1 problem

Let the first crow ate „x" gram of cheese. Then the fox stole „100 − x" gram of cheese from the first crow. The second crow has found $2.100 = 200$ gram of cheese. It ate twice less cheese than the first crow, so the second crow ate „x : 2" gram of cheese. The fox stole „200 − x:2" gram of cheese from the second crow. On the other hand that's three times the cheese, which the fox stole from the first crow, i.e.:

$$3.(100-x) = 200 - x:2 \qquad /.2$$

$$300 - 3x = 200 - x:2$$

$$600 - 6x = 400 - x$$

$$600 - 6x + x = 400$$

$$600 - 5x = 400$$

$$600 - 400 = 5x$$

$$5x = 200$$

$$x = 40$$

Consequently the first crow ate 40 gram of cheese and the fox stole $100 - 40 = 60$ gram of cheese from it.
The second crow ate $x:2 = 40:2 = 20$ gram of cheese. And then the fox stole $200 - 20 = 180$ gram of cheese from the second crow. Also $180 = 3.60$.

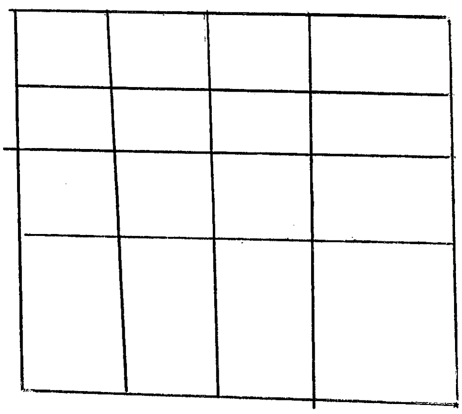
Consequently the fox stole $60 + 180 = 240$ gram of cheese altogether.

Answer: 240 gram of cheese

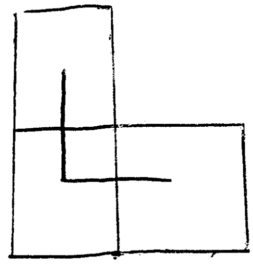
Работа 2

2.problem

Board



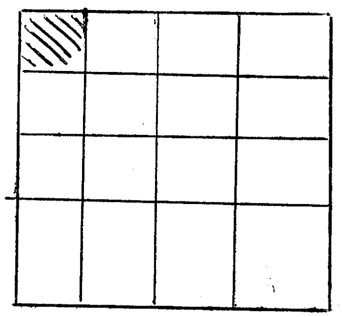
L-shaped piece:



 → painted square

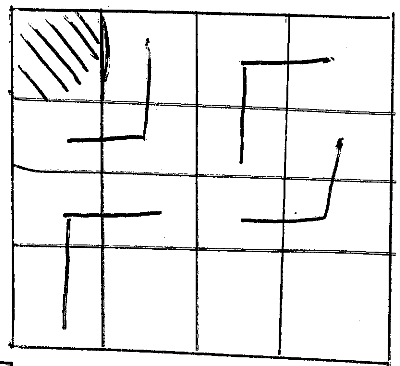
If Peter paints „0" squares, it is sure that he can't prevent Alex from doing the operation.

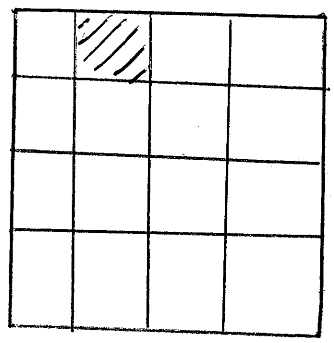
If Peter paints „1" square:



I st case, if the square is „in the corner".

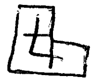
But Alex can make this:

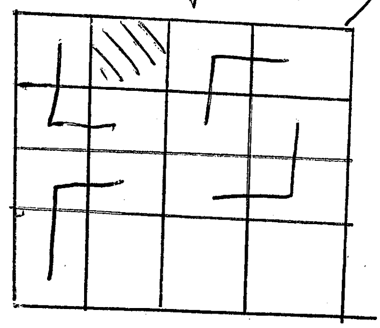


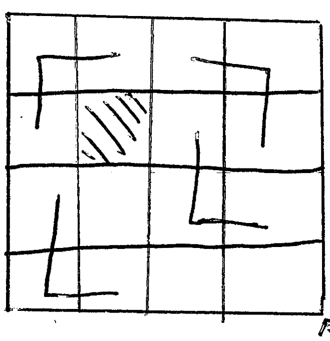


II nd case, if the square is like that.

But Alex can make this

( → L-shaped piece)





↖ Alex can make this

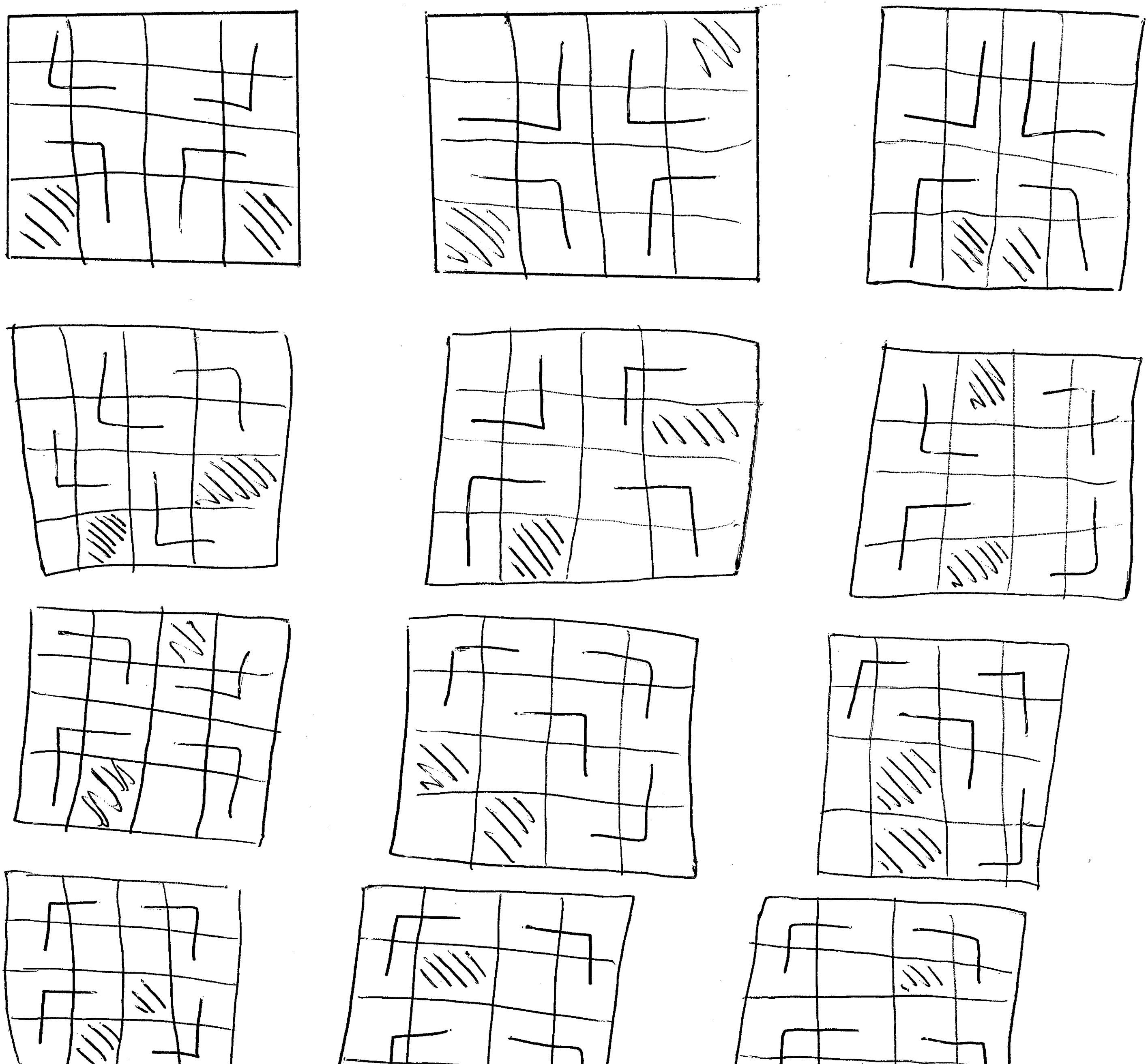
III rd case (the final) There are not other cases, which are different from those.

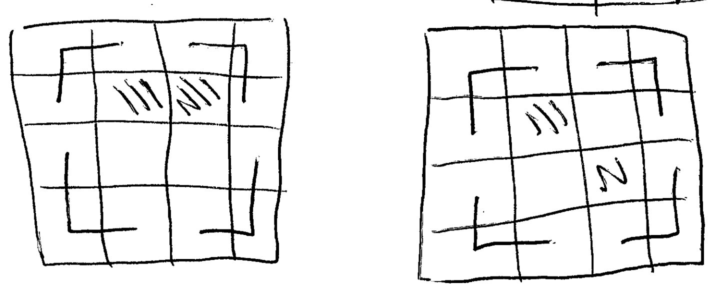
So if Peter paints „1" square, he can't prevent Alex.

Then, the smallest number of squares that must be painted is 2.

2problem. Работа 2

If Peter paints 2squares, he can't prevent Alex. Here are all the cases with 2 painted squares:



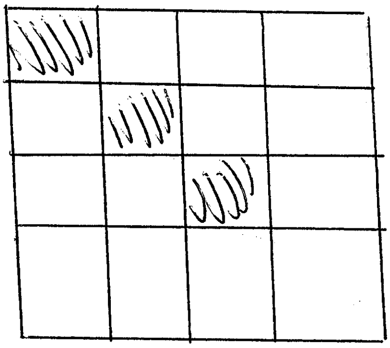


There are not other different cases.

Then the smallest number of squares that should be painted is 3.

2problem. Работа 2

There is an example with 3 painted squares:

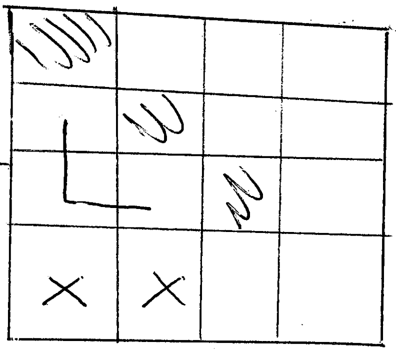


Let's say that Alex can fill the board with 4 L-shaped pieces

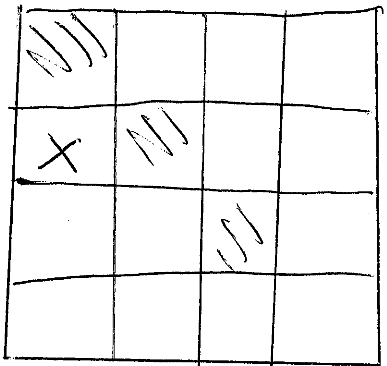
We have:

16 − 3 = 13 whithe squares.

4.3 = 12 and 13 − 12 = 1 whithe square with „x" → we can't "cut" it



if Alex cut this, he can't cut the 2 „x". 2 > 1 ↯



⇒ Alex can't cut that piece

It is obviously, that Alex can't cut 4 L-shaped pieces ~~with~~ in this example.

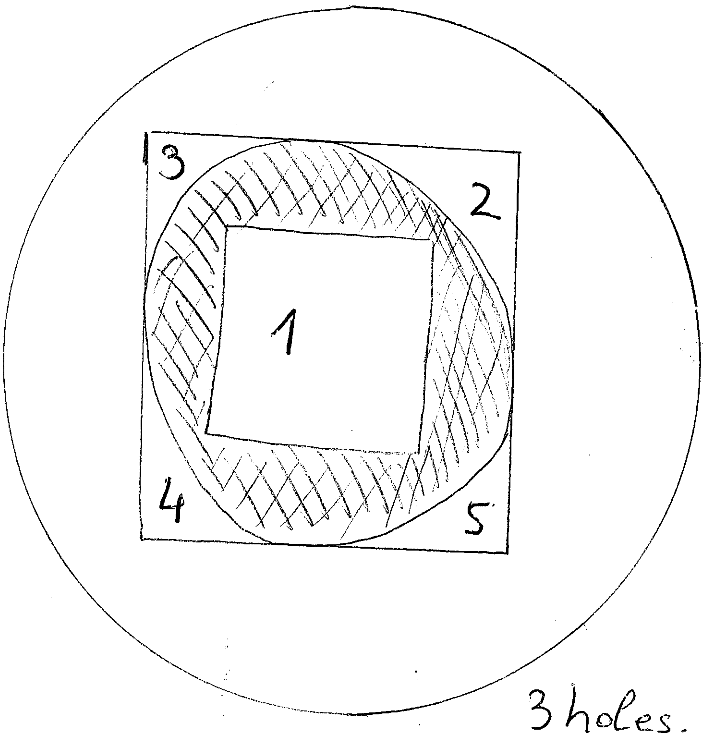
⇒ The smallest number of squares Peter should paint is 3.

Answer: 3 squares.

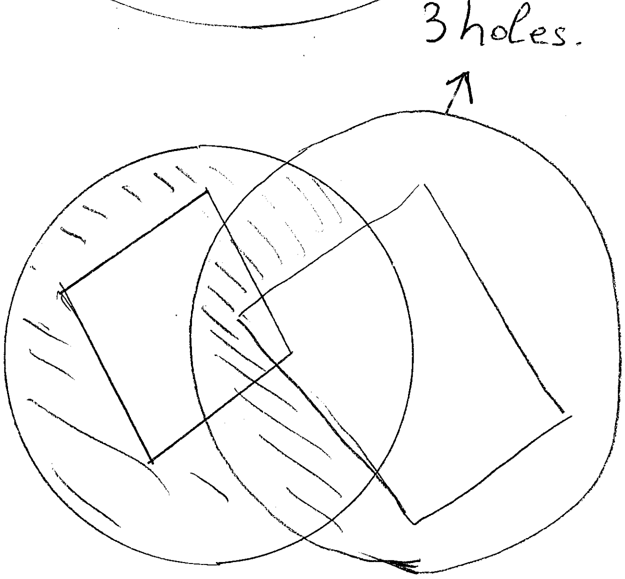
3 problem

Работа 2

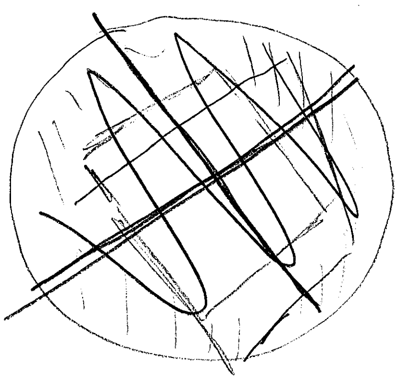
There is an example with exactly 5 holes. ~~The~~



→ 5 holes.



It is possible to make an example with more than 5 holes:



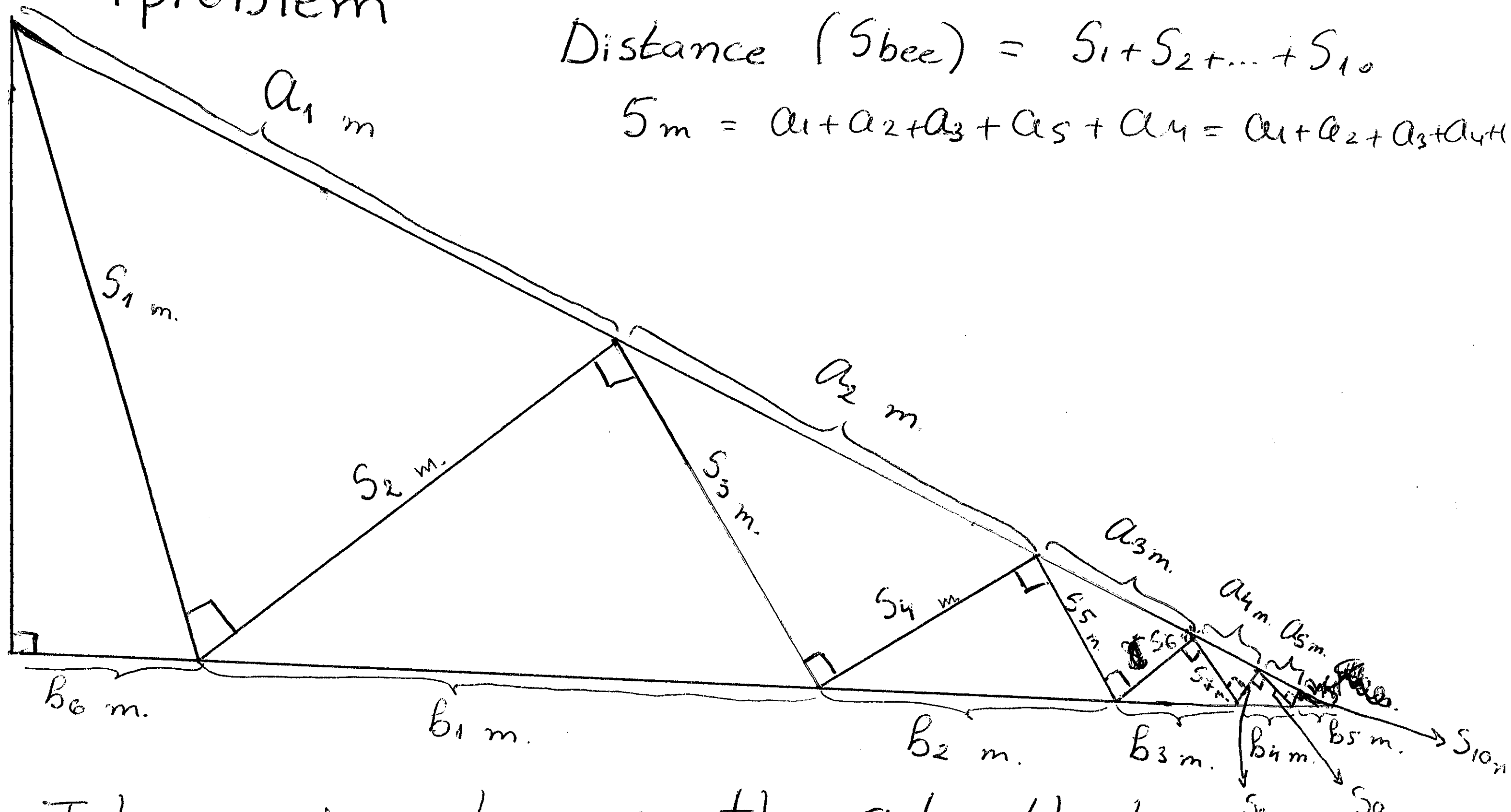
Answer: It is possible.

4 problem

Работа 2

Distance (S_{bee}) $= S_1 + S_2 + \dots + S_{10}$

$5m = a_1 + a_2 + a_3 + a_5 + a_4 = a_1 + a_2 + a_3 + a_4 + ($



I'm going to use the rule that in a right triangle the longest side is the hypotenuse, i.e:

$S_1 < a_1$
$S_2 < a_1$
$S_3 < a_2$
$S_4 < a_2$
$S_5 < a_3$
$S_6 < a_3$
$S_7 < a_4$
$S_8 < a_4$
$S_9 < a_5$
$S_{10} < a_5$

→ a_1, a_2, a_3, a_4 and a_5 are hypotenuses in the triangles.

+

$S_1 + \dots + S_{10} < 2(a_1 + a_2 + a_3 + a_4 + a_5)$

$S_1 + \dots + S_{10} < 2 \cdot 5$

$S_1 + \dots + S_{10} < 10$ m. ⇒ The bee can't fly more than 10 metres - it is not possible.

Answer: It's not possible.

5 problem: Работа 1

If „x" is an odd number => $x+2, x+4, \ldots,$ $x+16$ and $x+18$ are odd numbers => they're not divisible by an even number. We can make „x" to be an odd number and:

„$\vdots$" = is divible by

$(x+1) \vdots 19$
$(x+3) \vdots 17$
$(x+5) \vdots 15$
$\vdots$
$(x+15) \vdots 5$
$(x+17) \vdots 3$

So, =>

$(x+20) \vdots 19$
$(x+20) \vdots 17$
$\vdots$
$(x+20) \vdots 3$
$\Downarrow$

$$(x+20) \vdots GCD(\underbrace{3, 5, 7, \ldots, 19}_{\text{odd number}})$$

=> If $x = GCD(3, 5, 7, \ldots, 19) - 20$ exactly half of the propositions are correct.

If $x = GCD(3; 5; 7; \ldots; 19) - 20$ we have that exactly half of the propositions are correct.

There is a such positive number „x". Example: $x = GCD(3; 5; \ldots; 19) - 20$

The 1^{st}, 3^{rd}, 5^{th}, ..., 17^{th} propositions are correct.

Answer: There is.