5 $m^3 = n^3+13n-273$

$n>21$ bo'lsa $n^3 < n^3+13n-273$

$\forall n\in N$ uchun $3n^2-10n+274>0 \Rightarrow n^3+13n-273 < n^3+3n^2+3n+1=(n+1)^3$

ya'ni $n^3<m^3<(n+1)^3$ zidlik

Demak, $n\le 21$, $n^3+13n-273>0 \Rightarrow n\ge 6$

$n>8$ da $3n^2+10n-272>0 \Rightarrow n^3-3n^2+3n-1 < n^3+13n-273 \Rightarrow (n-1)^3<m^3<(n+1)^3$

$\Rightarrow m=n$ $\quad \cancel{n^3}=\cancel{n^3}-13n+273 \Rightarrow \underline{n=21}$

$6\le n\le 8$ da $\quad n=6 \Rightarrow m^3=21 \quad \varnothing$

$n=7 \Rightarrow m^3=7\cdot(7^2+13)-273=7\cdot 62-273=434-273=161 \quad \varnothing$

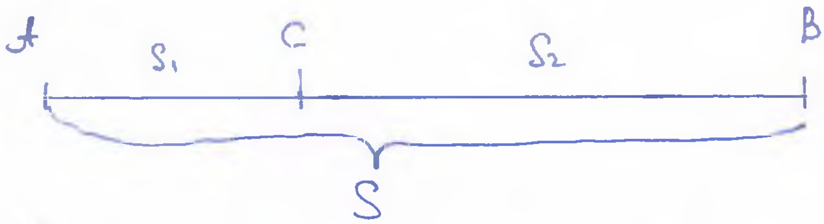
$n=8 \Rightarrow m^3 = 8\cdot(8^2+13)-273=8\cdot77-273=7\cdot(88-39)=7\cdot49=7^3$

$\underline{n=8}$

Demak $n_1=8$, $n_2=21$ $\quad n_1+n_2=29$

Javob: 29.

3 1



Boshlang'ich tezligi: v km/soat $=90$ km/soat

tezlanish: v_0 km/soat deylik.

Demak $v+5v_0=110 \Rightarrow v_0=4$

$S=3\cdot(v+v+3v_0)/2 = 3\cdot(90+90+12)/2=3\cdot96=288$ km

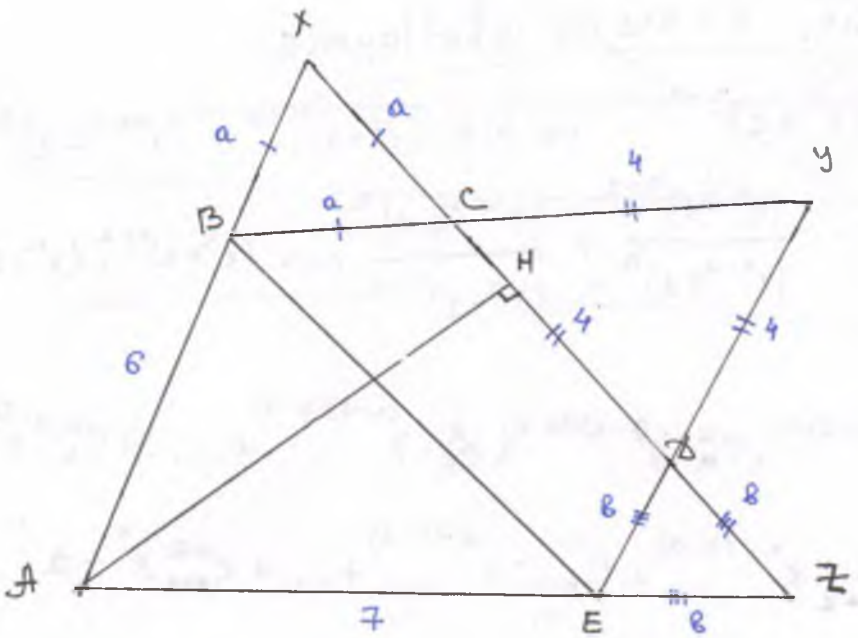
$S_2=2\cdot(v+3v_0+v+5v_0)/2 = 2\cdot(90+12+90+20)/2=90+12+90+20=$

$=180+32=212$

Demak $S_1=288-212=76$

Javob: 76 km.

(2)



$\angle A = 60^\circ$

$\angle A + \angle B + \angle C + \angle D + \angle E = 540^\circ$

Demak, $\angle B = \angle C = \angle D = \angle E = 120^\circ$

$AB \cap DC \equiv X$
$BC \cap ED \equiv Y$
$CD \cap AE \equiv Z$ bōlsin. $\angle XBC = 180^\circ - 120^\circ = 60^\circ = \angle XCB \Rightarrow \Delta BXC$ - muntazam

$\angle YCD = 180^\circ - 120^\circ = 60^\circ = \angle YDC \Rightarrow \Delta CYD$ - muntazam

$\angle ZDE = 180^\circ - 120^\circ = 60^\circ = \angle ZED \Rightarrow \Delta DZE$ - muntazam

va ΔAXZ da $\angle A = \angle X = \angle Z = 60^\circ \Rightarrow \Delta AXZ$ - muntazam

$\begin{cases} BX = XC = BC = a \\ ZD = ZE = DE = b \end{cases} \Rightarrow a + 4 + b = 7 + b$ ya'ni $XZ = AZ \Rightarrow a = 3$ va

$AX = AZ \Rightarrow 6 + a = 7 + b \Rightarrow b = a - 1 = 2 \Rightarrow AX = XZ = ZA = 9$

A nuqtadan CD gacha bōlgan masofa $AH = \frac{9\sqrt{3}}{2}$

Javob: $\frac{9\sqrt{3}}{2}$

(3) Biz masalani $\forall k$ uchun isbotlaymiz: $(a^{k-1} + b^{k-1})^k > (a^k + b^k)^{k-1}$ bunda $\forall k \in N$. Demak $\frac{(a^{k-1} + b^{k-1})^k}{b^{k(k-1)}} > \frac{(a^k + b^k)^{k-1}}{b^{k(k-1)}} \Leftrightarrow \left(\left(\frac{a}{b}\right)^{k-1} + 1\right)^k > \left(\left(\frac{a}{b}\right)^k + 1\right)^{k-1}$

ni isbotlash yetarli. $\frac{a}{b} = x$ deylik, bunda $x \in R^+$

Demak $(x^{k-1}+1)^k > (x^k+1)^{k-1}$ ni isbotlashimiz kerak.

Induksiya metodidan foydalanamiz

$k=1$ da $(x^0+1)^1=2 > (x^1+1)^0=1 \Rightarrow k=1$ da ōrinli

$k\le n$ uchun ōrinli bōlsin. $k=n+1$ da isbotlaymiz.

Demak $(x^{n-1}+1)^n > (x^n+1)^{n-1}$ va biz $(x^n+1)^{n+1} > (x^{n+1}+1)^n$ - ni

isbotlashimiz kerakda $\dfrac{(x^n+1)^{n+1}}{(x^{n-1}+1)^n} > \dfrac{(x^{n+1}+1)^n}{(x^n+1)^{n-1}} \Leftrightarrow (x^n+1)^{n+1}\cdot(x^n+1)^{n-1} >$

$> (x^{n-1}+1)^n\cdot(x^{n+1}+1)^n$

Umuman, $(x^{k-1}+1)^k = x^{(k-1)\cdot k} + C_k^1 x^{(k-1)\cdot(k-1)} + C_k^2\cdot x^{(k-1)(k-2)} + \dots + C_k^{k-1} x^{(k-1)\cdot 1} + 1$

$$(x^k+1)^{k-1} = x^{(k-1)k} + C_{k-1}^1 x^{k\cdot(k-2)} + C_{k-1}^2\cdot x^{k\cdot(k-3)} + \dots + C_{k-1}^{k-2} x^k + 1$$

$$(x^{k-1}+1)^k = (x^{(k-1)k}+1) + (C_k^1 x^{(k-1)(k-1)} + C_k^{k-1} x^{k-1}) + \dots$$

$$(x^k+1)^{k-1} = (x^{(k-1)k}+1) + (C_{k-1}^1 (x^{k(k-2)}) + C_{k-1}^{k-2} x^k) + \dots$$

$k=2019$ da

$$(x^{2018}+1)^{2019} = ((x^{2018})^{2019}+1) + (C_{2019}^1((x^{2018})^{2018} + (x^{2018})^1) + \dots$$

$$(x^{2019}+1)^{2018} = ((x^{2019})^{2018}+1) + C_{2018}^1((x^{2019})^{2017} + x^{2019}) + \dots$$

Umuman $n\le \frac{2018}{2}$ da $C_{2019}^n((x^{2018})^n + (x^{2018})^{2019-n})$ va

$C_{2018}^n((x^{2019})^n + (x^{2019})^{2018-n})$ larni taqqoslash yetarli

$C_{2019}^n > C_{2018}^n \Rightarrow x^{2018n} + x^{2018\cdot(2019-n)}$ va $x^{2019n} + x^{2019(2018-n)} \Leftrightarrow$

$\Leftrightarrow x^{2018\cdot(2019-n)} - x^{2019(2018-n)} = x^{2019(2018-n)}(x^n-1)$ va

$x^{2019n} - x^{2018n} = x^{2018n}(x^n-1) \Leftrightarrow x^{2019(2018-n)}$ va x^{2018n} larni

taqqoslash yetarli $2018\cdot 2019 > 2019n + 2018n \Rightarrow (x^{2018}+1)^{2019} > (x^{2019}+1)^{2018}$

va $(a^{2018}+b^{2018})^{2019} > (a^{2019}+b^{2019})^{2018}$ Isbotlandi!!!