

Shoshana Elgart, Final Round Solutions (R6)

#1. The fox stole 240 grams altogether.

To determine this, let us assume that the first crow ate ' x ' grams of cheese. Then, ' $100 - x$ ' grams are stolen by the fox.

The second crow ate twice less cheese, or $\frac{x}{2}$. However, it started with twice as much (200 grams), so the fox stole $200 - \frac{x}{2}$ from the second crow.

This is three times as much as $100 - x$, so:

$$3(100 - x) = 200 - \frac{x}{2}$$

$$300 - 3x = 200 - \frac{x}{2}$$

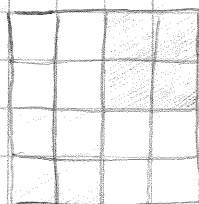
$$\frac{5x}{2} = 100$$

$x = 40$, so the first crow ate 40 grams of cheese. Altogether, the fox would steal $(100 - x) + (200 - \frac{x}{2})$, or $60 + 180$, or 240 grams of cheese. Thus, the answer is 240. ■

#2. Peter needs to color two squares in order for Alex's task to be impossible.

To approach this problem, let us first determine why one is impossible, and then determine why particular combination of two works.

Below, I've drawn one such square board.



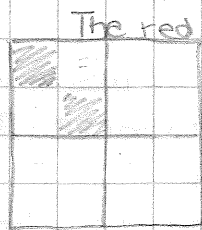
To prove why one square is impossible, I've divided the board into four quadrants.

In each quadrant, there are several ways to create an 'L-shape'.

If only one square is taken (turned red) it will belong to one quadrant and the others will remain untouched (allowing us to take three 'L-shapes' from them). In the fourth quadrant, however, only three squares will remain, enough for a fourth

L-piece. Thus, this is impossible with 1.

To show how Peter can do this with two, let's take two squares from one of the quadrants instead of one (two diagonal squares).

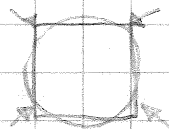


The red squares are marked gray.

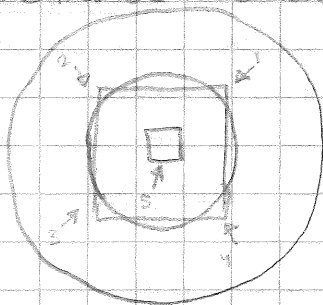
Looking at this board, it is clearly impossible to create 4 'L'-shaped pieces. Three can be formed, but a fourth one will not fit.

Thus, Peter needs to take two squares.

3. No, this is impossible. 'Holes' will occur when squares intersect with each other, or with circles. The maximal number of intersection holes occur when a square and a circle intersect (4 holes, see below).

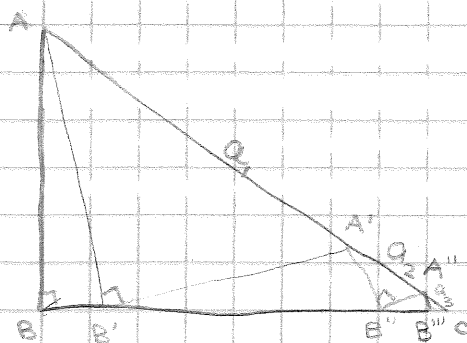


Since the centers of the square and the circle coincide, only one more hole is possible. This makes five holes (see below).



More holes are impossible, as a circle and square cannot intersect in more places. Thus, only five holes are possible.

#4. It is not possible. In the picture below, I have labelled the regions necessary to determine this.



Since the bee will only hit ten times, the length $q_1 + q_2 + q_3 + \dots + q_5$ is less than the hypotenuse of this triangle or 5.

$AB' + BA' + AB'' + \dots + A'''B$ (the length that the bee flies) is the very greatest $\sqrt{2}$ as much as $q_1 + q_2 + q_3 + q_4 + q_5$. (This is because, as a right triangle, $AB' + B'A'$ is at most $\sqrt{2} \cdot q_1$).

Thus, the length that the bee flies is LESS than $5\sqrt{2}$, and most certainly less than

10. $\square$

#5. Yes, there is.

Before commencing to solve this problem, it is important to note that -20 (though it is not positive) fulfils all of the criteria. Suppose a number 'x' does this for all odd numbers.

$x+1$ is divisible by 19

$x+3$ is divisible by 17

$x+5$ is divisible by 15

and so on and so forth.

As this is also true for -20 , $x - (-20)$ will be divisible by every odd number on the list (3, 5, 7, 9, ..., 15, 17, and 19). In order to minimize $x + 20$, it has got to be the least common multiple of all of those numbers (5 · 7 · 9 · 11 · 13 · 17 · 19).

Thus, x exists and it is:

$$(5 \cdot 7 \cdot 9 \cdot 11 \cdot 13 \cdot 17 \cdot 19) - 20.$$