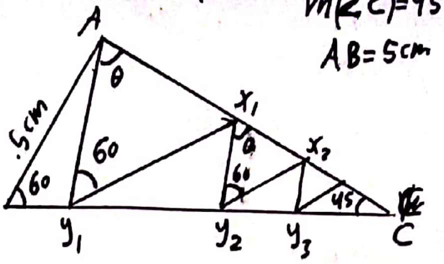
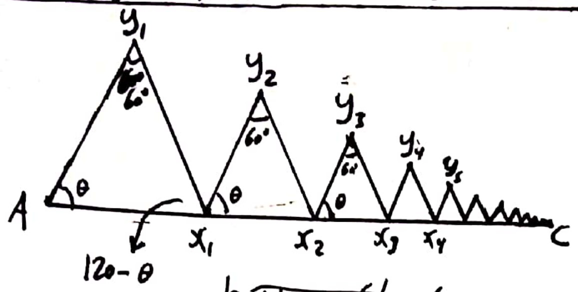
Q:1

$m\angle B = 60$ $\quad m\angle C = 45$ $\quad AB = 5cm$



From the drawing we can notice that all the triangles that have a base on $\overline{AC}$ segment are all similar due to the similarity of their angels

lets have a closer look on these triangels



Let the distance that the bee cover be (d) So $d = \overline{Ay_1} + \overline{y_1x_1} + \overline{x_1y_2} + \overline{y_2x_2} \cdots\cdots$

but as all triangles are similar we have that $\overline{Ay_1} + \overline{y_1x_1} = K(\overline{Ax_1})$ and $\overline{x_1y_2} + \overline{y_2x_2} = K(\overline{x_1x_2})$, $\overline{x_2y_3} + \overline{y_3x_4} = K(\overline{x_2x_3})$

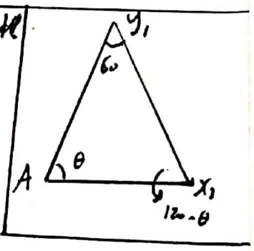
where K is a constant So we can say that

$d = K(\overline{Ax_1}) + K(\overline{x_1x_2}) + K(\overline{x_2x_3}) + \cdots = K(\overline{Ax_1} + \overline{x_1x_2} + \overline{x_2x_3} + \cdots\cdots) = K(\overline{AC})$

using sine rule $\frac{\overline{AC}}{\sin(B)} = \frac{\overline{AB}}{\sin(C)}$ $\therefore \frac{\overline{AC}}{\sin(60)} = \frac{5}{\sin(45)}$ $\therefore \frac{2}{\sqrt{3}}\overline{AC} = 5\sqrt{2}$ $\therefore \overline{AC} = \frac{5\sqrt{6}}{2}$

So we have that $d = \frac{5\sqrt{6}}{2}K$ $\quad \boxed{d = \frac{5\sqrt{6}}{2}K}$

now we should find (K) so that it be as maximum as possible



* From the left most triangle we have $K = \frac{Ay_1 + y_1x_1}{Ax_1}$

using sine rule $\frac{Ax_1}{\sin(60)} = \frac{x_1y_1}{\sin\theta} = \frac{Ay_1}{\sin(120-\theta)}$

$\therefore x_1y_1 = \frac{Ax_1\sin\theta}{\sin 60}$ $\quad Ay_1 = \frac{Ax_1\sin(120-\theta)}{\sin 60}$ $\quad \therefore K = \frac{Ay_1 + y_1x_1}{Ax_1} = \frac{\sin\theta}{\sin 60} + \frac{\sin(120-\theta)}{\sin 60}$

$\therefore K = \frac{2}{\sqrt{3}}\sin\theta + \frac{\sin(120)\cos\theta - \sin\theta\cos(120)}{\sin 60} = \frac{2}{\sqrt{3}}\sin(\theta) + \cos\theta + \frac{1}{\sqrt{3}}\sin\theta = \frac{3}{\sqrt{3}}\sin\theta + \cos\theta = \sqrt{3}\sin\theta + \cos\theta$

$\boxed{K = \sqrt{3}\sin\theta + \cos\theta}$ we can find the maximum value of K using derivative

$K' = \sqrt{3}\cos\theta - \sin\theta$ $\quad$ let $K' = 0$ $\quad \therefore \sqrt{3}\cos\theta = \sin\theta$ $\quad \therefore \tan\theta = \sqrt{3}$ $\quad \therefore \theta = 60°$

$f'(x)$ +++ ---
x $\quad \frac{\pi}{3}$

$\therefore$ the maximum value of (K) is at $\theta = 60°$ $\quad K = \sqrt{3}\sin(60) + \cos(60)$

$\therefore K = \sqrt{3} \times \frac{\sqrt{3}}{2} + \frac{1}{2} = \frac{3}{2} + \frac{1}{2} = 2$ $\quad$ so $K_{max} = 2$

$\therefore d_{max} = \frac{5\sqrt{6}}{2} \times K_{max} = \frac{5\sqrt{6}}{2} \times 2 = 5\sqrt{6}$ $\quad \boxed{d_{max} = 5\sqrt{6} \simeq 12.2}$

So yes it is possible for the bee to fly more than 12 meters

$\boxed{\text{yes, it is possible}}$

$\boxed{Q:2}$ $\overline{x_1} = x_1$ $\overline{x_n} = \left(\overline{x_{n-1}} + \frac{1}{n-1} x_n\right) \times \frac{n-1}{n}$

from 2 to 6 $\overline{x_k} < x_k$

$\star$ $\overline{x_2} < x_2$ $\frac{1}{2}x_1 + \frac{1}{2}x_2 < x_2$ $x_1 < x_2$ $\boxed{\overline{x_1} < x_2}$

$\star$ $\overline{x_3} < x_3$ $\frac{1}{3}x_1 + \frac{1}{3}x_2 + \frac{1}{3}x_3 < x_3$ $x_1 + x_2 < 2x_3$ $\frac{x_1+x_2}{2} < x_3$

$\boxed{\overline{x_2} < x_3}$

$\star$ $\overline{x_4} < x_4$ $\frac{1}{4}x_1 + \frac{1}{4}x_2 + \frac{1}{4}x_3 + \frac{1}{4}x_4 < x_4$ $x_1 + x_2 + x_3 < 3x_4$ $\frac{x_1+x_2+x_3}{3} < x_4$

$\therefore$ $\boxed{\overline{x_3} < x_4}$ and so on $\boxed{\overline{x_4} < x_5}$ $\boxed{\overline{x_5} < x_6}$

$\overline{x_2} = (\overline{x_1} + x_2) \times \frac{1}{2}$ $\therefore$ $\overline{x_2} > (\overline{x_1} + \overline{x_1}) \times \frac{1}{2}$ $\therefore \overline{x_2} > \overline{x_1}$

$\overline{x_3} = (\overline{x_2} + \frac{1}{2}x_3) \times \frac{2}{3}$ $\therefore$ $\overline{x_3} > (\overline{x_2} + \frac{1}{2}\overline{x_2}) \times \frac{2}{3}$ $\therefore \overline{x_3} > \overline{x_2} \longrightarrow \overline{x_3} > \overline{x_2} > \overline{x_1}$

and so on $\boxed{\overline{x_6} > \overline{x_5} > \overline{x_4} > \overline{x_3} > \overline{x_2} > \overline{x_1} \quad (1)}$

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from 7 to 12 $\overline{x_k} > x_k$

$\overline{x_7} > x_7$ $\frac{6}{7}(\overline{x_6} + \frac{1}{6}x_7) > x_7 \longrightarrow \boxed{\overline{x_6} > x_7}$

$\overline{x_8} < x_8$ $\frac{7}{8}(\overline{x_7} + \frac{1}{7}x_8) > x_8 \longrightarrow \boxed{\overline{x_7} > x_8}$

and so on $\boxed{\overline{x_8} > x_9}$ $\boxed{\overline{x_9} > x_{10}}$ $\boxed{\overline{x_{10}} > x_{11}}$ $\boxed{\overline{x_{11}} > x_{12}}$

$\star$ $\overline{x_7} = \frac{6}{7}(\overline{x_6} + \frac{1}{6}x_7)$ $\therefore$ $\overline{x_7} < \frac{6}{7}(\overline{x_6} + \frac{1}{6}\overline{x_6})$ $\therefore$ $\overline{x_7} < \overline{x_6}$

$\star$ $\overline{x_8} = \frac{7}{8}(\overline{x_7} + \frac{1}{7}x_8)$ $\therefore \overline{x_8} < \frac{7}{8}(\overline{x_7} + \frac{1}{7}\overline{x_7})$ $\therefore$ $\overline{x_8} < \overline{x_7} \rightarrow \overline{x_8} < \overline{x_7} < \overline{x_6}$

and so on ~~$\overline{x_{12}} < \overline{x_{11}} < \overline{x_{10}} < \overline{x_9} < \overline{x_8} < \overline{x_7} < \overline{x_6}$~~

$\boxed{\overline{x_{12}} < \overline{x_{11}} < \overline{x_{10}} < \overline{x_9} < \overline{x_8} < \overline{x_7} < \overline{x_6} \quad (2)}$

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from (1) and (2) $\overline{x_{12}} < \overline{x_{11}} < \overline{x_{10}} < \overline{x_9} < \overline{x_8} < \overline{x_7} < \overline{x_6} > \overline{x_5} > \overline{x_4} > \overline{x_3} > \overline{x_2} > \overline{x_1}$

so $\overline{x_6}$ is the maximal average

so at the 6th month (June) there will be the highest average

$\boxed{Q:3}$ every one can give one gift ~~to~~ at max to any one and all of them gave diffrent number so ~~they~~ every one of them must give a number from 0 to $N-1$ gifts and it cant be repeated

so the ~~max number of~~ sum of all gifts $0+1+2+3 \dots + (N-1)$

number of gifts $= \frac{(N-1)^2 + (N-1)}{2} = \frac{N^2 - 2N + 1 + N - 1}{2} = \frac{N^2 - N}{2}$

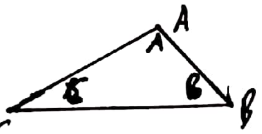
while all of them must recive the same number of gifts which is $= \frac{N^2 - N}{2} \div N = \frac{N-1}{2}$

So this can happen if and only if N is odd So that $N-1$ can be divisable by 2

So ~~it~~ this can happen when N is odd

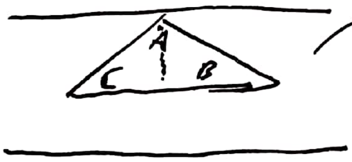
Q: 4 [1]

suppose the perfect case where we have the smallest triangle ~~is a triangle~~ is at least one triangle of area (2) and if this case ~~didnt~~ have any triang that has area(3) then every other case will have a similar triang as well because it is the same as this case but ~~scalled~~ scalled up

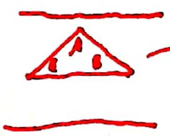
So we suppose that our triang of area 2 is $\triangle ABC$  every other triangle must have area bigger than or equal to (2)

So we have some limits that every ~~triangle~~ point inside it make area smaller than (2) and any ~~triangle~~ point outside it ~~make area~~ its area bigger than (2) so ~~all the points~~ the other 2 points must be outside this limits

we can easily form this limit by drawing 2 parallel straight line around each side of the triangle in condition that the both ~~par~~ straight lines are as far away from this side as the hight that is perpendicular to this side (Because our triangle is already of area 2 so any other triangle must be bigger than it)

 → example of the limit using $\overline{BC}$ side

there will also be another limit that any point put inside it ~~is~~ make a triangle of area less than (2) and any point ~~inside~~ put outside it makes a triangle of area more than (3) we can construct this limit by doing the same thing but this time the distance of the 2 parallel line is $\frac{3}{2}$ the hight of this side

 → example of the limit using side $\overline{CB}$

(if we are able to put the other 2 points outside the Black limit and inside the Red limit we will have no triangle of are three But we cant and thats what we will show)

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Q:4[2] Black ~~bim~~ parts are the parts in which there will be a triangle less than (2) if any point is put inside it

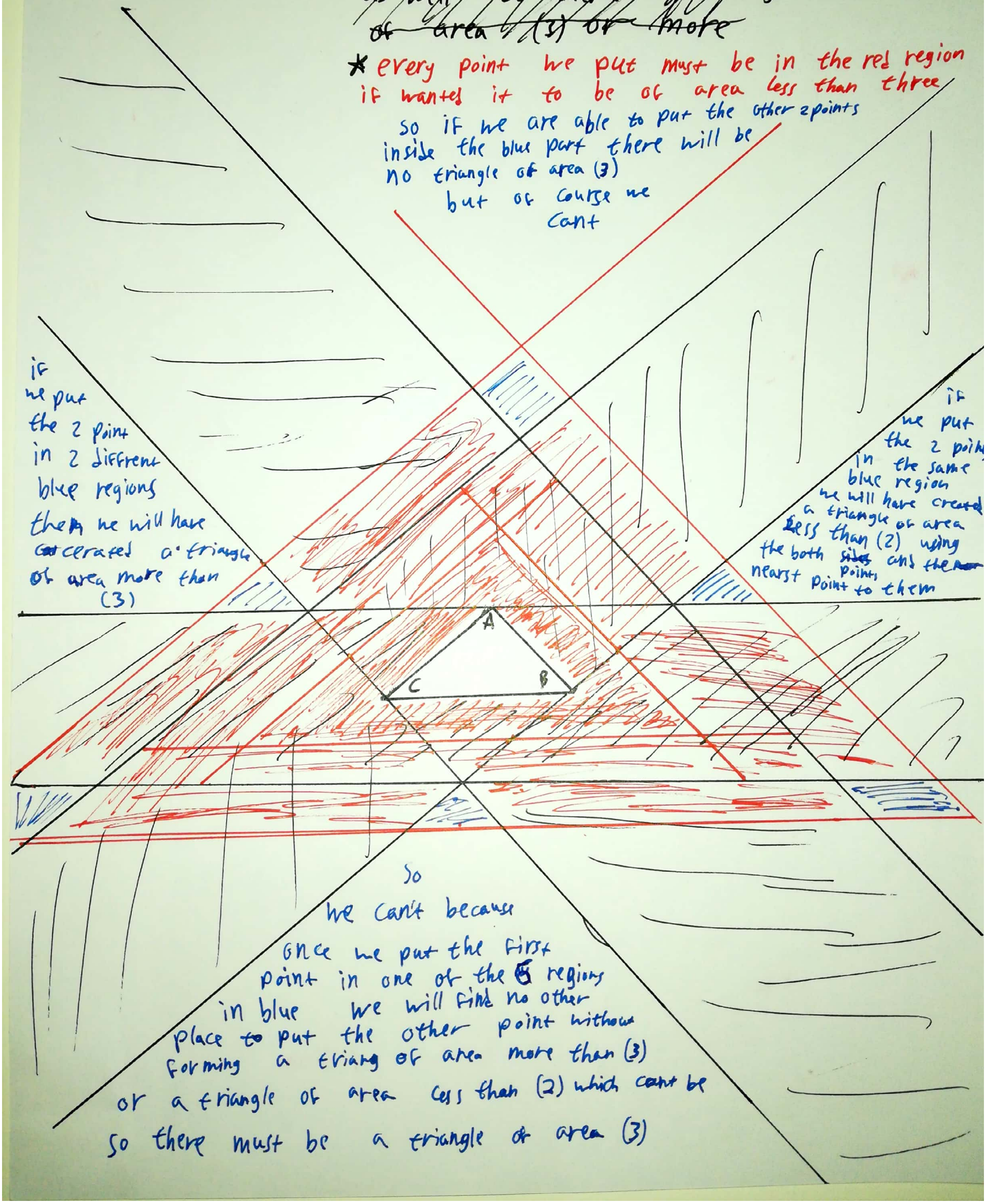
red part is the part in which ~~any~~ a triangle of area 3 or more will be formed if any point is put outside it

Blue parts is outside Black and inside red

* every point we put must be outside the Black part ~~so it is outside the red part as well, so there will always be a triangle of area (3) or more~~

* every point we put must be in the red region if wanted it to be of area less than three

so if we are able to put the other 2 points inside the blue part there will be no triangle of area (3) but of course we cant



if we put the 2 point in 2 diffrent blue regions then we will have ~~cr~~ cerated a triangle of area more than (3)

if we put the 2 points in the same blue region we will have created a triangle of area less than (2) using the both ~~sides~~ points and the nearst point to them

so we can't because once we put the first point in one of the 6 regions in blue we will find no other place ~~to~~ put the other point without forming a triang of area more than (3) or a triangle of area less than (2) which cant be

so there must be a triangle of area (3)

$\boxed{Q:5}$ 1-) $xy-2y-x=106$ 2-) $yz+3y-z=39$ 3-) $zx+3x-2z=438$

3-) $x=\frac{2y+106}{y-1}$ 4-) $y=\frac{z+39}{z+3}$ 5-) $z=\frac{438-3x}{x-2}$

from (5,4) $\boxed{4 \quad y=\frac{z+39}{z+3}=1+\frac{36}{z+3}}$ $\boxed{5 \quad z=-3+\frac{\cancel{438}\,432}{x-2}}$

$\therefore y=1+\frac{36}{\left(\frac{432}{x-2}\right)}=1+\frac{x-2}{12}$ $\quad$ I

from (I, 3) $\boxed{3 \quad x=2+\frac{108}{y-1}}$ $\boxed{I \quad y=1+\frac{x-2}{12}}$

$\therefore x=2+\frac{108}{\left(\frac{x-2}{12}\right)}=2+\frac{1296}{x-2}$ $\quad x-2=\frac{1296}{x-2}$

$(x-2)^2=1296$ $\quad x-2=\pm 36$ $\quad x=38$ or $x=-34$

if $x=38$ $\quad y=1+\frac{38-2}{12}=1+3=4$ $\quad z=-3+\frac{432}{38-2}=-3+12=9$

if $x=-34$ $\quad y=1+\frac{-34-2}{12}=-2$ $\quad z=-3+\frac{432}{-34-2}=-15$

So $x=38$ $y=4$ $z=9$

or $x=-34$ $y=-2$ $z=-15$

So solution set is $\{(38,4,9),(-34,-2,-15)\}$