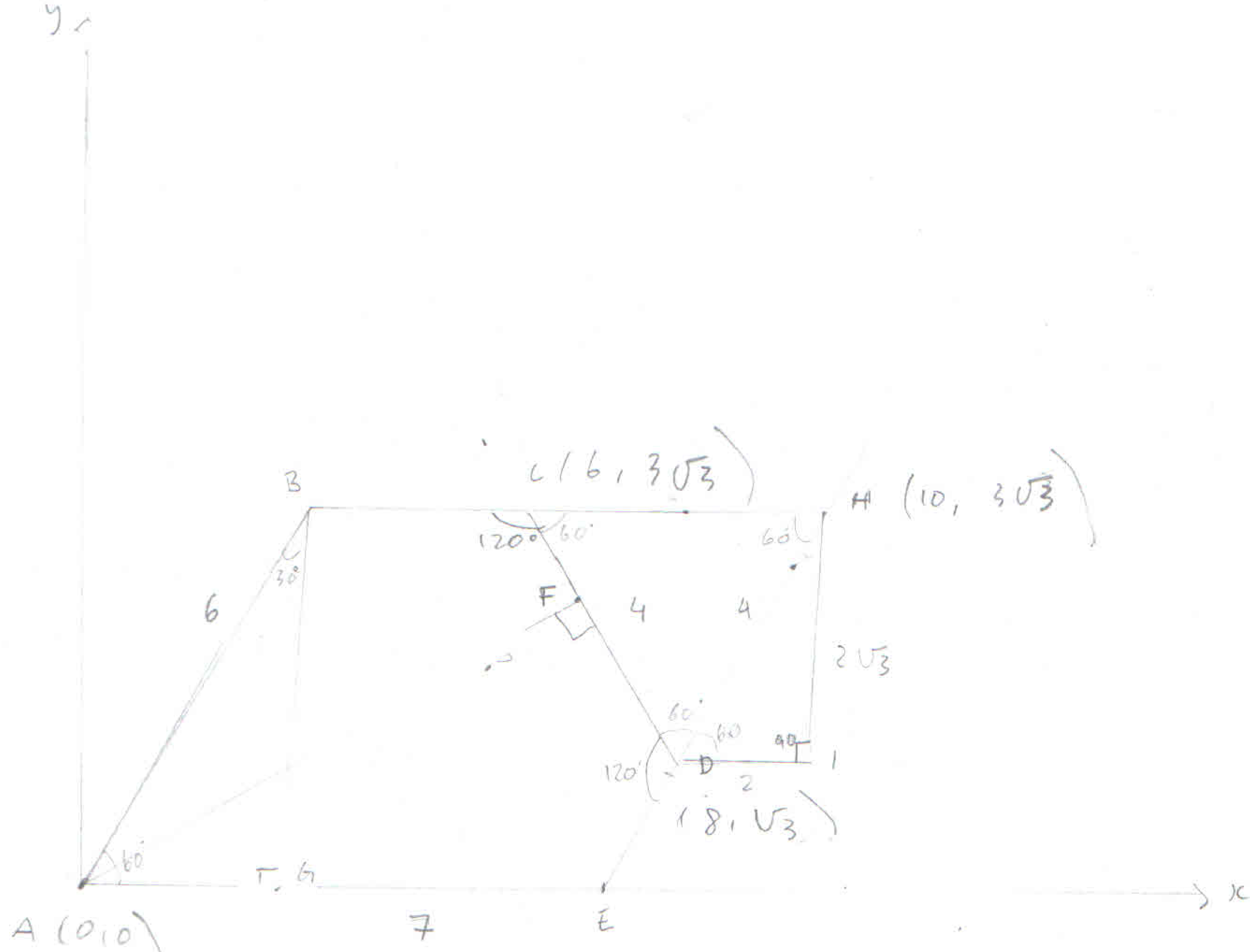
1) Note that all triangles will be isoceles

Triangles with 2 sides of length: 12 → 2 triangles
11 → 6 triangles
10 → 12 triangles
9 → 20 triangles
8 → 30 triangles
7 → 42 triangles
6 → 56 triangles
5 → 72 triangles
4 → 90 triangles
3 → 110 triangles
2 → 102 triangles
1 → 120 triangles

total: 22
\+ 6
12
20
30
42
56
72
3 90
110
102
120

Answer: 664 triangles

2) All the remaining angles will have an angle of $120°$ since $\frac{540-60}{4} = \frac{480}{4} = 120°$



$m\overline{BG}$:

$\sin 60 = \frac{m\overline{BG}}{6}$

$mBG = \sin 60 \cdot 6$

$= \frac{\sqrt{3}}{2} \cdot 6$

$= \frac{6\sqrt{3}}{2} = 3\sqrt{3}$

$m\overline{AG}$:

$\cos 60 = \frac{m\overline{AG}}{6}$

$m\overline{AG} = \cos 60 \cdot 6$

$= \frac{1}{2} \cdot 6$

$= 3$

This gives us the coordinates of H $(10, 3\sqrt{3})$

Note that $\triangle CDH$ is equilateral

By similar triangles $\triangle ABG$ and $\triangle DHI$,

we have $\frac{6}{4} = \frac{m\overline{AG}}{m\overline{DI}} = \frac{m\overline{BG}}{m\overline{HI}}$ which gives us the coordinates of D

2)(continuation)

A similar process gives us the coordinates of $C\left(6, \frac{6\sqrt{3}}{2}\right)$

$\left(6, 3\sqrt{3}\right)$

The equation of $\overline{CD}$:

$$m = \frac{\Delta y}{\Delta x} = \frac{\sqrt{3} - 3\sqrt{3}}{8-6} = \frac{-2\sqrt{3}}{2} = -\sqrt{3}$$

$y = mx + b$

$\sqrt{3} = -\sqrt{3} \cdot 8 + b$

$b = 9\sqrt{3}$

$y = -\sqrt{3}x + 9\sqrt{3}$

equation of $\overline{AF}$

$y = mx + b$

$y = \frac{1}{\sqrt{3}}x$ since $\overline{AF} \perp \overline{CD}$

coordinates of F

$$\begin{cases} y = -\sqrt{3}x + 9\sqrt{3} \\ y = \frac{1}{\sqrt{3}}x \end{cases}$$

$\frac{x}{\sqrt{3}} = -\sqrt{3}x + 9\sqrt{3}$

$\cancel{\frac{1+\sqrt{3}}{\sqrt{3}}x = 9\sqrt{3}}$

$\cancel{(1+\sqrt{3})x = 27}$

$x = \sqrt{3}\left(-\sqrt{3}x + 9\sqrt{3}\right)$

$x = -3x + 27$

$4x = 27$

$$x = \frac{27}{4}$$

$$y = -\sqrt{3}\left(\frac{27}{4}\right) + 9\sqrt{3}$$

$$= \frac{-27\sqrt{3}}{4} + \frac{36\sqrt{3}}{4}$$

$$= \frac{9\sqrt{3}}{4}$$

$$m\overline{AF} = \sqrt{x^2+y^2}$$

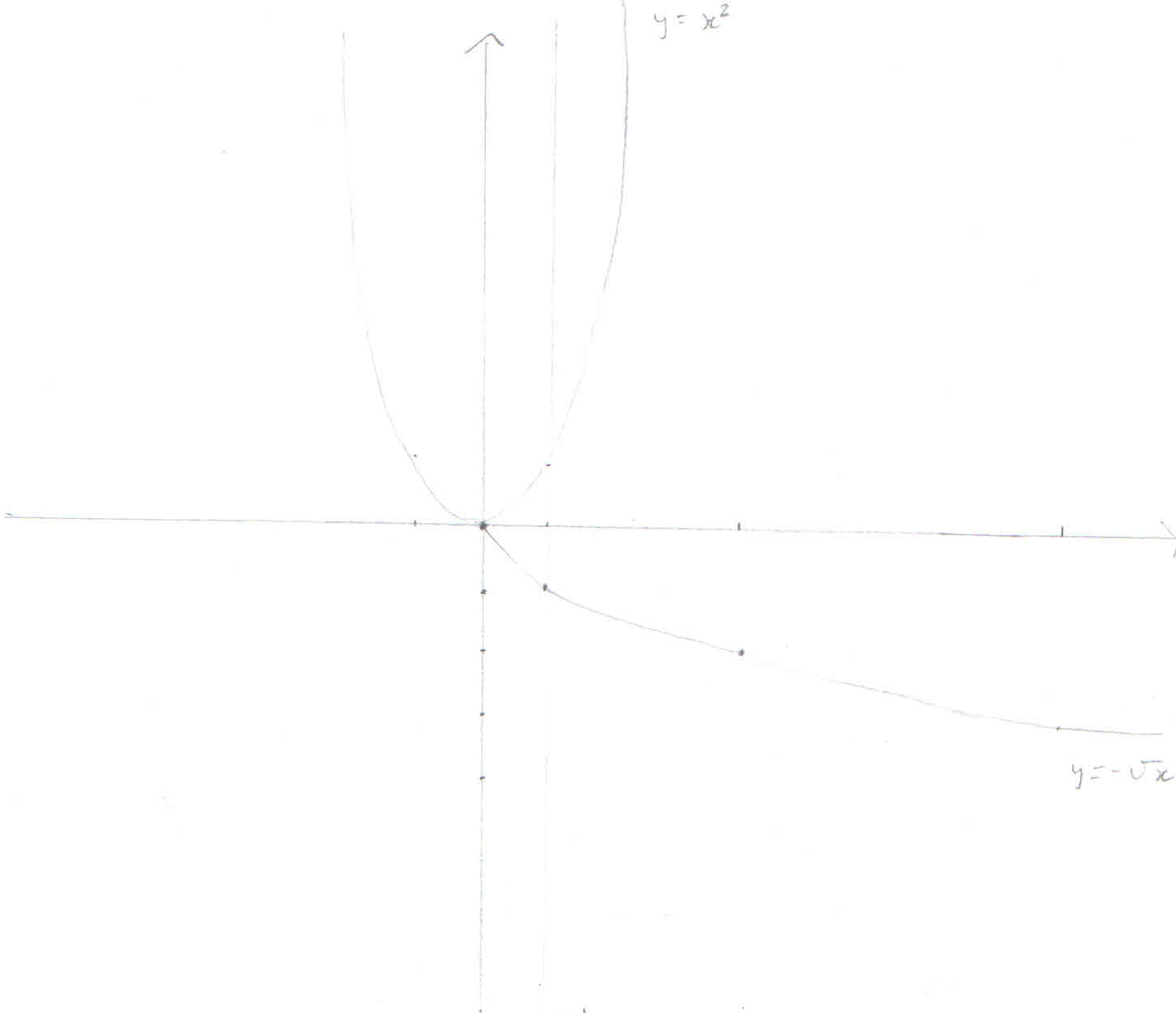
$$= \sqrt{\left(\frac{27}{4}\right)^2 + \left(\frac{9\sqrt{3}}{4}\right)^2}$$

$$= \sqrt{\frac{729+243}{16}}$$

$$= \sqrt{\frac{972}{16}} =$$

Answer: $\sqrt{62}$

3) 1)



$(y + \sqrt{x})(y - x^2)\sqrt{1-x} \leq 0$

if we look at $(y+\sqrt{x})(y-x^2)\sqrt{1-x} = 0$

$y = -\sqrt{x}$ or $y = x^2$ or $\sqrt{1-x} = 0$

$1 - x = 0$

$x = 1$

It seems that there will be an infinity of points satisfying the inequality since for instance, if we take $x = 1$, y may have any $\mathbb{R}$ value and the inequality will hold true.

4) Note that if there are N kids, there will be a kid that gives $N-1$ gifts, one that gives $N-2$ gifts, ..., 1 gift and 0 gift.

As such the total amount of gift is defined by $\frac{(N-1)N}{2}$

The goal will be that $\frac{(N-1)N}{2}$ is a multiple of N, so that each kid will an equal number of gifts.

$$\frac{(N-1)N}{2} = kN \quad \text{where } k \text{ is some natural number}$$

$$N-1 = 2k$$

$$N = 2k-1$$

$2k-1$ is the form of any positive odd number.

As such, any positive odd number will satisfy the problem (except 1)

Answer: Any positive odd number except 1.

5) Lets factorize $n^3+13n-273=0$

By the natural root theorem,

the roots $\in \{-1, 3, 7, 13, 21, 91, 273\}$

if the roots $\in \mathbb{Z}$

One cubo is if $n=21$ and $m=21$

It is interesting to note that 21 is a divisor of 273

$$m^3 = n^3 + 13n - 273$$

$$m^3 - n^3 - 13n + 273 = 0$$

$$(m-n)(m^2+n^2+nm) - 13(n+21) = 0$$

$$(m-n)((m+n)^2 - m + n) = 13(n+21)$$

Answer: 21