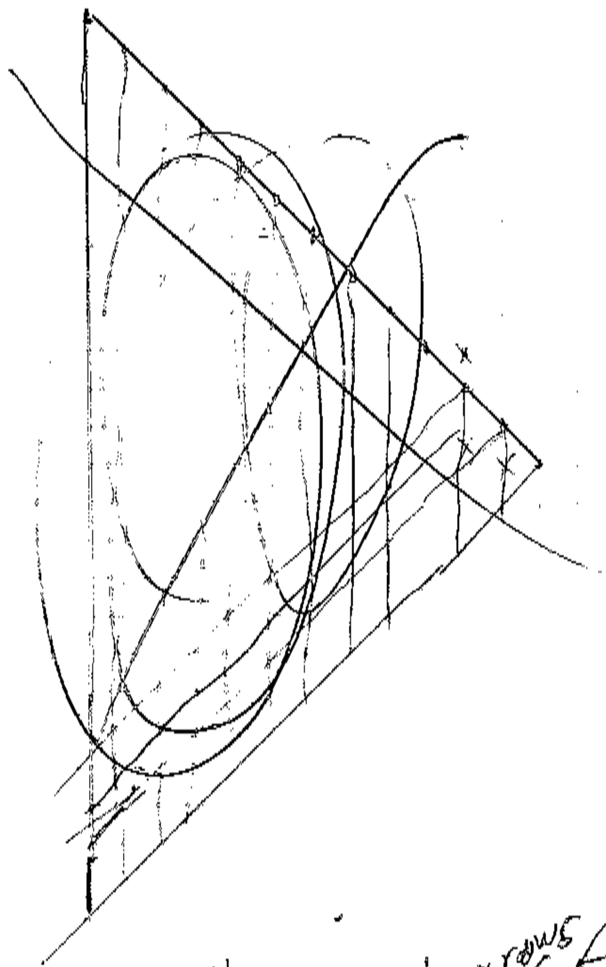
Question 1

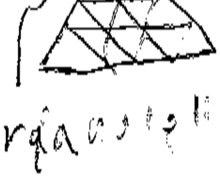
(1) Lets look only for the number of triangles like this ▷

(from symetrey its the same as the number of ◁ triangles)

We know that all of this type of triangles are in the big ▷

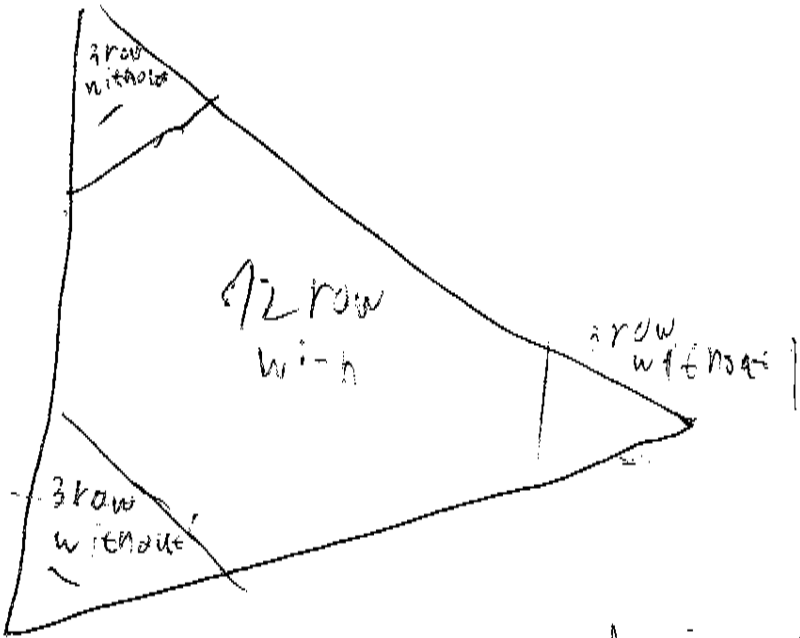


We will mark



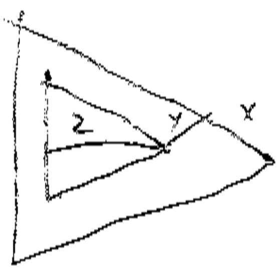
as an "x row triangle"

(2) we can see that each ▷ triangle is in the big 12 row triangle but not in the 3 little 3 row triangles



now all we got to do is calculate the number of ▷ in a ~~x~~ row ▷ triangle.

lets say there are n rows each triangle we can ~~show~~ represent as



such as x, y, z are integers

$x, y \geq 0 \quad z \geq 1$ and $x+y+z \leq n$

the number of options for that are $\binom{n+2}{3}$

and that is beacuse we can declare intgers
u, v such that u is $z-1$ an $v = n-x-y-2$

and will get that its the same as sharing
(3) $n-1$ objects bettwen 4 piles

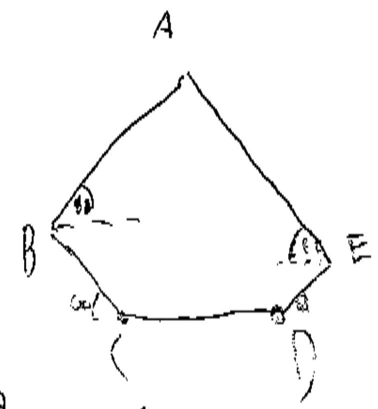
so the number of Δ is

$$\binom{14}{3} - \cancel{4}\, 3 \cdot \binom{5}{3} = \frac{14 \cdot 13 \cdot 12}{1 \cdot 2 \cdot 3} \cancel{4} - 30 = 334$$

so we get a total of $\boxed{667}$
triangles

Question 2

(4) lets mark $C=(0,0)$ $D=(4,0)$



$\mathrm{Dif}\,x\,\overrightarrow{CB} = -\frac{1}{2}BC \quad \mathrm{Dif}\,x\,\overrightarrow{BA} = \frac{1}{2}AB$

$\mathrm{Dif}\,x\,\overrightarrow{AE} = \frac{1}{2}AE \quad \mathrm{Dif}\,\overrightarrow{ED} = -\frac{1}{2}DE$

$AB + AE - BC - DE = 2(\mathrm{Dif}\,x\,\overrightarrow{CB} + \cdots \mathrm{Dif}\,x\,\overrightarrow{ED}) =$

$2\,\mathrm{Dif}\,x\,\overrightarrow{CD} = 2CD$

$BC + DE = 5$

$\mathrm{Dif}\,y\,\overrightarrow{CB} = \frac{\sqrt{3}}{2}BC \quad \mathrm{Dif}\,y\,\overrightarrow{BA} = \frac{\sqrt{3}}{2}AB$

$\mathrm{Dif}\,y\,\overrightarrow{AE} = -\frac{\sqrt{3}}{2}AE \quad \mathrm{Dif}\,y\,\overrightarrow{ED} = -\frac{\sqrt{3}}{2}DE$

$AB - AE + BC - DE = \frac{2}{\sqrt{3}}(\mathrm{Dif}\,y\,\overrightarrow{CB} + \cdots + \mathrm{Dif}\,y\,\overrightarrow{ED}) =$

$\frac{2}{\sqrt{3}}\,\mathrm{Dif}\,y\,CD = 0$

$BC = DE + 1$

$DE = 2$

$BC = 3$

Dity $\overrightarrow{CA} = \frac{\sqrt{3}}{2}\left(BC + AB\right) = \boxed{\frac{9\sqrt{3}}{2}}$

(5)

Question 3

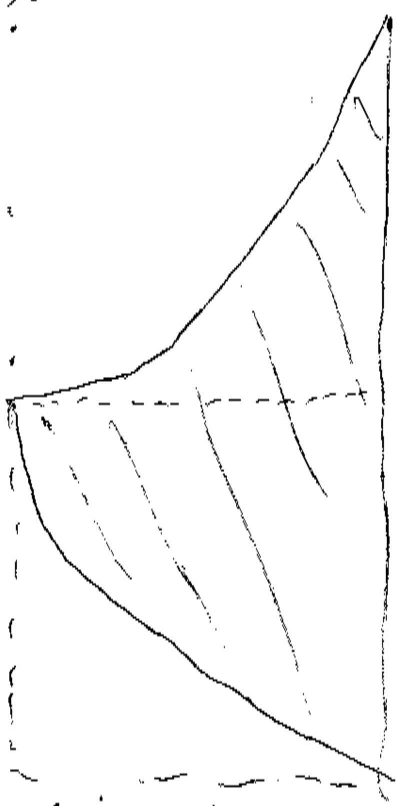
(6) for $\sqrt{x}$ and $\sqrt{1-x}$ to be defiened

x is beetwen 0 and 1

for the inequality to be true

y is beetwen $-\sqrt{x}$ and x^2

so the map will look like



the upper part formula is $x^2 = y$

if we rotate it 90° clock wise we will get

$(y-1)^2 = x \rightarrow y - 1 = \sqrt{x}$ since y is beetwen 0 and 1

then if we ~~retate~~ will move it down

we will $y = -\sqrt{x}$ so the two parts will connect and form a square of area

$\boxed{1}$

Question 4

(8) We know thet the children gave exactly all of the numbers beetwen 0 and $N-1$ so at average they gave $\frac{N-1}{2}$ gift so they all got $\frac{N-1}{2}$ gifts

If N is even then its not possible

for every odd N I will show how it is possible

we will mark children from 1 to N

child i will give:

1) if $i<\frac{N}{2}$ to every child j such as $i<j$

3) if $i>\frac{N}{2}$ to every child j such as $j\leq N-i$

every child i will get $\frac{N-1}{2}$.

1) if $i<\frac{N}{2}$ he will get from $i-1$ children before him and $\frac{N+1}{2}-i$ from the children between $\frac{N+1}{2}$ to $N-i$

2) if $i>\frac{N}{2}$ he will get from every children beetween 1 to $\frac{N-1}{2}$

and beacuse every one gives $N-i$ gifts it is okay and $\boxed{N \text{ is ODD}}$

Questionr lets find all the pairs (m,n)

(8) ~~n can not be bigger~~

n can not be lower then 6 or m will be negative

m can not be lower then n or

$$m^3 \geq n^3+3n^2 \overset{n>5}{>} n^3+13n > n^3+13n-273$$

lets check $m=n$

we get $13n=273$

$n=21$

$(21,21)$

$1+m=n$

$$m^3 = m^3+3m^2+3m+1+13m+13-273 = m^3+3m$$

$$3m^2+16m-259=0$$

beacuse its monotone and $m=7$ works then $(7,8)$ works

$m+2=n$

$$m^3 = m^3+6m^2+12m+8+13m+26-273$$

$$6m^2+25m-239=0$$

$m=4$ is to little $m=5$ is to much

$m+3=n$

(9) $$9m^2+40m-207$$
$$3<m<4$$

$m+4=n$
$$12m^2+61m-157$$
$$1<m<2$$

if $n \geq m+5$

$$n^3+13n-2\cancel{33} \geq m^3+15m^2+75m+125+13m+65-233=$$
$$m^3+15m^2+87m-83 \overset{m\geq 1}{>} m^3$$

then this are all the solutions

then the sum is $\boxed{29}$