Problem 1.

Solution:

Работа 5

Let the ~~fir~~ amount of cheese the first crow has eaten is X grams. This means that the fox stole the rest - this is 100 - X grams. As the second crow found twice as much cheese than the first one, and the first found 100 grams of cheese, the second crow found $100 \cdot 2 = 200$ grams. As the second crow ate twice less cheese than the first crow, the second crow ate $\frac{x}{2}$ grams. ⇒ The fox stole $200 - \frac{x}{2}$ grams of cheese from the second crow.

We know that the fox stole ~~ate~~ 3 times ~~more~~ as much cheese from the second crow as it stole from the first. That's how we get the equation

$$200 - \frac{x}{2} = 3(100 - x) \Rightarrow$$

$$200 - \frac{x}{2} = 300 - 3x \;\Big|\cdot 2 \Rightarrow$$

$$\cancel{2} \; 400 - x = 600 - 6x \Rightarrow$$

$$6x - x = 600 - 400 \Rightarrow$$

$$5x = 200 \Rightarrow$$

$$x = 40g. \Rightarrow$$

As the fox stole $(200 - \frac{x}{2}) + (100 - x)$ grams of cheese in total and we found $x = 40$, the fox stole $(200 - \frac{40}{2}) + (100 - 40) = 200 - 20 + 60 \Rightarrow 240$g. in total.

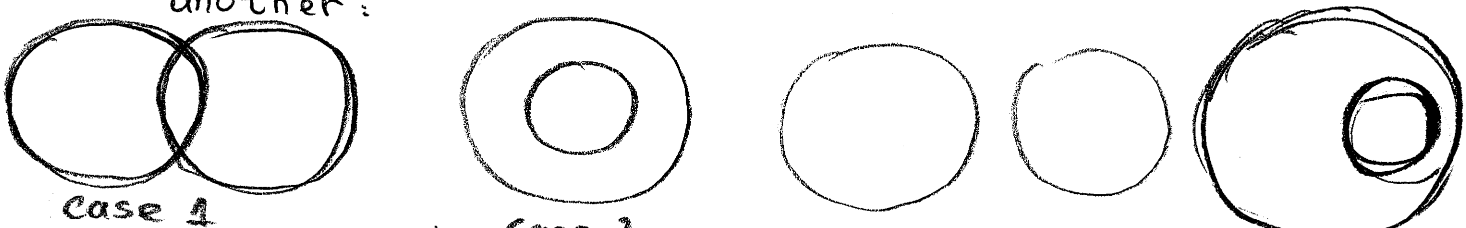
Problem 2.

Solution:

Работа 5

To prove that it is not possible to cut out two strange rings and to place them on a table so that the resulting figure would have more than 5 holes, we will analise all the possible possisions and placements of the two strange rings:

First lets see that for the ~~six~~ circles, from which the strange rings are cut out, we have 4 possibilities for placement one another:

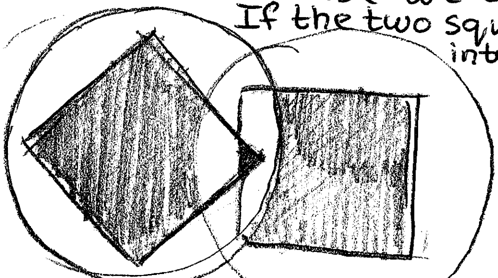


case 1 case 2 case 3 case 4

Now we will analyse the possibilities for the placement of the squares inside the circles in all of the cases:

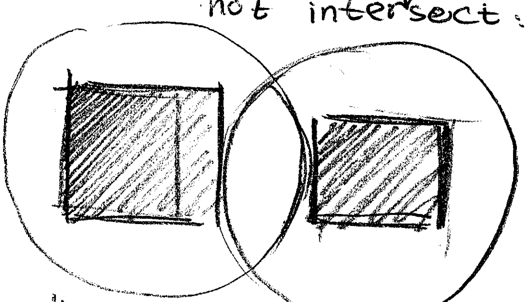
1) In first case we can have:

If the two squares intersect



Here we have 3 holes (the shaded regions)

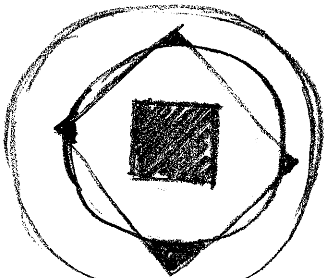
If the two squares do not intersect:



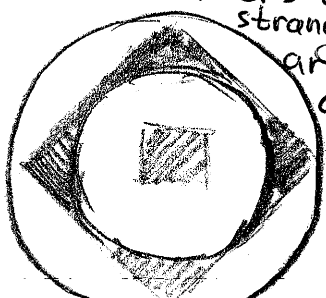
Here we have 2 holes

2) In the second case we can have:

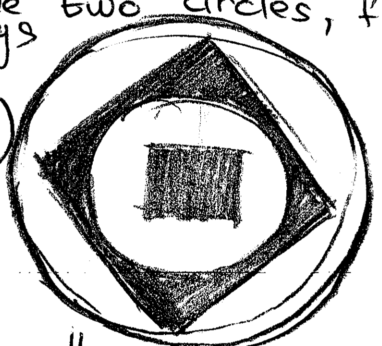
(The difference between the second and the fourth case is that in the second one the centers of the two circles, from which the strange rings are cut out coincide)



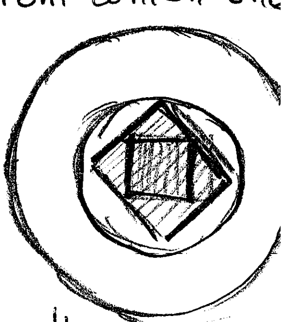
Here we have 5 holes



Here we have 5 holes

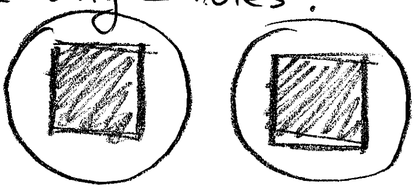


Here we have 2 holes



Here we have 2 holes no matter if the inner square is cut out of the large or out of the small circle

3) In the third case we obviously have only 2 holes:



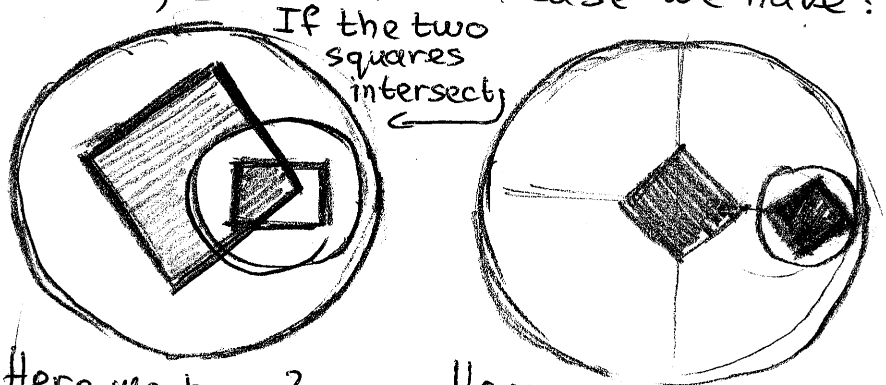
Problem 2.

Работа 5

Solution: ~~part~~

(part 2)

~~№~~ 4) In the fourth case we have:



Here we have 2 holes

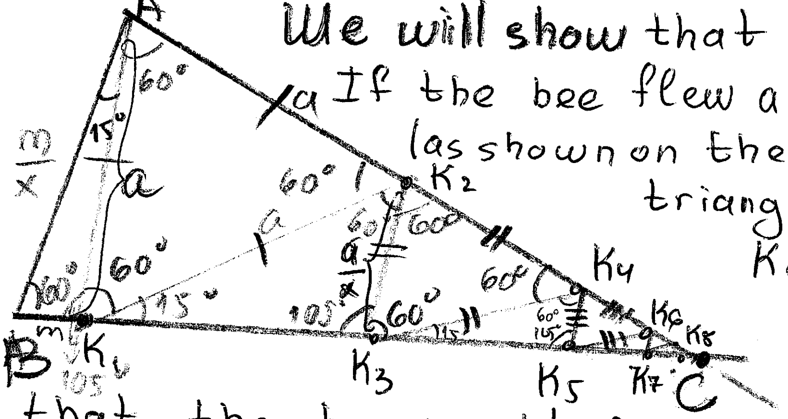
Here we have 2 holes

Now we have examined all the possible ~~solutions~~ positions of the two strange rings and as ~~in~~ the largest number of holes we found is 5, than in no case we can have more than 5 holes.

Problem 3

Работа 5

Solution:



We will show that it is possible. If the bee flew at 15° at first ~~(as on the~~ (as shown on the picture), then all the triangles AK_1K_2 and $K_1K_2K_3$ and $K_2K_3K_4$ and so on would be equilateral. This means that the bee would fly twice as much as the lenght of AC, which is 5m. long.

Let $AK_1 = a$ m. $\Rightarrow$

$K_2K_3 = \frac{a}{x}$

$K_3K_4 = \frac{a}{x^2}$

$\vdots$

As the triangles ~~$ABK_1, K_1K_2K_3K_4$~~ ABK_1, $K_1K_2K_3$, $K_3K_4K_5$ .... have equal angles.

We will prove $a + \frac{a}{x} + \frac{a}{x^2} + \dots \geq \frac{9.9}{2}$ or

$a + \frac{a}{x} + \frac{a}{x^2} + \dots \geq 4.95 \Rightarrow$ As $5 > 4.95$,

It is possible.

Problem 4.

Solution: Работа 5

Let the 3 perfect squares, written on the board be a^2, b^2 and c^2.

We know that when divided by 9 perfect squares give a remainder of 0, 1, 4 or 7. ~~To for~~

Let L be the six-digit perfect square $\Rightarrow$

$L = a^2 \cdot 10000 + b^2 \cdot 100 + c^2 \equiv a^2 + b^2 + c^2 \pmod 9$.

$\Rightarrow$ For the numbers a^2, b^2 and c^2 we can have these possibilities for remainder when divided by 9:

$0 = 0+0+0 = 4+4+1 = 4+1+4 = 1+4+4 = 1+7+1 =$
$= 1+1+7 = 7+1+1$

$1 = 0+0+1 = 0+1+0 = 1+0+0$

$4 = 4+0+0 = 0+4+0 = 0+0+4$.

Therefore we have this possibilities for the 6-digit number:

363636; 363681; 368136; 368181; 813636; 818136; 813681; 818181; 494964; 496449; 644949; 166464; 641664; 646416; 646425; 256464; 642564; 363664; 366436; 366464; 493636; 364936; 363649; 818149; 814981; 498181; 258181; 812581; 818125; 253636; 362536; 363625; 163636; 361636; 168181; 811681; 818116.

With a direct look through this numbers we get that perfect squares are:

166464,

Problem 5

Работа 5

Solution :

First of all we will see that if x is an odd number none of the propositions

„ $x+2$ is divisible by 2018"

„ $x+4$ is divisible by 2016"

⋮

„ $x+2018$ is divisible by 2"

can be true, as if x is an odd number every number $x+2$, $x+4$, ..., $x+2018$ is odd and ~~can't~~ therefore can't be divisible by an even number.

Now lets ~~as~~ establish ~~d~~ that the number

$W = 3.5.7. \dots .2017 - 2020$ is odd, therefore at least half of the propositions wouldn't be true if $x = W$.

Now we will prove that for any $a = 2k+1$ the proposition „ $x + a$ is divisible by $2020 - a$" will be true ($a = 1, 3, 5, \dots, 2017$) –

As $2020-a$ is also between the numbers $1, 3, 5, \dots, 2017$, than $1.3.5. \dots .2017$ is divisible by $2020-a$ and $a \neq 2020-a$ as a is an odd number $\Rightarrow$

$W+a = 3.5.7. \dots .2017 - (2020 - a)$ is divisible by $2020-a$ as $3.5. \dots 2017$ is divisible by $2020-a$ and $2020-a$ is divisible by itself