R10 Problem 1

$AC - ?$

Let the rate of speed's increase be $- a$

We know $t_1 = 3h$; $t_2 = 2h$; $v_0 = 90 \frac{Km}{h}$ $v = 110 \frac{Km}{h}$

• $v = v_0 + a(t_1 + t_2) \Rightarrow$

$$a = \frac{v - v_0}{t_1 + t_2} = \frac{110 \frac{Km}{h} - 90 \frac{Km}{h}}{3h + 2h} = 4 \frac{Km}{h^2}$$

• $AB = v_0 t_1 + \frac{a t_1^2}{2}$ (1)

↑ the formula for the motion at constant increase rate

$BC = v_B \cdot t_2 + \frac{a t_2^2}{2}$

$v_B = a t_1 + v_0$ $\Rightarrow$ $BC = a t_1 t_2 + v_0 t_2 + \frac{a t_2^2}{2}$ (2)

From (1) and (2)

So.

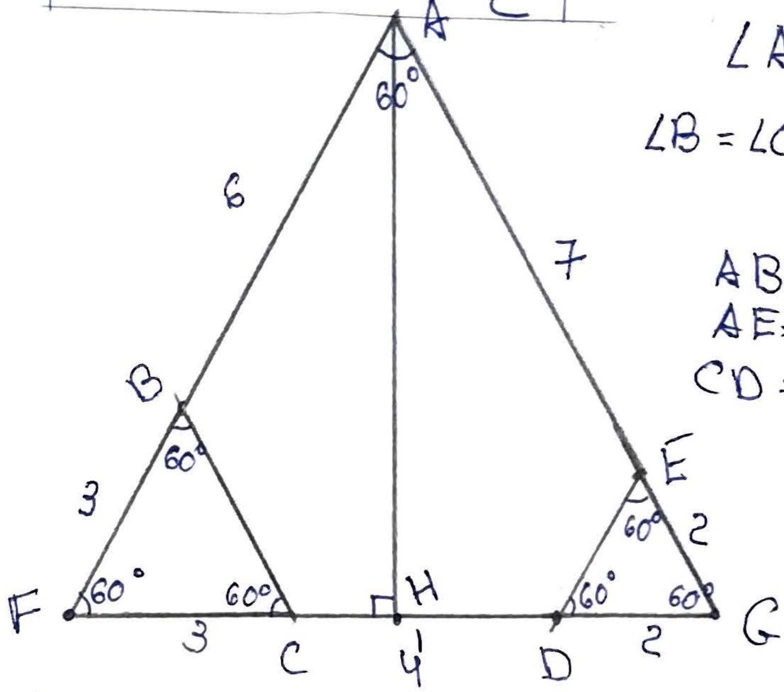
$AB - BC = AC = v_0 t_1 + \frac{a t_1^2}{2} - a t_1 t_2 - v_0 t_2 - \frac{a t_2^2}{2}$.

$AC = v_0 (t_1 - t_2) + \frac{a}{2}(t_1^2 - t_2^2) - a t_1 t_2$

$AC = 90 \frac{Km}{h}(3h - 2h) + \frac{4 Km}{2 h^2}(9h^2 - 4h^2) - 4 \frac{Km}{h^2} \cdot 3h \cdot 2h = 90 Km + 10 Km - 24 Km$

$\boxed{AC = 76 Km}$

R10 Problem 2



$\angle A = 60°$

$\angle B = \angle C = \angle D = \angle E$

$AB = 6$
$AE = 7$
$CD = 4$

$AH \perp CD$

$AH - ?$

$\angle A + \angle B + \angle C + \angle D + \angle E = \angle A + 4 \cdot \angle B = 540°$

$4\angle B = 480° \Rightarrow \quad \angle B = \angle C = \angle D = \angle E = 120°$

Let $AB \cap CD = \{F\}$ and $AE \cap CD = \{G\}$

In triangle $\triangle BFC$: $\angle FBC = \angle FCB = 60° \Rightarrow \triangle FBC$ is equilateral

and in $\triangle DEG$: $\angle DEG = \angle EDG = 60° \Rightarrow \triangle DEG$ is equilateral too

$\angle AFG = \angle AGF = \angle FAG = 60° \Rightarrow \triangle AFG$ is equilateral

so $AF = AG = GF$

$$\begin{cases} AG = 7 + EG = 7 + DG \\ GF = CF + 4 + DG \\ AG = GF \end{cases} \Rightarrow \quad CF = 3 \Rightarrow AF = 6 + 3 = 9$$

SO $AH = AF \cdot \sin 60° = 9 \cdot \frac{\sqrt{3}}{2}$

$$\boxed{AH = \frac{9\sqrt{3}}{2}}$$

R 10 Problem 3

$a, b > 0$

Prove: $(a^{2018} + b^{2018})^{2019} > (a^{2019} + b^{2019})^{2018}$

Let's assume that $a \geq b$. Then the inequality is equivalent to:

$$\cancel{a^{2018 \cdot 2019}} \cdot \left(1 + \left(\frac{b}{a}\right)^{2018}\right)^{2019} > \cancel{a^{2018 \cdot 2019}} \left(1 + \left(\frac{b}{a}\right)^{2019}\right)^{2018}$$

$$\left(1 + \left(\frac{b}{a}\right)^{2018}\right)^{2019} > \left(1 + \left(\frac{b}{a}\right)^{2019}\right)^{2018}$$

because

$$a \geq b \Leftrightarrow 1 \geq \frac{b}{a} \Rightarrow$$

$$\left(\frac{b}{a}\right)^{2018} \geq \left(\frac{b}{a}\right)^{2019} \Rightarrow 1 + \left(\frac{b}{a}\right)^{2018} \geq 1 + \left(\frac{b}{a}\right)^{2019}$$

because $1 + \left(\frac{b}{a}\right)^{2018} > 1$ we have that

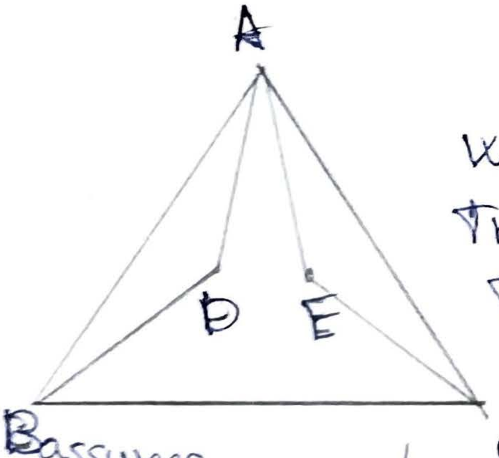
$$\left(1 + \left(\frac{b}{a}\right)^{2018}\right)^{2019} > \left(1 + \left(\frac{b}{a}\right)^{2018}\right)^{2018} \geq \left(1 + \left(\frac{b}{a}\right)^{2019}\right)^{2018}$$

Q.E.D

R10 Problem 4

· We will consider the convex hull of this 5 points, They we will have 3 cases.

Case 1: A triangle with 2 points inside it



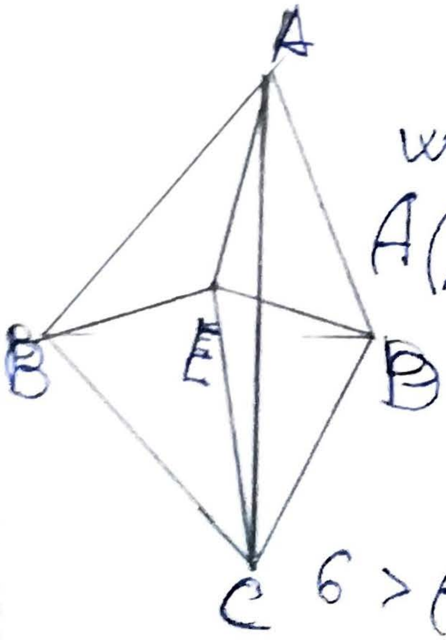
assume D is closer to AB than E

We know that the area of the triangles ABD and AEC is greater than 2

$\mathcal{A}_{(ABC)} > \mathcal{A}_{(ADB)} + \mathcal{A}_{(AEC)} > 2+2>3$

So triangle ABC satisfies.

Case 2: A convex quadrilateral with a point inside it



if we assume that $A_{(ABC)}, A_{(ACD)} < 3$

we will have: $A(ABCD) < 3+3 = 6$, but

$A(ABCD) = A(ABE) + A(BEC) + A(CED) + A(AED)$

$> 2+2+2+2 = 8$. So

$6 > A(ABCD) > 8$ Contradiction.

$\boxed{\text{R10 Problem 5}}$

n - cubo if $\exists\, m$ s.t. $\quad n, m \in \mathbb{N}$

$m^3 = n^3 + 13n - 273$

Now we will check by hand the cases $n = 1, 2, 3, 4, 5, 6, 7$

$1^3 + 13\cdot 1 - 273 = -259 \neq m^3$

$2^3 + 13\cdot 2 - 273 = -239 \neq m^3$

$3^3 + 13\cdot 3 - 273 = -207 \neq m^3$

$4^3 + 13\cdot 4 - 273 = -157 \neq m^3$

$5^3 + 13\cdot 5 - 273 = -83 \neq m^3$

$6^3 + 13\cdot 6 - 273 = 21 \neq m^3$

$7^3 + 13\cdot 7 - 273 = 161 \neq m^3$

and see that there are no solutions. Now supause that $n > 7$.

We have that:

$$(n+1)^3 > n^3 + 13n - 273 > (n-2)^3$$

- $(n+1)^3 > n^3 + 13n - 273 \iff 3n^2 - 10n + 274 > 0$

  $3n^2 - 10n + 274 > 3n\cdot 7 - 10n + 274 = 11n + 274 > 0$
- $n^3 + 13n - 273 > (n-2)^3 \iff 6n^2 + n - 265 > 0$

  $6n^2 + n - 265 > 6\cdot 7^2 + 7 - 265 = 36 > 0$

So we have only 2 cases:

- $n^3 + 13n - 273 = n^3 \iff 13n = 273 \iff \underline{n = 21}$ ← cubo
- $n^3 + 13n - 273 = (n-1)^3 \iff 3n^2 + 10n - 272 = 0$

  $\iff (n-8)(3n+34) = 0 \iff \underline{n = 8}$ ← cubo

So the sum of all the cubos is:

$$21 + 8 = \boxed{29}$$