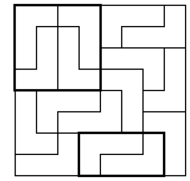
Problems for grades R5 – R6

1. A square 8×8 divided into 16 L-tetrominoes (L-shaped 4-square figures) is shown on the picture. Several tetrominoes make smaller rectangles (two such rectangles are highlighted). Is it possible to divide an 8×8 square into 16 L-tetrominoes without such small rectangles? (*A. Tesler*)

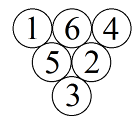


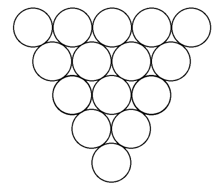
2. Peter writes a poem. First day he wrote first few lines of the poem, and each next day he writes one more line than during the previous day (for example, if the poem had 3 lines at the end of the first day, then it has 7 lines after the second day, 12 lines after the third day, and so on).

   a) Is it possible for the total number of lines to end with 4 after a few days (more than one)?

   b) Is it possible for the total number of lines to end with 4 after a few days (more than one), and with digit 7 a few days after that? (*I. Tumanova*)

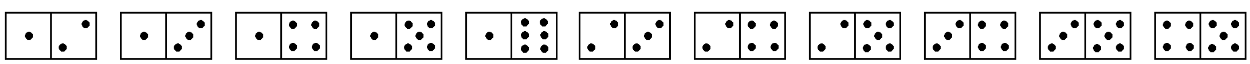
3. Six numbers from 1 to 6 are arranged such that each number is equal to the difference of the two numbers above it (see the left picture). Such an arrangement is called *nice*. Write numbers from 1 to 15 into the circles on the right picture (each number one time) to form a *nice* arrangement. (*A. R. Arab*)





4. Felix and Sue play a game. They have a chocolate bar of 2019×2020 square pieces. Every turn a player breaks off a rectangular piece and eats it (after that, the bar stays rectangular but has less squares). Felix starts the game, and then they move one after another. A player who makes the bar to have perimeter 10 is the winner. Who can win regardless of the opponent’s game? How should he or she act? (*O. Pyaive*)

5. There is a set of domino tiles shown on the picture.



   a) Is it possible to join them all into one chain according to the domino rules?

   b) Is it possible to remove one tile so that all other tiles cannot form a chain? (*A. Tesler*)

6. Four types of people live on an island: knights (cannot say false statements), liars (cannot say true statements), ordinary people (can say anything) and shy people (they don’t say statements at all). Once several islanders come together, and each of them said one of these phrases: “Who are you?”, “I am a knight”, “I am a liar”, “I am ordinary”, “I am shy”. Each phrase was pronounced exactly 10 times. Is it possible that the biggest group there were knights? (*A. Tesler*)

7. ***Only for grade R5.*** There are three barrels. The first one is full while the other two are empty. At 12:00, water starts to flow from the first barrel to the other two: every minute 2 liters flow into the second barrel and 4 liters flow into the third one. At 13:00, the volumes of water in the first barrel and in the second one are equal. When the first barrel will become empty? (*A. Tesler*)

   Only for grade R6. There are 3 types of tea in the shop: green, black and fruit. At first, the proportion of the number of packs of these types was $4:5:8$. After some tea had been sold and a new portion had arrived, the proportion became $5:7:12$. It is known that the number of fruit tea packs increased in 60%, and the amount of green tea increased not more than in 20 packs. How many packs were in the shop at the beginning? (*L. Koreshkova*)

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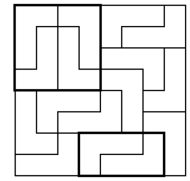
International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2019/2020. Qualifying round

Problems for grade R7

1. A square 8×8 divided into 16 L-tetrominoes (L-shaped 4-square figures) is shown on the picture. Several tetrominoes make smaller rectangles (two such rectangles are highlighted). Is it possible to divide an 8×8 square into 16 L-tetrominoes without such small rectangles? (*A. Tesler*)

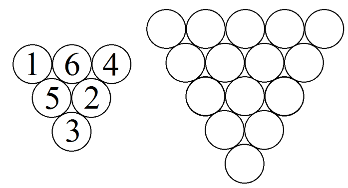


2. Peter writes a poem. First day he wrote first few lines of the poem, and each next day he writes one more line than during the previous day (for example, if the poem had 3 lines at the end of the first day, then it has 7 lines after the second day, 12 lines after the third day, and so on).

   a) Is it possible for the total number of lines to end with 4 after a few days (more than one)?

   b) Is it possible for the total number of lines to end with 4 after a few days (more than one), and with digit 7 a few days after that? (*I. Tumanova*)

3. Six numbers from 1 to 6 are arranged such that each number is equal to the difference of the two numbers above it (see the left picture). Such an arrangement is called *nice*. Write numbers from 1 to 15 into the circles on the right picture (each number one time) to form a *nice* arrangement. (*A. R. Arab*)



4. There are 3 types of tea in the shop: green, black and fruit. At first, the proportion of the number of packs of these types was $4 : 5 : 8$. After some tea had been sold and a new portion had arrived, the proportion became $5 : 7 : 12$. It is known that the number of fruit tea packs increased in 60%, and the amount of green tea increased not more than in 20 packs. How many packs were in the shop at the beginning? (*L. Koreshkova*)

5. There are two tanks of 2020 m^3 each. At midnight, the first one contains 100 m^3 of water, and the second one is full. Every hour, 110 m^3 of water comes into the first tank (until it is full), and 50 m^3 of water are pumped from the second tank (until it is empty). At which moments is the difference between volumes of water in the tanks equal to the half of the initial difference? (*I. Ibatulin*)

6. Harry Potter has a $10 \times 10 \times 10$ cm box. When he casts a spell, one of the dimensions of the box (length, width, or height) increases by 50%, and each of two other dimensions decreases by 20%. Is it possible that, after several spells, Harry's box is of size $20 \times 20 \times 20$ cm? (*A. Tesler*)

7. Four types of people live on an island: knights (cannot say false statements), liars (cannot say true statements), ordinary people (can say anything) and shy people (they don't say statements at all). Once several islanders come together, and each of them said one of these phrases: "Who are you?", "I am a knight", "I am a liar", "I am ordinary", "I am shy". Each phrase was pronounced exactly 6 times. Is it known that all types were represented by different and nonzero number of people. Knights were the most numerous type of the people. How many knights were there? (Find all possible answers to this question and prove that other answers are impossible.) (*A. Tesler*)

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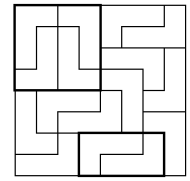
Problems for grade R8



1. A square 8×8 divided into 16 L-tetrominoes (L-shaped 4-square figures) is shown on the picture. Several tetrominoes make smaller rectangles (two such rectangles are highlighted). Is it possible to divide an 8×8 square into 16 L-tetrominoes without such small rectangles? (*A. Tesler*)

2. There are 3 types of tea in the shop: green, black and fruit. At first, the proportion of the number of packs of these types was $4 : 5 : 8$. After some tea had been sold and a new portion had arrived, the proportion became $5 : 7 : 12$. It is known that the number of fruit tea packs increased in 60%, and the amount of green tea increased not more than in 20 packs. How many packs were in the shop at the beginning? (*L. Koreshkova*)

3. Two hackers wrote two programs to analyse changes of numbers after some operations.
The first program in one iteration multiplies a positive integer by 3, subtracts the result’s sum of digits from the result; and after that the program repeats 7 more iterations with the new result. The output of the first hacker’s program is the ratio of the final number to the initial one.
The program of the second hacker starts from a number which consists only of digits 9; in one iteration, the program divides the number by its sum of digits (if it is divisible), or subtracts this sum of digits (otherwise); and then the program performs another 7 iterations. The output of the second hacker’s program is the ratio of the initial number to the final one.
The hackers decided to play a game: each of them chooses an initial number for himself; the hacker with bigger output is the winner. Who can win regardless of the opponent’s choice? (*I. Ibatulin*)

4. There are two tanks of 2020 m^3 each. At midnight, the first one contains 100 m^3 of water, and the second one is full. Every hour, 110 m^3 of water comes into the first tank (until it is full), and 50 m^3 of water are pumped from the second tank (until it is empty). At which moments is the difference between volumes of water in the tanks equal to the half of the initial difference? (*I. Ibatulin*)

5. ABC and CDE are two isosceles right triangles with the hypotenuse lengths $BC = 7$ and $CE = 14$. C lies on the segment BE, and points A and D are at the same side of line BE. O is the intersection point of segments AE and BD. Find the area of the triangle ODE. (*A. R. Arab*)

6. Four types of people live on an island: knights (cannot say false statements), liars (cannot say true statements), ordinary people (can say anything) and shy people (they don’t say statements at all). Once several islanders come together, and each of them said one of these phrases: “Who are you?”, “I am a knight”, “I am a liar”, “I am ordinary”, “I am shy”. Each phrase was pronounced exactly 6 times. Is it known that all types were represented by different and nonzero number of people. Knights were the most numerous type of the people. How many knights were there? (Find all possible answers to this question and prove that other answers are impossible.) (*A. Tesler*)

7. 12 coins of 1 cm radius lie on a square table with side 1 m without overlapping. Prove that it is always possible to choose 4 different coins with centers A, B, C, D such that $1 \le CD : AB < 1.1$ or $1 \le AC : AB < 1.1$. (*A. Tesler*)

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International Mathematical Olympiad
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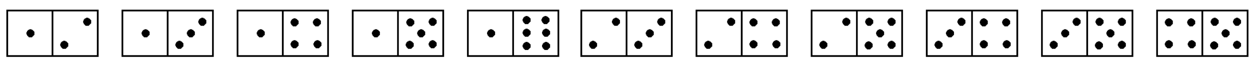
Problems for grade R9

1. There are 3 types of tea in the shop: green, black and fruit. At first, the proportion of the number of packs of these types was $4 : 5 : 8$. After some tea had been sold and a new portion had arrived, the proportion became $5 : 7 : 12$. It is known that the number of fruit tea packs increased in 60%, and the amount of green tea increased not more than in 20 packs. How many packs were in the shop at the beginning? (*L. Koreshkova*)

2. There is a set of domino tiles shown on the picture.



   a) Is it possible to join them all into one chain according to the domino rules?

   b) Is it possible to remove one tile so that all other tiles cannot form a chain? (*A. Tesler*)

3. There are two tanks of 2020 m^3 each. At midnight, the first one contains 100 m^3 of water, and the second one is full. Every hour, 110 m^3 of water comes into the first tank (until it is full), and 50 m^3 of water are pumped from the second tank (until it is empty). At which moments is the difference between volumes of water in the tanks equal to the half of the initial difference? (*I. Ibatulin*)

4. A triangle is inscribed into a circle of diameter 5. Find all possible values of the perimeter of the triangle if each its side has integer length (and prove that other values are impossible). (*P. Mulenko*)

5. Do there exist four different numbers a, b, x, y such that the representation of x in the base-a numeral system is just the same as the representation of y in the base-b numeral system, and vice versa: the representation of x in the base-b system is just the same as the representation of y in the base-a system? (*V. Fedotov*)

6. 12 coins of 1 cm radius lie on a square table with side 1 m without overlapping. Prove that it is always possible to choose 4 different coins with centers A, B, C, D such that $1 \le CD : AB < 1.1$ or $1 \le AC : AB < 1.1$. (*A. Tesler*)

7. Let a and b be two real numbers such that $2a^3 + 2b^3 + 3a^2b + 3ab^2 + 60ab = 16000$. Find all possible values of $a + b$. (*A. R. Arab*)

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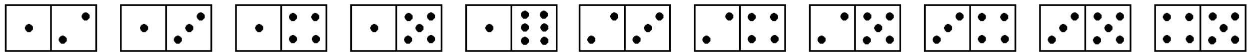
Problems for grades R10 – R11

1. There is a set of domino tiles shown on the picture.



   a) Is it possible to join them all into one chain according to the domino rules?

   b) Is it possible to remove one tile so that all other tiles cannot form a chain? (*A. Tesler*)

2. In $\triangle ABC$, $AB = 6, BC = 4, AC = 8$. A point M is chosen on the side AC so that the incircles of the triangles ABM and BCM have a common point. Find the ratio of the areas of these triangles. (*L. Koreshkova*)

3. Four types of people live on an island: knights (cannot say false statements), liars (cannot say true statements), ordinary people (can say anything) and shy people (they don't say statements at all). Once several islanders come together, and each of them said one of these phrases: "Who are you?", "I am a knight", "I am a liar", "I am ordinary", "I am shy". Each phrase was pronounced exactly 6 times. Is it known that all types were represented by different and nonzero number of people. Knights were the most numerous type of the people. How many knights were there? (Find all possible answers to this question and prove that other answers are impossible.) (*A. Tesler*)

4. The surface of a wooden 1 m^3 cube is painted. From each vertex, a pyramid is cut off; as a result, a polyhedron with 14 faces is obtained. Each painted face is a rectangle, and each unpainted face is an equilateral triangle (the triangles are not necessarily equal). Find the total area of the painted faces of the polyhedron if it is $\sqrt{3}$ times less than the total area of the unpainted ones. (*A. Tesler*)

5. By a command k, robots Niner and Zerro write down all integers from 1 to $37k$. Then Niner chooses a number with maximal quantity of digits 9, and Zerro chooses a number with maximal quantity of digits 0. If one of the robots has more corresponding digits than the other one, he earns a point. Find the final score after a match consisting of commands $k\ldots$
   (a) for k from 1 to 2019; (b) for k from 1 to 10^{2019}. (*V. Fedotov*)

6. Is it possible to write seven consecutive positive integers (in some order) instead of the underscores so that the equality $(x - \_)(x - \_)(x - \_) = (x - \_)(x - \_)(x - \_) + \_$ would hold for all x? (*A. Tesler*)

7. There are three tanks. Water flows out of the first tank and flows into other two with some constant speeds. Initially, the amount of water in the first tank equals to the total amount in two other tanks; after few hours, the amount of water in the second tank equals to the total amount in two other tanks; and after few more hours, the amount of water in the third tank equals to the total amount in two other tanks. Is it possible that all the tanks are non-empty during all the time? (*A. Tesler*)

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