1. Număr triunghiurile după ~~nr.~~ lungimea laturii lor:

Sunt $24+96=120$ cu latura 1

$18+24+36+24+6+6=114$ cu lat 2

latura 3: $42+22+10+5+5+4+4=92$

latura 4: $6(4+3+2+1)+2+4+6=72$

latura 6: $\cancel{6\cdot 6}+6+12+18+2+12+3=53$

latura 7: $6+12+12+6=36$

latura 8: $6+12+12+3=33$

latura 9: $6+12+2=20$

latura 10: $6+6=12$

latura 11: 6

latura 12: 2

$\Rightarrow$ În total sunt:

$120+114+92+72+53+36+33+20+12+6+2+90=650$

234 326 398 451 487 520 540 552 558 560

$\Rightarrow$ 650 de triunghiuri

2. Dacă primul pătrat este 16 =>

$N^2 = 16/\_\_/\_\_$

$N \geq 400$ deoarece $160000 \leq N^2 \leq 169999$

$N \leq 412$

Verific pe rând $400^2$, $401^2$, $402^2$, ... $412^2$ =>

$408^2 = 166464$

Dacă primul este 25 =>

$N^2 = 25/\_\_/\_\_$

Verific $500^2$, $501^2$, ... $511^2$. Nu verifică niciunul.

Dacă primul este 36 =>

$N^2 = 36/\_\_/\_\_$.

=> Verific $600^2$, $601^2$, ... $611^2$. Nu verifică niciunul.

Dacă primul este 49 =>

$N^2 = 49/\_\_/\_\_$.

=> Verific $700^2$, $701^2$, ... $711^2$. Nu verifică niciunul.

Dacă primul este 64 =>

$N^2 = 64/\_\_/\_\_$.

=> Verific $800^2$, $801^2$, ... $811^2$. =>

$804^2 = 646416$.

Dacă primul este 81 =>

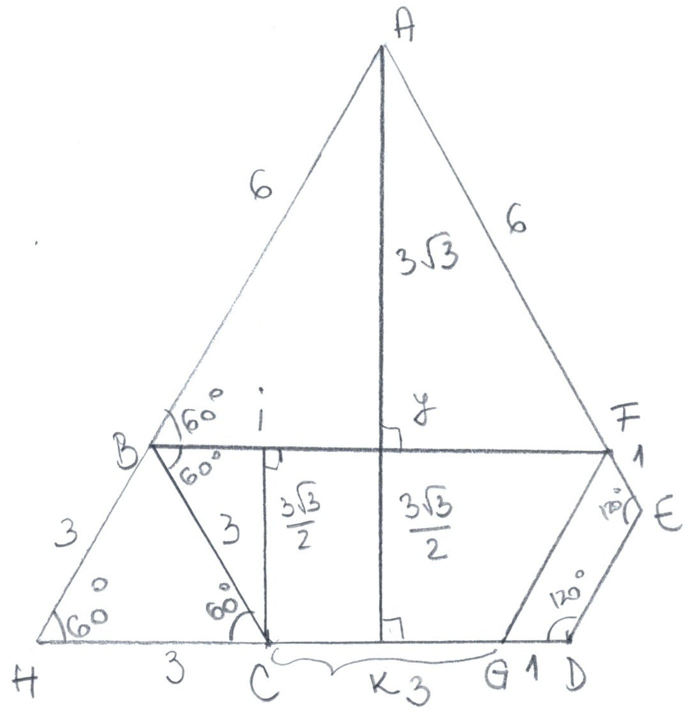
$N^2 = 81/\_\_/\_\_$.

=> Verific $900^2$, $901^2$, ... $911^2$. Nu verifică niciunul.

=> Singurele valori ale numărului de 6 cifre sunt 646416 și 166464.

3. $m(\hat{A}) + m(\hat{B}) + m(\hat{C}) + m(\hat{D}) + m(\hat{E}) = \cancel{540}^\circ\ 540^\circ$

$\Rightarrow m(\hat{A}) + 4m(\hat{B}) = 540^\circ \Rightarrow m(\hat{B}) = 120^\circ$




Fie $F \in (AE)$ astfel încât $AF = 6$, $FE = 1$.

$\Rightarrow \Delta ABF$ echilateral ($AB = AF = 6$ și $m(\sphericalangle BAF) = 60^\circ$)

$\Rightarrow \left.\begin{array}{l} m(\sphericalangle FBA) = 60^\circ = m(\sphericalangle FBC) \\ m(\sphericalangle BCD) = 120^\circ \end{array}\right\} \Rightarrow CD \parallel BF$ ($\sphericalangle$ interne de aceeași parte a secantei)

Fie $FG \parallel DE$, $G \in (CD)$.

$\Rightarrow \left.\begin{array}{l} FEDG \text{ trapez} \\ m(\sphericalangle GDE) = m(\sphericalangle DEF) = 120^\circ \end{array}\right\} \Rightarrow FEDG$ trapez isoscel $\Rightarrow GD = FE = 1 \Rightarrow CG = 3$

Cum $\left.\begin{array}{l} m(\sphericalangle AED) = 120^\circ \\ m(\sphericalangle BAE) = 60^\circ \end{array}\right\} \Rightarrow DE \parallel AB$ ($\sphericalangle$ interne de aceeași parte a secantei).

$\Rightarrow DE \parallel FG \parallel AB$

Fie $\{H\} = AB \cap CD$. $\Rightarrow \left.\begin{array}{l} BH \parallel FG \\ BF \parallel GH \end{array}\right\} \Rightarrow BFGH$ paralelogram $\Rightarrow$ $BF = HG$

$BF = 6$ deoarece $\Delta ABF$ echilateral

$\Rightarrow HG = 6$, $CG = 3 \Rightarrow HC = 3$.

$m(\sphericalangle HBC) = m(\sphericalangle HCB) = 180^\circ - 120^\circ = 60^\circ \Rightarrow \Delta HBC$ echilateral

$\Rightarrow HC = BC = 3$

Fie $AK \perp CD$, $K \in CD$, $\{y\} = BF \cap AK$, $Ci \perp BF$, $i \in (BF)$.

$\Rightarrow \Delta ABy: \left.\begin{array}{l} m(\sphericalangle ABy) = 60^\circ \\ m(\sphericalangle AyB) = 90^\circ \end{array}\right\} \Rightarrow Ay = AB \sin 60^\circ = 3\sqrt{3}$.

$CIJK$ dreptunghi $\Rightarrow JK = CI$

$\Delta BCI$ : $\left.\begin{array}{l} m(\sphericalangle BIC) = 90^\circ \\ m(\sphericalangle IBC) = 60^\circ \end{array}\right\} \Rightarrow CI = BC \sin 60^\circ = \frac{3\sqrt{3}}{2}$

$\left.\begin{array}{l} \left.\begin{array}{l} AJ = 3\sqrt{3} \\ JK = \frac{3\sqrt{3}}{2} \end{array}\right\} \Rightarrow AK = \frac{9\sqrt{3}}{2} \\ AK \perp CD,\ K \in CD \end{array}\right\} \Rightarrow d(A, CD) = \frac{9\sqrt{3}}{2}$

4. Da, de exemplu $x = 1\cdot3\cdot5\cdot7\cdot\ldots\cdot2019 - 2020$.

Cum $1\cdot3\cdot5\cdot7\cdot\ldots\cdot2019 > 2\cdot2019 > 2020 \Rightarrow x > 0 \Rightarrow$

$x \in \mathbb{N}^* \Rightarrow x$ este un întreg pozitiv.

(1) Demonstrez că propozițiile de forma:

$[x + (2019-2k)] \vdots (2k+1)$ sunt adevărate, $k \in \{1, \ldots 1009\}$

$x + 2019 - 2k = x + 2020 - (2k+1) = 1\cdot3\cdot5\cdot7\cdot\ldots\cdot2019 - (2k+1)$

Cum $1\cdot3\cdot5\cdot\ldots\cdot2019 \vdots (2k+1)$, deoarece $k \in \{1, 2, \ldots 1009\}$

$\Rightarrow x + 2019 - 2k \vdots (2k+1)$, deoarece $1\cdot3\cdot\ldots\cdot2019 \vdots (2k+1)$
$(2k+1) \vdots (2k+1)$

$\Rightarrow$ propoziția este adevărată.

(2) Demonstrez că propozițiile de forma:

$(x + 2020 - 2k) \vdots 2k)$ sunt false, $k \in \{1, 2, \ldots 1009\}$

Cum $x = \underbrace{1\cdot3\cdot5\cdot\ldots\cdot2019}_{\not\vdots 2} - \underbrace{2020}_{\vdots 2} \not\vdots 2$ și $2020 - 2k \vdots 2 \Rightarrow$

$x + 2020 - 2k \not\vdots 2 \Rightarrow$

$(x + 2020 - 2k) \not\vdots (2k) \Rightarrow$ propoziția este falsă

Din (1) și (2) $\Rightarrow$ 1009 propoziții adevărate, 1009 propoziții false. $\Rightarrow$

pentru $x = 1\cdot3\cdot5\cdot7\cdot\ldots\cdot2019 - 2020 = \left[\prod_{k=1}^{1009} (2k+1)\right] - 2020$,

jumătate din afirmații sunt corecte.

5. Da, există o cale cu care B câștigă mereu:

Fie $a_1 \le a_2 \le \ldots \le a_8$ nr. alese de A.

$\Rightarrow \begin{cases} a_1+a_2 \text{ este cea mai mică sumă} \\ a_1+a_3 \text{ este a doua cea mai mică sumă} \\ a_7+a_8 \text{ este cea mai mare sumă} \\ a_6+a_8 \text{ este a doua cea mai mare sumă} \end{cases}$

$\Rightarrow$ Știu $a_1+a_2$; $a_1+a_3$; $a_7+a_8$; $a_6+a_8$. (1)

Dacă adun toate cele 28 de sume, obțin $7(a_1+\ldots+a_8) \Rightarrow$ știu și $a_1+a_2+\cdots+a_8$. (2)

Din (1) și (2) $\Rightarrow$ știu $\begin{cases} a_2+a_4+a_5+a_6=n_1 \\ a_2+a_4+a_5+a_7=n_2 \\ a_3+a_4+a_5+a_6=n_3 \\ a_3+a_4+a_5+a_7=n_4 \end{cases}$

Văd ~~ce s~~ care 2 sume adunate dau $n_1, n_2, n_3, n_4$ și ce sume sunt comune $\Rightarrow$ $a_4+a_5$ este una dintre sumele comune.

Iau pe cazuri cât poate fi $a_4+a_5$.

$\Rightarrow$ Știu $a_4+a_5$ $\Rightarrow$ știu $\begin{cases} a_2+a_6 \\ a_2+a_7 \\ a_3+a_6 \\ a_3+a_7 \end{cases}$ $a_1+a_8$

Iau pe cazuri cât poate fi $a_1+a_7$.

$\Rightarrow$ Știu $a_1+a_7$, $a_3+a_7$ $\Rightarrow$ știu $a_3-a_1$, $a_1+a_3$ $\Rightarrow$ știu $a_3, a_1$ $\Rightarrow$ știu $a_2, a_6, a_7, a_8$.

$\Rightarrow$ știu $a_4+a_1$, $a_4+a_2$, ~~$a_4+a_4+a_8$~~ $a_4+a_3$, ~~$a_4+a_5$~~, $a_4+a_6$, $a_4+a_7$, $a_4+a_8$ $\Rightarrow$ știu $6a_4+a_1+a_2+a_3+a_6+a_7+a_8$ $\Rightarrow$ știu $a_4$ $\Rightarrow$ știu $a_5$

$\Rightarrow$ Obțin nr. $a_1, a_2 \dots a_8$.

Verific nr. din fiecare caz și îmi rămân nr. reale ($a_1, a_2, \dots a_8$). (Verific nr. din fiecare caz, adică refac ale 28 de sume și le compar cu cele bune).

$\Rightarrow$ B știe nr. ($a_1, \dots a_8$)

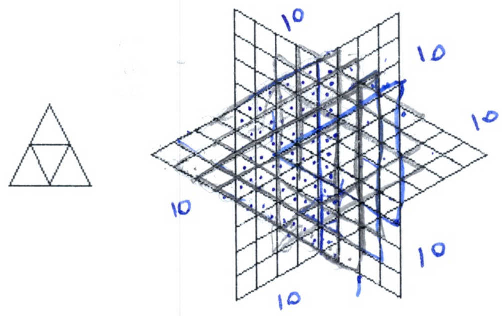
$\Rightarrow$ B câștigă. (Nu există 2 mulțimi de 8 nr. care să dea aceleași 28 de sume)

7/7

International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2018/2019. Final round

# Problems for the class R8

1. In the picture on the left, you can find five triangles (four small and one big). And how many triangles can you find in the picture on the right?




$60 + 96 +$
$4+2+18+8$
$36$

2. Three students write on a whiteboard 3 two-digit perfect squares next to each other. Surprisingly the 6-digit number obtained is also a perfect square! Find all possible values of this number.

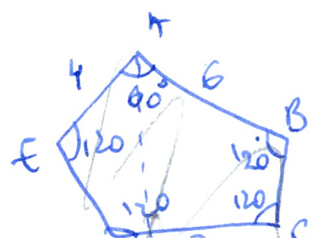
3. In a convex pentagon $ABCDE$, $\angle A = 60°$, and all other angles are equal. It is known that $AB = 6, CD = 4, EA = 7$. Find the distance from $A$ to the line $CD$.

4. Is there a positive integer $x$ such that exactly half of the propositions "$x+1$ is divisible by 2019", "$x+2$ is divisible by 2018", "$x+3$ is divisible by 2017", ..., "$x+2017$ is divisible by 3", "$x+2018$ is divisible by 2" are correct?

5. Two players $A$ and $B$ are playing the following game. $A$ chooses 8 real numbers. (Some of these numbers could be equal to each other.) On a piece of paper, $A$ writes sums of all possible sets of 2 of these numbers in an arbitrary order. Next, $A$ gives the paper to $B$. (This paper contains 28 sums; some of these sums could be equal to each other.) $B$ wins if he can figure out the 8 original numbers on the first guess. Is there a way for $B$ to definitely win the game?

$540 + 60$
$480$

$\overline{ab}\ \overline{cd}\ \overline{ef}$




$1+2+\ldots+f = 28$

- The paper should not contain personal data of the participant, so **you should not sign your paper** (the personal data should be written in the questionnaire).
- Please solve the problems by yourself. Solving together or cheating is not allowed.
- Using calculators, books, or Internet is not allowed.
- The results will be published at `formulo.org` before April 10.

$\frac{9 \cdot 1,7}{2}$

$7,8$ $\frac{9\sqrt{3}}{2}$

$408^2$ $804^2$

# Rules of the final round of the Olympiad "Formula of Unity" / "The Third Millennium" 2018/19

1. Participants of the final round include the winners of the qualifying round as well as all those who received diplomas for winning in the Olympiad 2017/18. The locations and dates of the final round are listed on the page http://www.formulo.org/en/olymp/2018-math-en/

2. The round will last for 4 hours.

3. It is necessary to bring your pens and paper with you. The participants are not allowed to use calculators, computers, telephones, any other communication tools.

4. Solutions should be written in Esperanto, English, French, Georgian, German, Persian, Romanian, Russian, Spanish, Ukrainian, or Uzbek.

5. The participants are to fill in a participant form they receive before the beginning of the final round. (The time for filling in the participants form is not included into 4 hours.) The paper sheets with solutions should not include the participant's name and other personal data.

6. Since the date of the $2^{nd}$ round varies in different countries, the participants and organizers are asked not to publish the problems on the web before March 7.

7. Preliminary results of the Olympiad will be published on http://formulo.org before March 24, 2018. Appeals (requests to reconsider one's solutions) can be submitted within 3 days thereafter.

## Information for the organizers

1. The Organizing Committee asks the local organizers to ensure participants' compliance with the rules. The time necessary to fill in the participant form is not included into 4 hours provided for solving problems.

2. The Olympiad papers are to be scanned and sent to ***solv@formulo.org*** within 3 days after the date of the final round. The papers of participants of **different grades** should be e-mailed in **separate messages**. Participant forms are to be e-mailed along with the papers in the same messages. The subjects of the messages should include the words "Final round", the name of the host organization and the grade (R5, R6, etc). The file names should follow an example: solutions1.pdf, form1.pdf, solutions2.pdf, form2.pdf.

3. The papers of unofficial participants (not including the papers marked by the local organizers) should be sent in separate messages with subject lines such as "Final round, unofficial participants, University of Nankago, R5".

4. In case of any uncertainty, please contact the Organizing Committee by ***olimp@formulo.org***.