International mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
2018/2019 year, final round

Participant form

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7. Do you attend a mathematical circle (extracurricular group)? If yes, please indicate the organization and your teacher YES, THE SCHOOL MATH CIRCLE, TEACHER – TELEUCA MARCEL

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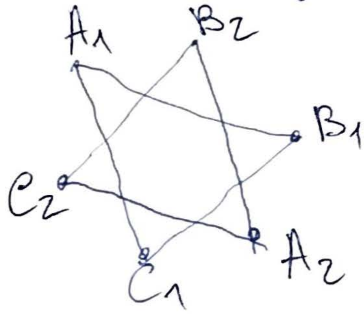
Problem 1

We ~~esepect~~ expect 2 kinds of triangles:

first type 

second type 

So 

$\triangle A_1B_1C_1$ is first type

$\triangle A_2B_2C_2$ is second type

The number of triangles of first type in $\triangle A_1B_1C_1$ will be equal with the number of triangles of second type in $\triangle A_2B_2C_2$

Let a - The ~~num~~ number of triangles of ~~$\triangle A_1B$~~ first type

b - The number of triangles of second type

$a=b$ and $a+b=$ all triangles

Let the segment ⊢⊣ be one (1)

By analogy ⊢⊢⊣ will be 2

The side can have maximum lengh 12 and the minimum

We will find how many triangles of first type are in $\triangle A_1B_1C_1$

K_{12} With side 12 $- 1$

K_{11} With side 11 $- \sum_{i=1}^{2} i$

K_{10} $10 - \sum_{i=1}^{3} i$

K_9 $9 - \sum_{i=1}^{4} i$

K_8 $8 - \sum_{i=1}^{5} i$

K_7 $7 - \sum_{i=1}^{6} i$

K_6 $6 - \sum_{i=1}^{7} i$

K_5 $5 - \sum_{i=1}^{8} i$

K_4 $4 - \sum_{i=1}^{9} i$

K_3 $3 - \sum_{i=1}^{9} i - 1 + 8$

K_2 $2 - \sum_{i=1}^{9} i - 2 - 1 + 8 + 7$

K_1 $1 - 60$

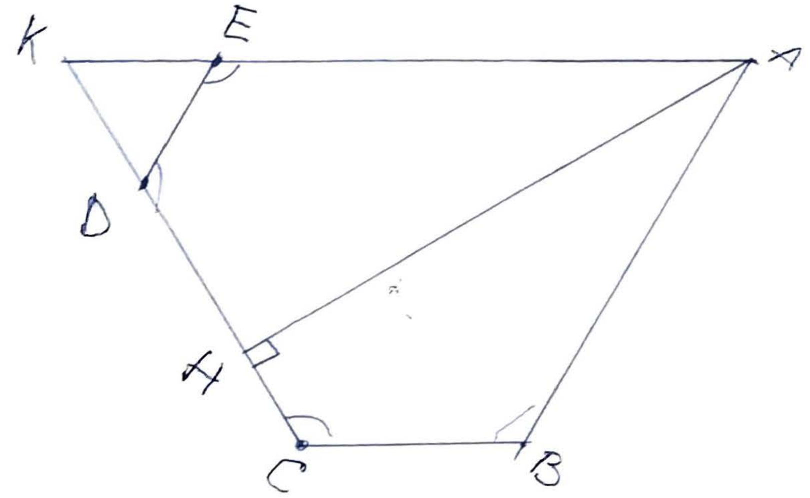
$$a = \sum_{i=1}^{12} K_i = 334$$

$$a = b = 334$$

$$a + b = 668$$

Answer: 668 triangles

Problem 2.



Let $AE \cap DC = \{K\}$ and $H \in (DC)$, $AH \perp DC$

$\angle K + \angle KAB + \angle ABC + \angle BCK = 360^\circ$

But $\angle DEA = \angle ABC = \angle BCD = \angle CDE = \frac{540^\circ - 60^\circ}{4} = 120^\circ$

$\angle K + 60^\circ + 120^\circ + 120^\circ = 360^\circ$

$\Rightarrow \angle K = 60^\circ \Rightarrow BC \parallel AK$ and $KC = AB = 6$

$KC = 6$, $DC = 4 \Rightarrow KC - DC = KD = 2$

$\angle KDE = 180^\circ - \angle EDC = 180^\circ - 120^\circ = 60^\circ$

$\Rightarrow \angle EKD = \angle KDE = \angle KED = 60^\circ$

$\Rightarrow KE = 2$

$AK = AE + EK = 2 + 7 = 9$

$\angle HKA = 60^\circ \Rightarrow \angle KAH = 30^\circ$

$\Rightarrow \cancel{2 \cdot AH}$ $2 \cdot AK = AH \cdot \sqrt{3}$

$AH = \frac{9\sqrt{3}}{2}$

Answer: $AH = \frac{9\sqrt{3}}{2}$.

Problem 3

$$(\sqrt{x}+y)(y-x^2)\sqrt{1-x} \le 0$$

$0 \le x \le 1$ because $\sqrt{1-x} \ge 0$ $\quad 1-x \ge 0$

$\sqrt{x} \ge 0$ $\quad x \ge 0$

Because $\sqrt{1-x} \ge 0$

I) $y \ge x^2 \quad y \le -\sqrt{x}$

$-\sqrt{x} \ge y \ge x^2$ False

II) $x^2 \ge y \ge -\sqrt{x}$

$\Rightarrow \quad 1 \ge y \ge -1$

if $\sqrt{1-x} = 0$

$x = 1 \quad , y \in R$

Exists points ~~and~~ at infinity so exist point $(0,0)$

$\Rightarrow$ the area will be a infinity.

Answer: infinity.

Problem 4

Let the children be

$a_1, a_2 \rightarrow a_{n-1}, a_n$

The maximum ~~~~ number of gifts that a child can give is $n-1$.

results that a child can give $\{0, 1, 2 \dots, n-2, n-1\}$

so a child have n options

results that the total number of gifts will be $\frac{(n-1)n}{2}$

After this every child will received k gifts

The total number of gifts will be $n \cdot k$

so $$nk = \frac{(n-1)n}{2}$$

$$k = \frac{n-1}{2} \qquad n = 2k+1$$

The strategy is: a_i give $i-1$ gifts. If one child receives k gifts no one will give him a gift more.

Answer: for all $n = 2k+1$ $k \in N^*$
every child will received k gifts.

Problem 5

$m^3 = n^3 + 13n - 273 \geq 0 \Rightarrow$

$\Rightarrow n \geq 6$

$m^3 \geq (n-6)^3$

But $(n+2)^3 \geq n^3 + 13n - 273$

$n^3 + 3 \cdot n^2 \cdot 2 + 3 \cdot 4 \cdot n + 8 \geq n^3 + 13n - 273$

$6n^2 - n + 281 \geq 0$

$12n^2 - 2n + 1 + 281 \cdot 2 - 1 \geq 0$

$(n-1)^2 + 11n^2 + 281 \cdot 2 - 1 > 0$

$m^3 \in \{(n-6)^3, (n-5)^3, \dots, n^3, (n+1)^3, (n+2)^3\}$

if $m^3 = (n-6)^3$

$n^3 + 13n - 273 = (n-6)^3$

$S = \emptyset$

if $m^3 = (n-5)^3$

$n^3 + 13n - 273 = (n-5)^3$

$S = \emptyset$

if $m^3 = (n-4)^3$

$n^3 + 13n - 273 = (n-4)^3$

$S = \emptyset$

if $m^3 = (n-3)^3$

$n^3 + 13n - 273 = (n-3)^3$

$S = \emptyset$

if $m^3 = (n-2)^3$

$n^3 + 13n - 273 = (n-3)^3$

$S = \emptyset$

if $m^3 = (n-1)^3$

$(n-1)^3 = n^3 + 13n - 273$

$S = \{8\}$

if $m^3 = n^3$ $n^3 + 13n - 273 = n^3$

$S = \{21\}$

if $m^3 = (n+1)^3$

$n^3 + 13n - 273 = (n+1)^3$

$S = \emptyset$

if $m^3 = (n+2)^3$

$n^3 + 13n - 273 = (n+2)^3$

$S = \emptyset$

$n = (8, 21)$

The Sum of cubes is $8 + 21 = 29$

Answer: 29.