1) Let an x-triangle be a triangle of x units sides

By mere counting it can be seen that there are 1-triangles

Moving further we get the following sequence or pattern

Number of 1-triangles $\Rightarrow$ 120
Number of 2-triangles $\Rightarrow$ 114
Number of 3-triangles $\Rightarrow$ 104
Number of 4-triangles $\Rightarrow$ 90
Number of 5-triangles $\Rightarrow$ 72
Number of 6-triangles $\Rightarrow$ 56
Number of 7-triangles $\Rightarrow$ 42
Number of 8-triangles $\Rightarrow$ 30
Number of 9-triangles $\Rightarrow$ 20
Number of 10-triangles $\Rightarrow$ 12
Number of 11-triangles $\Rightarrow$ 6
Number of 12-triangles $\Rightarrow$ 2

Hence, the total number of triangles is $(120+114+104+90+72+56+42+30+20+12+2) = 668$

There are 668 triangles in the picture

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5) Setting $n=m$, we get

$n^3 = n^3 + 13n - 273 \Rightarrow 13n - 273 = 0 \Rightarrow 13n = 273 \Rightarrow n = 21$

Giving us the solution, $n=21$, $m=21$

$m^3 = n^3 + 13n - 273 \Rightarrow m^3 - n^3 = 13n - 273 = 13(n-21)$

$\Rightarrow (m-n)(m^2+mn+n^2) = 13(n-21)$

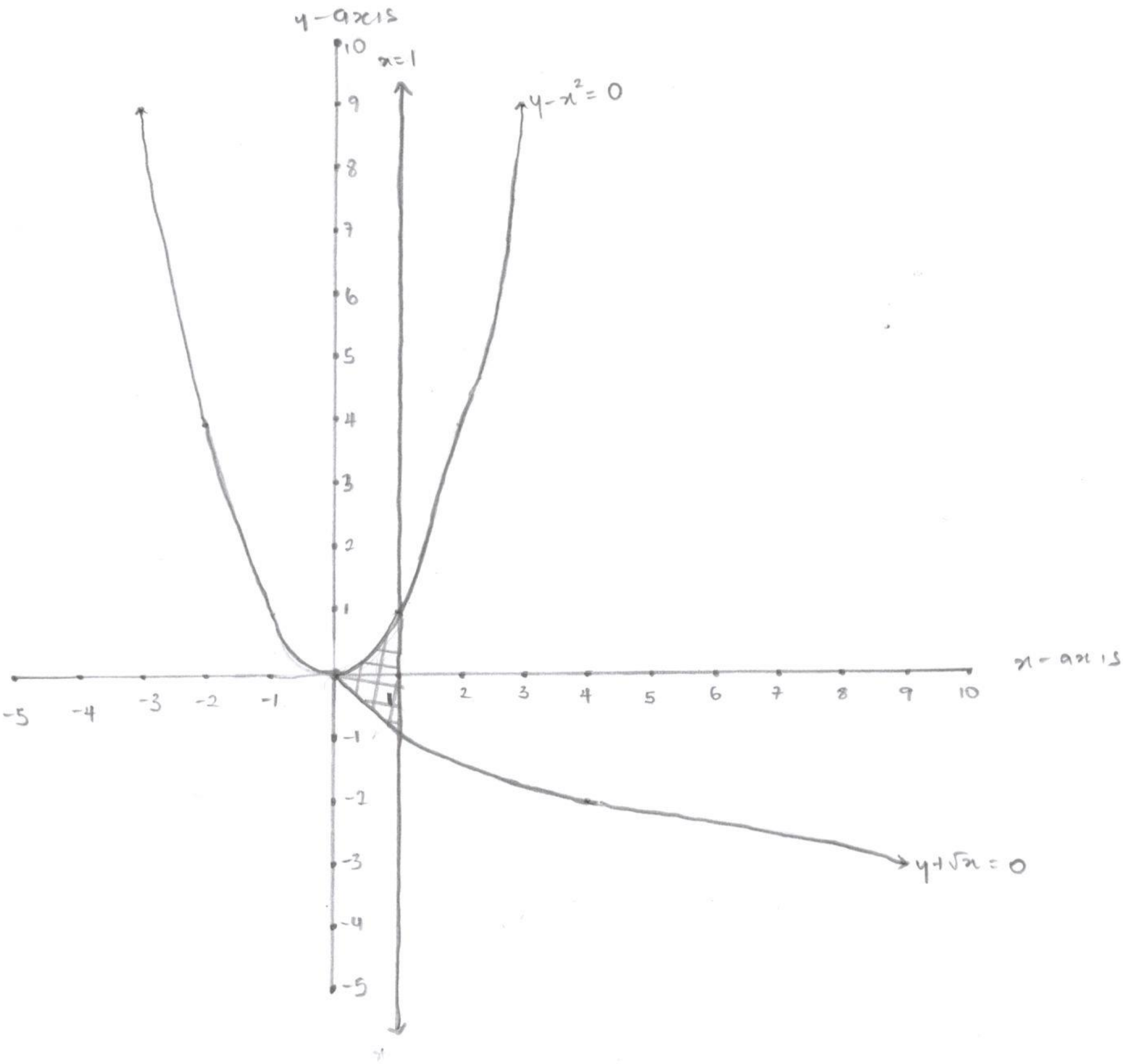
it can now easily be seen that for $n>21$ $m>n$ and for $n<21$, $m<n$
because, both sides have to be both positive or both negative and $m^2+mn+n^2>0$
for $n>21$, we get $m>n \Rightarrow m \geq n+1 \Rightarrow m^3 \geq (n+1)^3 = n^3+3n^2+3n+1$

Hence $m^3-n^3 \geq 3n^2+3n+1 > 13n-273 = m^3-n^3$ which is a contradiction

Therefore, $0<n \leq 21$, testing numbers in this range, we get $n=8$, $m=7$ and $n=21$, $m=21$ as the only two solutions

Hence, the sum of all cubos is $8+21 = 29$

3) Let us first draw the graph of lines $(y+\sqrt{x})=0$, $(y-x^2)=0$ and $\sqrt{1-x}=0$



It is easy to see that the rest of the options lead to a contradiction

leaving us with, $y+\sqrt{x}\geq 0$, $y-x^2\leq 0$, $\sqrt{1-x}\geq 0$

<We can also see that the rest give infinite areas>

Giving us the shaded region.

$$\int_0^1 (x^2)\,dx - \int_0^1 (-\sqrt{x})\,dx = \left(\frac{x^3}{3}\Big|_0^1\right) - \left(-\frac{x^{3/2}}{3/2}\Big|_0^1\right)$$

$$= \left(\frac{1}{3}-0\right) - \left(-\frac{2}{3}-0\right) = \frac{1}{3} - \left(-\frac{2}{3}\right) = 1$$

Hence, the area of the set of points is 1

To see that the rest lead to a contradiction: ~~Obviously, $\sqrt{1-x}\geq 0 \Leftrightarrow 1-x\geq 0 \Rightarrow x\leq 0$~~ $\sqrt{1-x}\geq 0$
Because of $\sqrt{1-x}$, $x\leq 1$ and $\sqrt{x}$, $x\geq 0$; hence $0\leq x\leq 1$. Assumming that $y+\sqrt{x}\leq 0$ and $y-x^2\geq 0$, then $y+\sqrt{x}\leq y-x^2 \Rightarrow \sqrt{x}\leq -x^2$ but $\sqrt{x}$ is positive and x^2 is negative.
Hence, a contradiction.

4) Given that they all gave distinct numbers of gifts; the maximum of gifts each child can give is $(n-1)$ and the minimum is 0. Meaning that, there are n distinct numbers of gifts possible for each child to give and n children available. Since they all gave distinct numbers; then, every number was given by a particular child. Leaving us with the pattern

Child	A_1	A_2	A_3	......	A_{N-1}	A_N
No. of gifts given	0	1	2	......	$N-2$	$N-1$

where A_i $(1 \le i \le N)$ are the children

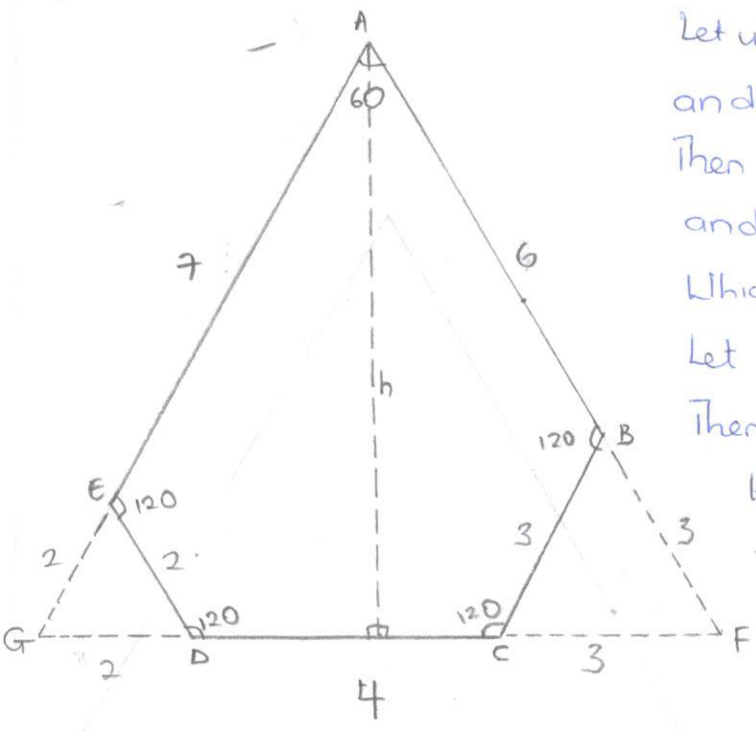
At this point, it is easy to see that the number of gifts can only be evenly distributed when $N \mid 0+1+2+\cdots+(N-1)$

$$\Rightarrow N \mid \frac{(N-1)(N)}{2}$$

Which is only possible when N is odd

Hence it works for only odd $N > 1$

2)



Let the height be h.

Let us extend $\overline{AB}$ and $\overline{CD}$ to meet at F and AE and CD to meet at G

Then $\angle GED = \angle EDG = \angle EGD = 60^\circ$

and $\angle BCF = \angle BFC = \angle FBC = 60^\circ$

Which implies that AGF is equilateral

Let $EG = GD = ED = x$ and $BC = CF = BF = y$

Then $7 + x = 4 + x + y = 6 + y \Rightarrow x = 2$ & $y = 3$

Hence, AGF has side length of 9

$$\sin 60 = h/9$$

$$\Rightarrow h = 9 \sin 60 = 9\sqrt{3}/2$$

Hence, the distance from A to CD is $\frac{9\sqrt{3}}{2}$