

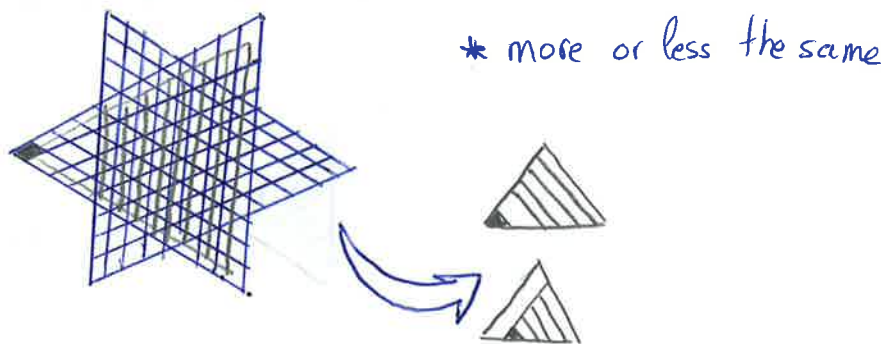
International mathematical Olympiad  
 "Formula of Unity" / "The Third Millennium"  
 2018/2019 year, final round

SOLUTIONS TO THE SECOND ROUND FOR PARTICIPANTS.

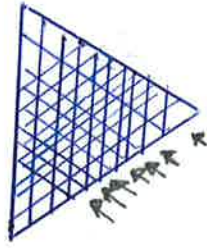
GRADE: 9 NUMBER: 23

40

The picture on the right is like two triangles displayed opposite, we can think about how many triangles can be formed with a little triangle, that contain that triangle, so we start in the left:



It seeing how many triangles can you form that contain a single (little) triangle so we know that from the left to the right we can obtain 9, 9, 9, 9, 8, 7, 6, 5, 5, 3, 2, 1 from the bottom, so now we need to see the "columns" Page: 4 of 9



All triangles that are in the same line (top down) can take part from the same number of triangle so if we multiply the number of the triangle form in the botton to the triangle that are in the same line with a lateral looking to the bottom of the side tha we're starting.



if we start from here, the triangle that I mencioned are this



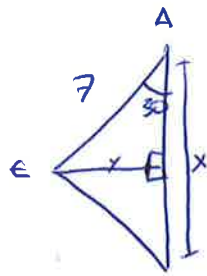
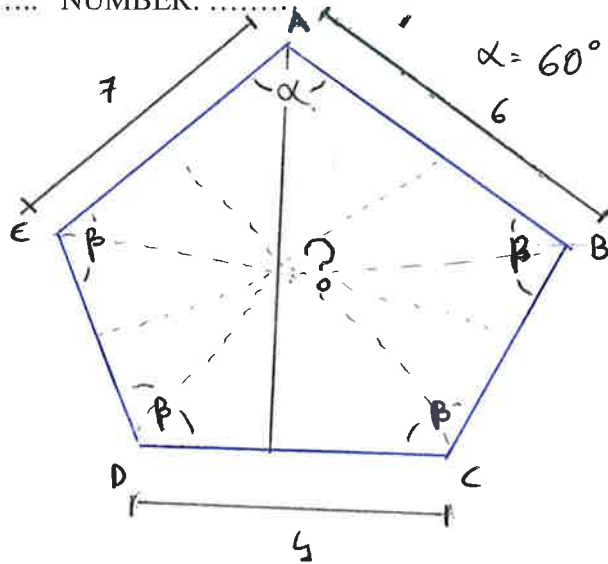
If we multiplied all we know that are 668 triangles in total.

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(2.0)



$$\operatorname{tg} 30^\circ = \frac{7}{x}$$



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SOLUTIONS TO THE SECOND ROUND FOR PARTICIPANTS.

GRADE: 9 NUMBER: 2

3.

$$(y + \sqrt{x})(y - x^2)\sqrt{1-x} \leq 0 ;$$

$$(y^2 - yx^2 + y\sqrt{x} - x^2\sqrt{x})\sqrt{1-x} \leq 0 ;$$

$$y^2\sqrt{1-x} - yx^2\sqrt{1-x} + y\sqrt{x-x^2} - x^2\sqrt{x-x^2} \leq 0 ;$$

$$(y^2 - yx^2)\sqrt{1-x} + (y - x^2)\sqrt{x-x^2} \leq 0 ;$$

$$(y^2 - yx^2)\sqrt{1-x} \leq -(y - x^2)\sqrt{x-x^2} ;$$

$$(y^5 + y^2x^4 - 2y^3x^2)(1-x) \leq -(y^2 + x^5 - 2yx^2)(x-x^2) ;$$

$$y^5 + y^2x^4 - 2y^3x^2 - xy^4 - y^2x^5 + 2y^3x^3 \leq -(y^2x + x^5 - 2yx^3 - y^2x^2 - x^6 + 2yx^4) ;$$

$$y^5 + y^2x^4 - 2y^3x^2 - xy^4 - y^2x^5 + 2y^3x^3 \leq -y^2x - x^5 + 2yx^3 + y^2x^2 + x^6 - 2yx^4 ;$$

$$y^5 + y^2x^4 - 2y^3x^2 - xy^4 - y^2x^5 + 2y^3x^3 + y^2x + x^5 - 2yx^3 - y^2x^2 - x^6 + 2yx^4 \leq 0$$



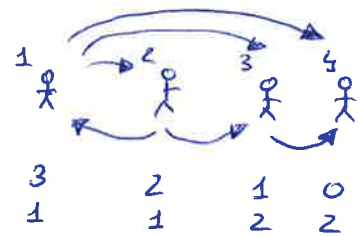
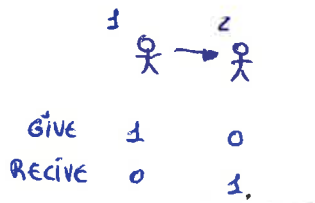
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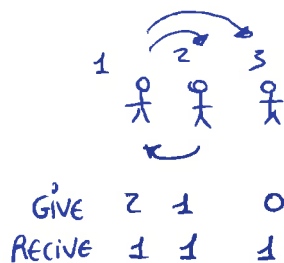
GRADE: ..... 9 ..... NUMBER: ..... 23 .....

4

Knowing that anybody can give more than one present to the same child, and that everybody has to give a different number of presents (a child can give 0 presents) so we're in the situation that in a group of 2 and in a group of 4



So the number of presents given in a group of  $N$  children is  $N-1, N-2, N-3 \dots 0$ , if we see the situation of  $N=3$



We see that if the sum of  $N-1, N-2, \dots, 0$  (present given) is a multiplied of  $N$ ,  $N$  is possible.

For example the  $N=11$  the sum of  $N-1, N-2, \dots, 0$  is 55, and if you do it, it is possible.