$\boxed{1}$ Ha, mumkin. A uchdan AC tomon bilan $60°$ li burchak hosil qiladigan chiziqli d) yo'nalishida uchib boshlasa bu masofa 12 dan oshadi. Shu masofani hisoblaylik. Bu masofalar geometrik progressiya tashqil qilganligi uchun b_1 deb eng kattasini olaylik.

$$\frac{BC}{AC}=\frac{5}{b_1} \Rightarrow b_1=\frac{5\cdot\sqrt{3}}{2\sin 75°}.$$

2-marotaba chizganda muntazam uchburchak hosil bo'ladi. Demak, $2b_1=\frac{5\sqrt{3}}{\sin 75°}$. $S=\frac{b_1}{1-q}$.

$$q=\frac{\sin 15°}{\sin 75°}=tg 15°=2-\sqrt{3}.$$

$$S=\frac{\frac{5\sqrt{3}}{\sin 75°}}{1-2+\sqrt{3}}=\frac{5\sqrt{3}\cdot(\sqrt{3}+1)}{2\sin 75°}=\frac{5\cdot\sqrt{3}\cdot(\sqrt{3}+1)}{\frac{\sqrt{3}+1}{\sqrt{2}}}=5\sqrt{6}>12$$

$(5\sqrt{6})^2>(12)^2$

$150>144.$ $\Rightarrow$ Javob: Ha, mumkin.

$\boxed{2}$ $$\frac{x_1+x_2+\ldots+x_k}{k}=\overline{x_k}$$

$\overline{x_1}<\overline{x_2}$. $\frac{x_1+x_2}{2}<x_2$ $\Rightarrow x_1<x_2$. Demak, $\overline{x_1}<\overline{x_2}$

$\frac{x_1+x_2+x_3}{3}<x_3 \Rightarrow x_1+x_2<2x_3$ (bor)

$\overline{x_2} \vee \overline{x_3}$

$\frac{x_1+x_2}{2} \vee \frac{x_1+x_2+x_3}{3} \Rightarrow x_1+x_2 \vee 2x_3 \Rightarrow \overline{x_2}<\overline{x_3}.$

Huddi shunday. $\overline{x_3}<\overline{x_4}<\overline{x_5}<\overline{x_6}$. Demak, $\overline{x_1}, \overline{x_2}, \overline{x_3}, \overline{x_4}, \overline{x_5}, \overline{x_6}$ lar ichida eng kattasi $\overline{x_6}$.

Endi $\overline{x_6}, \overline{x_7}, \overline{x_8}, \overline{x_9}, \overline{x_{10}}, \overline{x_{11}}, \overline{x_{12}}$ larni taqqoslaymiz.

$\overline{x_m}>x_m$ bo'lganligi uchun $x_1+x_2+\ldots+x_{m-1}>(m-1)\cdot x_m$

$\Rightarrow$ $\overline{x_m}>\overline{x_{m+1}}>\overline{x_{m+2}}$. Demak, $\overline{x_6}>\overline{x_7}>\overline{x_8}>\overline{x_9}>\overline{x_{10}}>\overline{x_{11}}>\overline{x_{12}}$

$\Rightarrow$ Eng kattasi $\overline{x_6}$. Javob: 6-oyda.

3 N ta bola har biri turli sondagi sovga berishi kerak. Boshqa tomondan har bir bola kōpi bilan $N-1$ ta sovga bera oladi. Demak, qaysidir bola sovga bermagan, qaysidir bola 1 ta sovga bergan, qaysidir bola 2 ta sovga bergan ... oxirgi bola $N-1$ ta sovga bergan.

Jami sovgalar soni $\frac{N\cdot(N-1)}{2}$ ta va u N ga bōlinishli kerak. Chunki, hamma bir xil sovga olgan.

$$\Rightarrow \quad \frac{N\cdot(N-1)}{2\cdot N} \in \mathbb{N} \Rightarrow \frac{N-1}{2} \in \mathbb{N}.$$

$\mathbb{N}$ – natural sonlar tōplami.

$\Rightarrow$ $N-1$ – juft. $\Rightarrow$ N – toq son. Demak, barcha toq sonlar uchun bu vaziyat ōrinli bōladi.

Javob: $N=2k+1,\ k\in\mathbb{N}$.

5
$$\begin{cases} xy-2y=z+106 \\ yz+3y=z+39 \\ zx+3x=2z+438 \end{cases}$$

Agar $x=2$ bōlsa $x+106=0$ ↯. $\Rightarrow x\neq 2$.

1-tenglamadan $y$ni x orqali topamiz.

$$y=\frac{x+106}{x-2}. \quad x\neq 2$$

3-tenglamadan $z$ni x orqali topamiz.

$$z=\frac{438-3x}{x-2}.$$

Bularni 2-tenglamaga qōysak,

$$\frac{x+106}{x-2}\cdot\frac{438-3x}{x-2}=\frac{438-3x}{x-2}+39-3\cdot\frac{x+106}{x-2}$$

$$\frac{x+106}{x-2}\cdot\frac{146-x}{x-2}=\frac{146-x}{x-2}+13-\frac{x+106}{x-2}.$$

$$\frac{(x+106)(146-x)}{(x-2)^2}=\frac{40-2x}{x-2}+13$$

Umumiy maxraj berib soddalashtirsak,

$x^2-4x-1292=0$ ga kelamiz.

$\Rightarrow D=4^2\cdot 18^2$

$\Rightarrow \left(x_{1,2}=\dfrac{12\pm 72}{2}= \begin{cases} 42 \\ -30 \end{cases} \right.$ $x_1=42$, $x_2=-30$

$y_1=\dfrac{x_1+106}{x_1-2}=\dfrac{42+106}{42-2}=\dfrac{148}{40}=\dfrac{27}{10}$.

$y_2=\dfrac{x_2+106}{x_2-2}=\dfrac{-30+106}{-32}=-\dfrac{76}{32}=-\dfrac{19}{8}$

$z_1=\dfrac{432-3x_1}{x_1-2}=\dfrac{438-126}{40}=\dfrac{312}{40}=\dfrac{78}{10}=\dfrac{39}{5}$.

$z_2=\dfrac{438-3x_2}{x_2-2}=\Big)$

$x_{1,2}=\dfrac{4\pm 72}{2}= \begin{cases} 39=x_1 \\ -34=x_2 \end{cases}$

$y_1=\dfrac{39+106}{37}=\dfrac{145}{37}$.

$y_2=\dfrac{-34+106}{-36}=\dfrac{72}{-36}=-2$

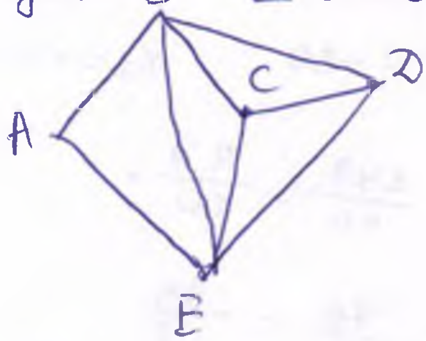
$z_1=\dfrac{438-117}{37}=\dfrac{321}{37}$.

$z_2=\dfrac{438+102}{-36}=\dfrac{540}{-36}=-15$

Javob: $\left(39;\ \dfrac{145}{37};\ \dfrac{321}{37}\right)$

$(-34;\ -2;\ -15)$

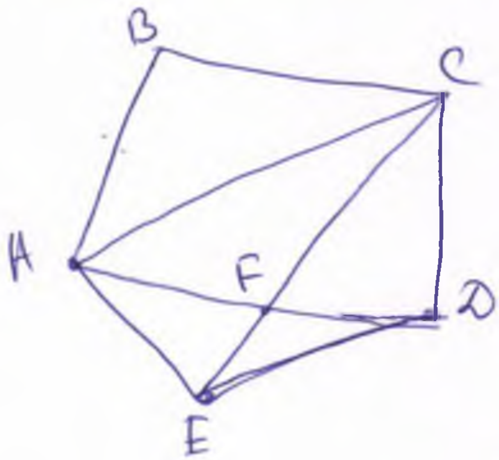
4 Agar bu 5 ta nuqtani ketma-ket tutashtirib chiqilgan paytda qo botiq 5 burchak hosil bo'lsa masala yechiladi. Chunki, qaysidir nuqta boshqa nuqtalardan hosil bo'lgan B Δ ichiga tushadi.



$$\Rightarrow + \begin{cases} S_{BCE} \geqslant 2. \\ S_{BCD} \geqslant 2. \\ S_{CDE} \geqslant 2 \end{cases} \Rightarrow S_{BED} \geqslant 6.$$

Isbotlandi.

Demak, biz qavariq holini qarashimiz yetarli. Qavariq 5 burchak uchun shunday 2 ta diagonal topiladiki, bu diagonallardan 1 tasi 2-sini $\frac{1}{3}$ dan kichik bo'lgan nisbatdagi kesmalarga ajratadi.



$$\frac{EF}{FC} \leq \frac{1}{3}.$$

$S_{ACE} \geqslant 2$

$\Rightarrow S_{AFC} \geqslant \frac{3}{2}.$ (1)

$S_{EDC} \geqslant 2.$

$\Rightarrow S_{CDF} \geqslant \frac{3}{2}$ (2)

(1) va (2) ga ko'ra $S_{ACD} \geqslant \frac{3}{2} + \frac{3}{2} = 3.$

$\Rightarrow S_{ACD} \geqslant 3. \Rightarrow$ shunday 3 ta nuqta topiladi.

Isbot tugadi.

5 davomi.

Umumiy maxraj bersak,

$$(x+106)(146-x) = (40-2x)\cdot(x-2) + 13\cdot(x-2)^2$$

$$146x + 146\cdot 106 - x^2 - 106x = 40x - 2x^2 - 80 + 4x + 13\cdot(x^2-4x+4)$$

$$40x + 146\cdot 106 - x^2 = 44x - 2x^2 - 80 + 13x^2 - 52x + 52$$

$$12x^2 - 48x - 106\cdot 146 - 28 = 0$$

$$12x^2 - 48x - 2364 = 0$$

$$6x^2 - 69x - 1182 = 0$$

$$2x^2 - 23x - 394 = 0$$

$$12x^2 - 48x - 15504 = 0$$

$$3x^2 - 12x - 3876 = 0$$

$$D = 144 + 12\cdot 3876 = 46656$$

$$x^2 - 4x - 1292 = 0$$

$$D = 16 + 1292\cdot 4 = 4^2\cdot 18^2$$

$$x_{1,2} = \frac{12 \pm 72}{6} = \frac{84}{6} = 14 = x_1$$

$$-\frac{60}{6} = -10 = x_2$$

$$y_1 = \frac{106+14}{4} = \frac{120}{4} = 30$$

$$y_2 = \frac{106-10}{-12} = \frac{96}{-12} = -8$$

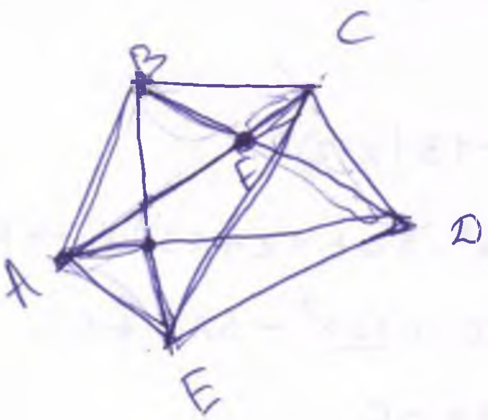
$$z_1 = \frac{438-42}{4} = \frac{396}{4} = 99$$

$$z_2 = \frac{438+30}{-12} = -\frac{468}{12} = -\frac{117}{3} = -39$$

$(14; 30; 99)$

$(-10; -8; -39)$

4



$\frac{7}{3}$