Problem 1

We take a general case when

$n = 4$

Then there

will be

Size	Up triangle	Size	Down triangle
4	1	4	0
3	3	3	0
2	6	2	1
1	10	1	6

So we observ the formula

$S = T_1 + T_2 + T_n + T_{n-1} + T_{n-3} + \dots D$

$T_1 = \cancel{1+2}$

$T_2 = 1+2$

so $T_3 = 1+2+3$

$S = \left[\frac{n(n+2)(2n+1)}{8} \right]$

In the figure there a

120 size 1 triangles

We apeey the rule fo

the two triangle

$S = \left[\frac{n \cdot 2 \cdot 14 \cdot 25}{4} \right] - 24 = 1026$ -triang

Problem 2

The two-digit perfect squares are:

16
25
36
49
64
81.

For every x:

$x \equiv \pm 1 \mod 9$ $x^2 \equiv 1 \mod 9$
$x \equiv \pm 2 \mod 9$ $x^2 \equiv 4 \mod 9$
$x \equiv \pm 3 \mod 9$ $x^2 \equiv 0 \mod 9$
$x \equiv \pm 4 \mod 9$ $x^2 \equiv 7 \mod 9$
$x \equiv \pm 5 \mod 9$ $x^2 \equiv 7 \mod 9$

A number is divisible by 9 when its sum of digits is divisible by 9.

so we obtain

363636 ×	646416 V
818181 ×	166464 V
368181 ×	641664 ×
368136 ×	
~~3636~~	
813681 ×	
818136 ×	
363681 ×	

Answer: 646416; 166464.

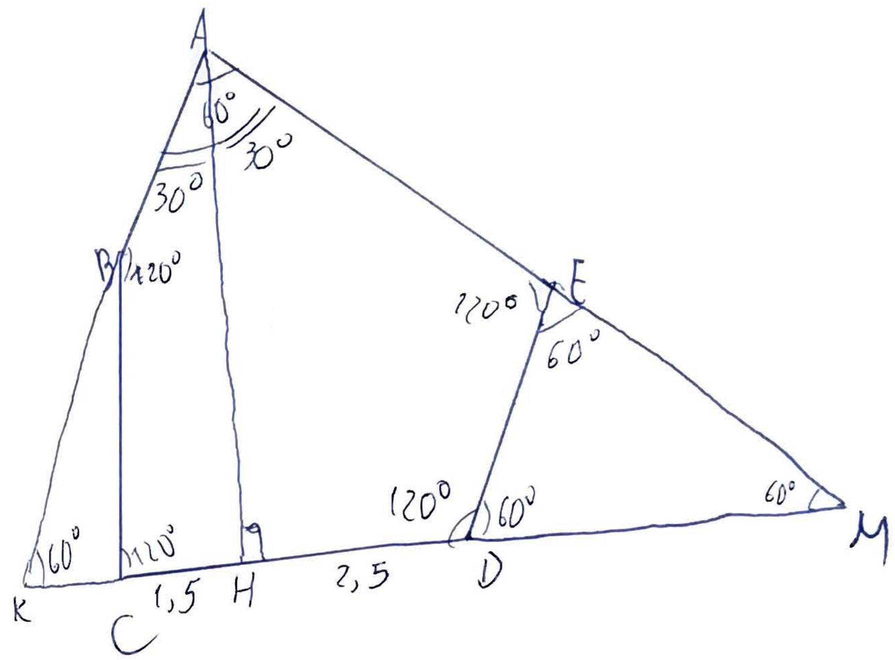
Problem 3

$\measuredangle A = 60°$

$AB = 6$

$CD = 4$

$EA = 7$



$m(\measuredangle DEM) = 180 - 120° = 60°$

$m(\measuredangle EDM) = 180 - 120 = 60°$

$\Rightarrow m(\measuredangle EMD) = 60°$

$\Downarrow$

ΔEMD - equilateral

From $AHED \Rightarrow m(\measuredangle HAE) = 360 - 120 - 120 - 90 = 30°$

$\Downarrow$

$m(\measuredangle HAB) = 30°$

$m(\measuredangle BCK) = 180 - 120° = 60°$

$m(\measuredangle KBC) = 60°$

$\Rightarrow \Delta BKC$ - equilateral

$KC = x \quad MD = z$

~~$\Delta BKC \sim \Delta$~~

$CH = y \quad HD = 4 - y$

$\Delta AHK \sim \Delta AHM$

$\frac{AH}{AH} = \frac{HK}{HM} = \frac{AK}{AM}$

$AK = AM$

$6 + x = 7 + z$

$x = z + 1$

$HK = HM$

$y + x = 4 - y + z$

$y + \cancel{z} + 1 = 4 - y + \cancel{z}$

$2y = 3 \quad \boxed{y = 1,5}$

$\Downarrow$

$\boxed{HD = 2,5}$

Problem 4

$x+1^{2018} \equiv \mod 2019$

We take half of propositions

$x+2018 \equiv 0 \pmod 2$

$x+2020 \equiv 0 \pmod 2$

$x+2020 \equiv 0 \pmod 3$

. . . .

$x+2020 \equiv 0 \pmod{1011}$

The smallest x values

is

$1009! - 2020$ which is the answer.

Answer: Yes, exists ($1009! - 2020$)

Problem 5

B can not be sure to win because for:

① 2, 4, 6, 10, 3, 7, 9, 11

② 1, 5, 7, 9, 4, 6, 8, 12

The two sets have the same pairwise sums.