Problem 1 Работа 3

Let the first crow has eaten x grams of its cheese.
Then the fox has stolen 100-x grams of the first cheese.
The second crow has found twice as much cheese as the first one, so it has found $100 \cdot 2 = 200$ grams of cheese and it has eaten $\frac{x}{2}$ grams of it. Then the fox has stolen $200 - \frac{x}{2}$ grams of the second cheese.

We know that the fox has stolen three times as much cheese from the second crow as from the first one.

So $200 - \frac{x}{2} = 3 \cdot (100 - x)$.

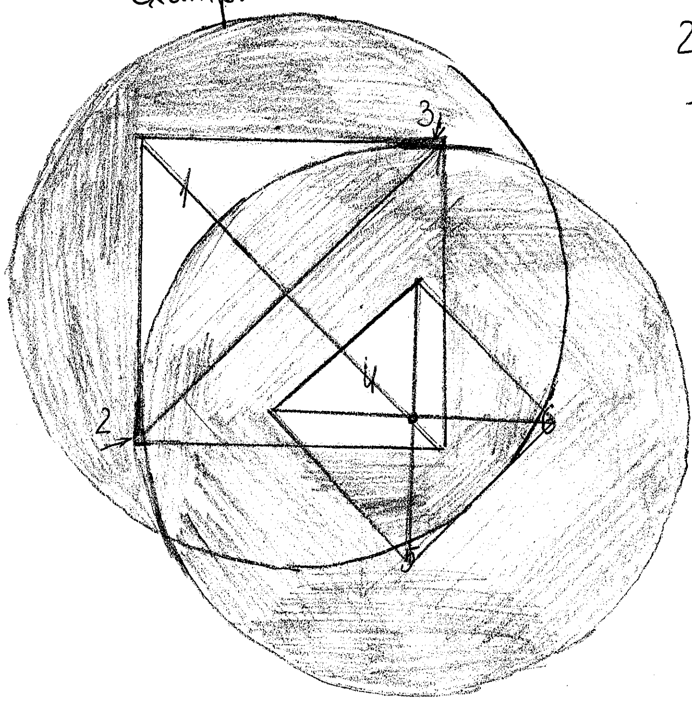
$$200 - \frac{x}{2} = 300 - 3x \quad / \cdot 2$$
$$400 - x = 600 - 6x$$
$$5x = 200$$
$$x = 40g$$

So the fox has stolen $(100-x)+(200-\frac{x}{2}) = (100-40) + (200 - \frac{40}{2}) = 60 + (200-20) =$
$= 60 + 180 = 240$ grams of cheese altogether.

Problem 2

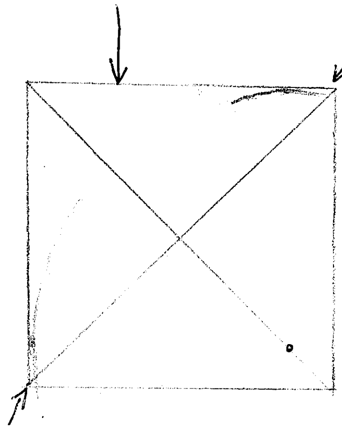
I think it's possible to cut out two strange rings and to place them on a table so that the resulting figure would have more than 5 holes.

Here's the example:

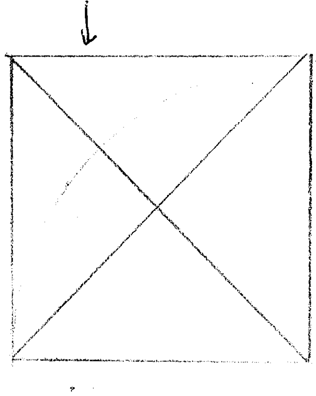


The second circle is touching the ends of the first square.

2→, 3↓: There are holes here, because there aren't holes when the center of the second strange ring is at the corner of the square hole of the first:



There are holes but they are very small. The radius of the circl is smaller than th side of the squar



There aren't holes, because the radius of the circle is equal to the side of the square.

Problem 3

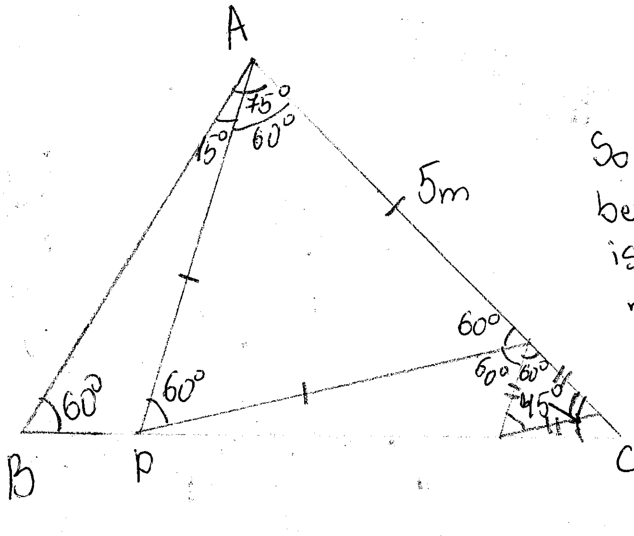
Работа 3

$\measuredangle A$ is 75°, because $\measuredangle A + \measuredangle B + \measuredangle C = 180^\circ$

$\measuredangle A + 60^\circ + 45^\circ = 180^\circ$

$\measuredangle A + 105^\circ = 180^\circ$

$\measuredangle A = 75^\circ$



So the bee can choose the path such that the angle between AC and ~~the~~ AP (the first point where it turns is 60°. So because the turning angle is 60° we get many equilateral triangles along the whole AC. Because the equilateral triangle has three equal sides then the path of the bee is twice as AC. So the path of the bee is $2 \cdot AC = 2 \cdot 5m = 10m$. $10m > 9{,}9m$, so it's possible for the bee to fly more than 9,9 meters.

Problem 4 Работа 3

The all possible values of the 6-digit number are 166464 and 646416.

The all 2-digit perfect squares are $16=4^2$, $25=5^2$, $36=6^2$, $49=7^2$, $64=8^2$, $81=9^2$.

The 6-digit number is in the form $(100x+y)^2 = 10000x^2+200xy+y^2$, where x is between 4,5,6,7,8,9 (we need the number begins with perfect square)

y must be between 4,5,6,7,8,9, too, because the number must ends with perfect square. If y is bigger than 9, so y is at least 10, but after checking for the x with y bigger than 9 there aren't any solutions. So $y^2 < 100$.

So $10000x^2+200xy+y^2 = x^2 \cdot 10000 + z^2 \cdot 100 + y^2$

$100z^2 = 200xy$

$z^2 = 2xy$

So $xy = 2 \cdot k^2$, because z is an integer.

$z^2 = 2 \cdot 2 \cdot k^2$

$z = 2k$

So z is even number, and it's between 4,5,6,7,8,9.

So z is:

- $z=4$, then $k=2$ and $xy=8$, but it's impossible, because $x \geq 4$ and $y \geq 4$
- $z=6$, then $k=3$ and $xy=18$, but it's impossible, because $18=2 \cdot 3 \cdot 3$ can't be a product of two numbers (x and y) that are bigger than 3 and smaller than 10 (between 4,5,6,7,8,9)
- $z=8$, then $k=4$ and $xy=32$, but the only one way to get 32 as a product of two numbers between 4,5,6,7,8,9 is $32=4 \cdot 8$, so we have two ~~ways~~ cases:

$x=4, y=8$: $(100x+y)^2 = 408^2 = 166464 \to 16=4^2, 64=8^2, 64=8^2$

$x=8, y=4$: $(100x+y)^2 = 804^2 = 646416 \to 64=8^2, 64=8^2, 16=4^2$

So the all possible values of the 6-digit number are two: 166464 and 646416

Problem 5 Работа 3

If x is an odd number then:

- wen notice that all the propositions are in the form „x+ (2020-a) is divisible by a"

So if x is an odd number all the propositions where a is even are wrong because we need odd number (odd + even = odd) to be divisible by even number. But the propositions where a is even are exactly half of all propositions (All are 2019-2+1 = 2018 and the even numbers between 2 and 2019 (including 2 and 2019) are exactly 1009). So we just need for x to ~~(perfor)~~ execute all the propositions where a is an odd number, so we just find as much as bigger x as we need for which all the propositions where a is odd are correct.

So the answer is „yes", there is a positive integer x such that exactly half of the propositions in the condition are correct