movement is equal to $N$ periods of rotation in a constant magnetic field:

$$t = NT = N\frac{2\pi m}{qB} \quad \Rightarrow \quad V_y = \frac{2\pi NE}{B}.$$

Introduce the speed increase factor $n$:

$$V = nV_x = \sqrt{V_x^2 + V_y^2} \quad \Rightarrow \quad n = \sqrt{\frac{(2\pi NE)^2}{(V_xB)^2} + 1} = 1.6.$$

**Criteria.** The wrong answer «1.7», obtained because of incorrect roundings, costs 1 point.

**11.9. (3 points)** A flat thin object was placed parallel to the plane of a thin divergent lens with a focal length of 10 cm. The lens was replaced with a converging lens with the same focal length.

**[9]** Where should the object be moved along the main optical axis so that the size of its image does not change? Give only one **symbol**: either «+» if the object should be moved to the lens, or «−» if otherwise.

**[10]** What distance should the object be moved along the main optical axis so that the size of its image does not change? Give your answer in centimeters with an accuracy of integers.

(*S. Starovoytov*)

**Answers: [9]** −. **[10]** 20.

**Solution.** Let's write down the thin lens equation For both cases.

In the first case, the image is an imaginary reduced direct image:

$$-\frac{1}{F} = \frac{1}{d_1} + \frac{1}{f_1}.$$

Transform by using the letter Γ to denote the magnification of the lens:

$$d_1 = \left(\frac{1-\Gamma}{\Gamma}\right) \cdot F.$$

In the second case, the image is a real reduced inverted image:

$$\frac{1}{F} = \frac{1}{d_2} + \frac{1}{f_2} \quad \Rightarrow \quad d_2 = \left(\frac{\Gamma+1}{\Gamma}\right) \cdot F.$$

The magnifications in both cases are the same, but with different signs. Therefore

$$\Delta d = d_2 - d_1 = 2F = 20 \text{ cm},$$

and the object must be moved away from the lens.

**Criteria.** Question **[9]** costs 1 point. Question **[10]** costs 2 points.