To solve this problem, we can assume a value for $\overline{ABC}$ such that it is a square of an integer, has different digits, and is no more than 3 digits long. Then we can test those digits in $\overline{CBA}$ and $\overline{CAB}$ to see if they are perfect squares as well. This method is efficient because we only need to test perfect squares under 1000 and just one example will prove that three such digits are possible.

Example: $A=9, B=6, C=1$.

$\overline{ABC} = 961 = 31^2$

$\overline{CBA} = 169 = 13^2$

$\overline{CAB} = 196 = 14^2$

Therefore, there are three different digits such that the $\overline{ABC}$, $\overline{CBA}$, and $\overline{CAB}$ are squares of integer numbers.

Let us first note how many adjacent squares each different type of square can have. A corner square can have 2, an edge square can have 3, and a square that is not touching the edge of the 10x10 can have 4. Since an edge square has an odd number of adjacent squares, it can never be balanced. Since there are 32 edge squares, the maximum number of balanced squares the board can have is the total number of squares subtract the number of edge squares. 100 - 32 = 68. Here is proof by an example:

•	o	•	o	•	o	•	o	•	o
•	o	•	o	•	o	•	o	•	o
•	o	•	o	•	o	•	o	•	o
•	o	•	o	•	o	•	o	•	o
•	o	•	o	•	o	•	o	•	o
•	o	•	o	•	o	•	o	•	o
•	o	•	o	•	o	•	o	•	o
•	o	•	o	•	o	•	o	•	o
•	o	•	o	•	o	•	o	•	o
•	o	•	o	•	o	•	o	•	o

(• = blue, o = white). Here, every corner square and square that is not touching the edge of the 10x10 is balanced. Therefore, the maximum amount of balanced squares the board can have is 68.

Let's denote the triple of numbers as $\{a, b, c\}$.
For each position, there are 9 possibilities: $\{1,1,1\}$, $\{2,2,2\}$, $\{3,3,3\}$, $\{1,2,3\}$, $\{1,3,2\}$, $\{2,1,3\}$, $\{2,3,1\}$, $\{3,1,2\}$, $\{3,2,1\}$. Since there are 4 positions for the nine triples, there would be 9^4 sets, which is 6561 sets. However this is not our final answer because it assumes that different permutations are different sets. Now, we must consider the cases:

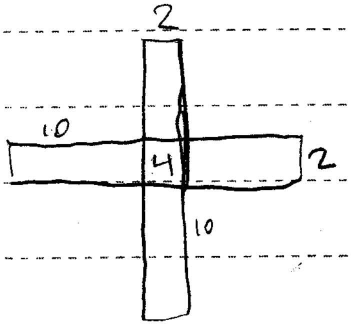
Case 1: $a=b=c$. For this case to occur, $\{a,b,c\}$ must have been formed using only $\{1,1,1\}$, $\{2,2,2\}$, or $\{3,3,3\}$ in all positions. Since there are 4 positions for the three triples, there would be $3^4 = 81$ sets in Case 1. Since $a=b=c$, there is only 1 permutation so there is no need to divide.

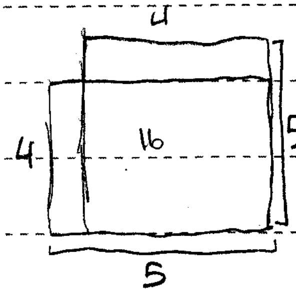
Case 2: $a=b$, $a,b \neq c$. This case is impossible because for a to be equal to b, only $\{1,1,1\}$, $\{2,2,2\}$, or $\{3,3,3\}$ could be used in all positions. However, c must then be equal to a and b, as shown in Case 1. Since this is a contradiction, Case 2 has 0 sets.

Case 3: $a \neq b$, $a \neq c$, $b \neq c$. Here, there are 6 permutations: $\{a,b,c\}$, $\{a,c,b\}$, $\{b,a,c\}$, $\{b,c,a\}$, $\{c,a,b\}$, $\{c,b,a\}$ Since the problem states that different permutations don't produce different sets, we should divide the number of sets in this case by 6. There are 6561 total sets and 81 of those belong in Case 1, so the rest must belong in this case. $6561-81=6480$. Now we divide 6480 by the 6 permutations. $\frac{6480}{6}=1080$. There are 1080 sets in Case 3.

Adding up all the cases, we should get Case 1 + Case 2 + Case 3 $= 81+0+1080 = 1161$. Therefore, there are 1161 different sets in the game.

First, let us note that we want the rectangles to be as close to a square as possible. Here are two examples:





In both cases, the rectangles' areas are 20. However, the 2x10 rectangles only had an overlap of 4, while the 4x5 rectangle had an overlap of 16. In the second case, the values of the side lengths were closer to each other, which gave it a greater common area.

We should apply this idea to the problem we face. Let us list all the prime factors of the numbers between 2010 and 2020.

$2011 = 1 \times 2011$ (prime)

$2012 = 2 \times 2 \times 503$

$2013 = 3 \times 11 \times 61$

$2014 = 2 \times 19 \times 53$

$2015 = 5 \times 13 \times 31$

$2016 = 2 \times 2 \times 2 \times 2 \times 2 \times 7 \times 3 \times 3$

$2017 = 1 \times 2017$ (prime)

$2018 = 2 \times 1009$

$2019 = 3 \times 673$

Now by using these prime factors, we should construct a rectangle for each number so that the side lengths are as close as possible.

$2011 = 1 \times 2011$

$2012 = 4 \times 503$

$2013 = 33 \times 61$

$2014 = 38 \times 53$

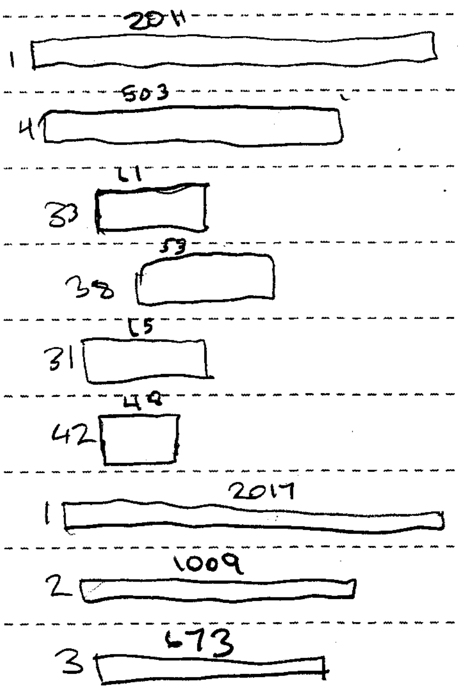
$2015 = 31 \times 65$

$2016 = 42 \times 48$

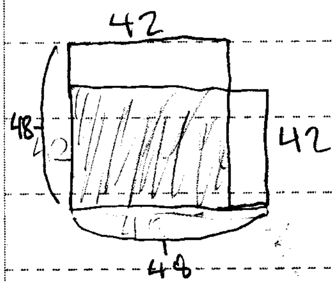
$2017 = 1 \times 2017$

$2018 = 2 \times 1009$

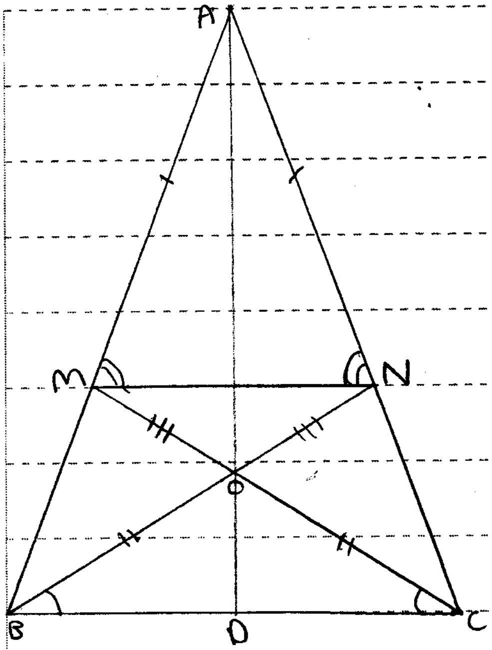
$2019 = 3 \times 673$



Of all these rectangles, the 42x48 rectangle is closest to a square, as its side lengths are only 6 units apart. Therefore, we should use two copies of this rectangles on the grid, which would give us the maximal possible area.



The common part of these rectangles is $42 \times 42 = 1764$. Therefore, the maximal common area of the rectangles is 1764 units.



Let us draw a line MN. Since $\triangle AMN$ is isoceles, $\angle AMN = \angle ANM$. Since $\triangle BOC$ is isoceles, $\angle BCO = \angle CBO$.

Let's assume that $\triangle ABC$ is isoceles. If this were true, $\overline{MN}$ would be parallel to BC, and $\triangle MNO$ would be similar to $\triangle BOC$. The point O is on AD, the altitude of $\triangle ABC$, which is isoceles, so MO must be equal to NO. The ratio $\frac{CO}{MO}$ is equal to $\frac{BO}{NO}$, and MN and BC are parallel, proving that $\triangle MNO$ is similar to $\triangle BOC$. If these two triangles are isoceles, it must make $\triangle ABC$ isoceles.

However, if $\triangle ABC$ was not isoceles, then point O would not be on AD the altitude of $\triangle ABC$. Then there would be no way to prove the equality of side lengths MO and NO, which means that there is no way to prove that $\triangle MNO$ is similar to $\triangle BOC$. Without this, there would be no way to prove that $\triangle ABC$ is isosceles. Therefore, $\triangle ABC$ must be isosceles.