Let the triangle have angles a, b, c s.t. $a \geq b \geq c$. Then, we have $\tan a + \tan b + \tan c = 2016$. It is well known that if $a+b+c = \pi$, then $\tan a \tan b \tan c = \tan a + \tan b + \tan c$. We now claim that $90 \geq a, b, c$ (the triangle is not obtuse). If not, we would have $180 > a > 90$ and $90 \geq b, c$. Note that we then have $\tan a < 0$ and $\tan b, \tan c \geq 0$. However, we have $2016 = \tan a + \tan b + \tan c$

$$= \tan a \tan b \tan c \leq 0$$ because $\tan a < 0$ and $\tan b, \tan c \geq 0$. 2016 is actually bigger than 0, so we have a contradiction. Thus, $90 \geq a, b, c$. We now claim that the closest value of a (to the nearest degree) is $90°$. ~~It is sufficient to show that~~ ~~$\tan(89.5°) < \frac{2016}{3} = 672$,~~

~~$2016 = \tan a + \tan b + \tan c$~~

Note that $2016 = \tan a + \tan b + \tan c \leq 3 \tan a$ (since $90 \geq a \geq b \geq c > 0$ means that $\tan a \geq \tan b \geq \tan c$), so $\tan a \geq \frac{2016}{3} = 672$. It is sufficient to show that $\tan 89.5° < 672$, for this then implies that a is closer to $90°$ than $89°$. To show this, first note that

R10
Problem: 1

$$\tan 89.5^\circ = \frac{1}{\tan(90-89.5)} = \frac{1}{\tan 0.5^\circ} = \frac{1}{\tan \frac{\pi}{360}}.$$

The Taylor series approximation about 0 for $\tan x$ is $\tan x = x + \frac{x^3}{3} + \frac{2x^5}{15} + \ldots$, where all terms are nonnegative if x is nonnegative (for $0 \le x < \frac{\pi}{2}$).

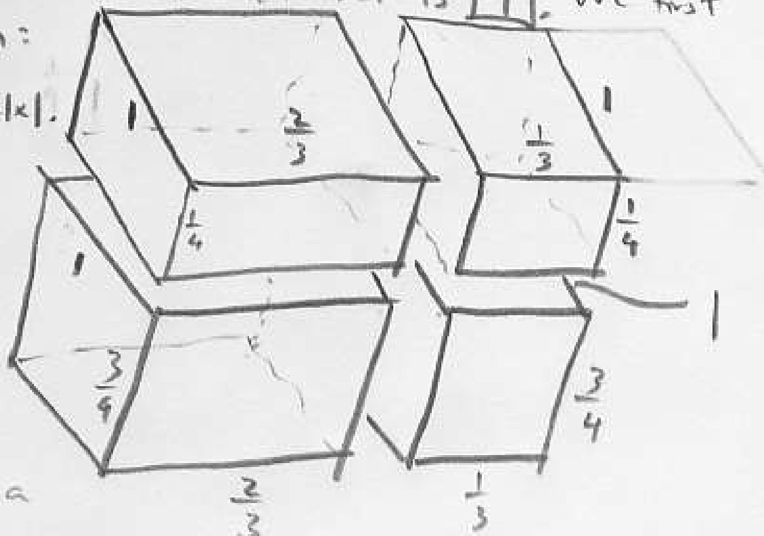
Clearly, $\tan x > x$ for positive $x < \frac{\pi}{2}$, so we have $\tan\frac{\pi}{360} > \frac{\pi}{360}$, so $\tan 89.5^\circ < \frac{1}{\frac{\pi}{360}} = \frac{360}{\pi}$.

$\frac{360}{\pi} < \frac{360}{3} = 120 < 672$, as desired.

Thus, a is closest to $\boxed{90^\circ}$.

R10
Problem: 2

We claim that the minimum number is $\boxed{4}$. We first give a construction:



WLOG, assume the cube is $1\times1\times1$. Consider a $1\times\frac{1}{4}\times\frac{2}{3}$ typical parallelepiped, a $1\times\frac{1}{3}\times\frac{1}{4}$ typical parallelepiped, a $1\times\frac{3}{4}\times\frac{2}{3}$ typical parallelepiped, and a $1\times\frac{3}{4}\times\frac{1}{3}$ typical parallelepiped, all glued together. Clearly, they form a $1\times1\times1$ cube.

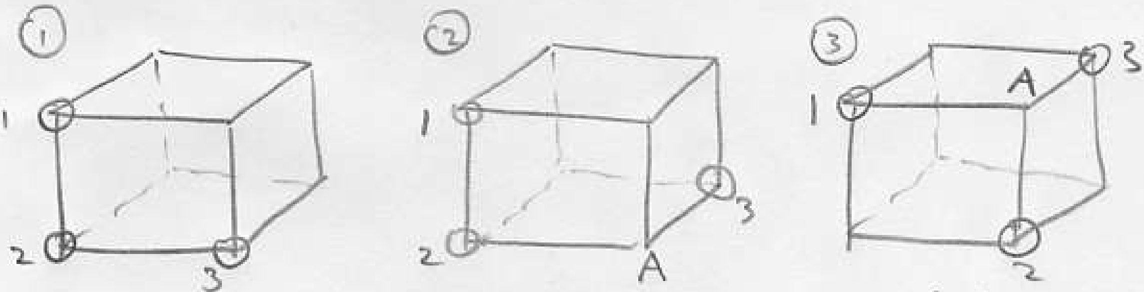
We now show that we cannot have fewer typical parallelepipeds. Assume otherwise and say we only have 3. ~~Consider some face of the cube. This face (which is a square) clearly has 4 vertices. Since we have at most 3 parallelepipeds, at least one of the parallelepipeds must contain at least two of these vertices by the Pigeonhole Principle. We then have two cases:~~

~~① ②~~

~~either the parallelepiped covers two adjacent vertices (like in ①) or it covers two opposite vertices (like in ②).~~

Problem: 2

We have 8 vertices in a cube and so by the Pigeonhole principle, one of the parallelepipeds will contain at least $\lceil \frac{8}{3} \rceil = 3$ vertices of the cube. Note that these vertices must also be vertices of the parallelepiped. We have several cases: ① the 3 vertices are all on one face, ② 2 vertices are adjacent and the third is not in any face that contains the other 2, ③ no two vertices are adjacent.
(the 3 vertices are denoted 1, 2, 3)



In ①, we automatically get that two of the dimensions (height and width, height and length, or width and length) both have length 1, so the parallelepiped is not typical $\Rightarrow\Leftarrow$

In ②, the only way for a parallelepiped to contain the vertices 1, 2, and 3 is if it also contains A. This then gives that side length containing 2 and A has the same length as the side length containing A and 3, which are different dimensions, so the parallelepiped is not typical $\Rightarrow\Leftarrow$

In ③, the only way for a parallelepiped to contain the vertices 1,2,3 is if it contains vertex A. However, this gives that the side length containing vertices 1 and A has the same length as the side length containing A and 2, which are different dimensions, so the parallelepiped is not typical $\Rightarrow\Leftarrow$

In all cases, we reach a contradiction and thus, it is impossible to form a cube with three or fewer typical parallelepipeds.

The minimum number is thus $\boxed{4}$ and we have given a construction for 4 that is valid.

QED

Let $f(n) = 2^n + n^{2016}$. Note that $f(0) = 1+0 = 1$, which is not prime, so 0 doesn't work.

Now, consider the case when n is a positive even. We let $n = 2k$, where k is a positive integer.

Then, $f(n) = f(2k) = 2^{2k} + (2k)^{2016}$

$$\geq 2^2 + 2^{2016}$$

$$> 2.$$

Clearly, $2 \mid f(2k)$, but $f(2k) > 2$, so $f(2k)$ is not prime. Thus, no positive even works.

It remains to check when n is a positive odd. Note that any odd can be written as $6k+1$, $6k+3$, or $6k+5$, where k is some nonnegative integer.

~~In the cases where $n = 6k+1$ or $6k+5$, we have~~

~~$f(n) = 2^n + n^{2016} \equiv (-1)^n + n \pmod 3$, because~~

~~$2 \equiv -1 \pmod 3$ and $n^{2016} \equiv (n^{672})^3 \equiv$~~

Let us first consider the cases when $n = 6k+1$ or $6k+5$.

In these cases, $3 \nmid n^{1008}$, so by Fermat's little theorem, $(n^{1008})^2 \equiv 1 \pmod 3$. Thus, $n^{2016} \equiv 1 \pmod 3$.

Then, $f(n) = 2^n + n^{2016} \equiv (-1)^n + 1 \pmod 3$. n is odd because $6k+1$ and $6k+5$ are odd, so $f(n) \equiv (-1)^n + 1$ $\equiv -1+1 \equiv 0 \pmod 3$, so $3 \mid f(n)$.

If $n=1$, then $f(n) = f(1) = 2^1 + 1^{2016} = 3$, which is prime, So $n=1$ works. However, if $n>1$, then we have

$f(n)=2^n+n^{2016} > 2^1+1^{2016}=3$, and since 3 also divides $f(n)$, that means that $f(n)$ is composite, so only $n=1$ works in the case where $n=6k+1$ or $6k+5$.

The final case is when $n=6k+3$.

We have $f(n)=2^n+n^{2016}=2^{6k+3}+(n^{672})^3$

$$=(2^{2k+1})^3+(n^{672})^3.$$

$$=(2^{2k+1}+n^{672})\left((2^{2k+1})^2-2^{2k+1}\cdot n^{672}+(n^{672})^2\right)$$

Clearly, $\underline{2^{2k+1}+n^{672}>1}$. Also,

$(2^{2k+1})^2-2^{2k+1}\cdot n^{672}+(n^{672})^2$

$$> 2^{2k+2}-2\cdot 2^{k+1}\cdot n^{672}+(n^{672})^2$$

$$=(2^{2k+1}-n^{672})^2$$

$$=(2^{2k+1}-(6k+3)^{672})^2$$

Clearly, $2\nmid(6k+3)^{672}$, so $2^{2k+1}\neq(6k+3)^{672}$,

so $(2^{2k+1}-(6k+3)^{672})^2\geqslant 1$.

Thus, $\underline{(2^{2k+1})^2-2^{2k+1}\cdot n^{672}+(n^{672})^2>1}$.

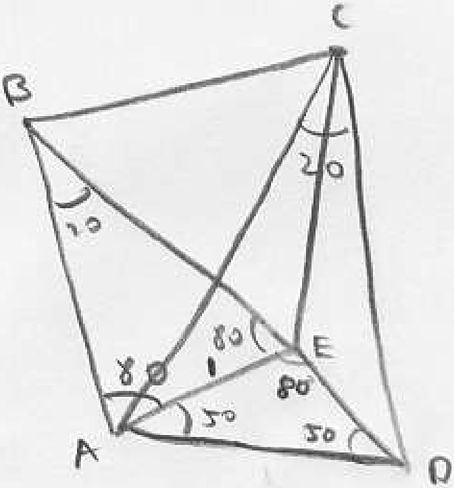
$f(n)$ is thus a product of two integers that are both greater than 1.

It is thus impossible for $f(n)$ to be prime.
To conclude, the only possible case where $f(n)$ is prime is when $n = \boxed{1}$.

QED

R10

Problem: 4



WLOG, assume $AE = 1$. $\triangle AED$ is isoceles, so if we drop a perpendicular from E to AD, it bisects AD, so $AD = 2 \cdot \cos 50°$. $\angle CAD = \angle CDA = 80°$, so $\triangle ACD$ is isoceles, so dropping an altitude from C to AD bisects AD, so $AC = \dfrac{\frac{1}{2} \cdot AD}{\cos \angle CAD} = \dfrac{\cos 50°}{\cos 80°}$. $\angle BAE = \angle BEA = 80°$, so dropping an altitude from B to AE bisects AE, so $AB = \dfrac{\frac{1}{2} AE}{\cos \angle BAE} = \dfrac{1}{2\cos 80°}$. Then,

$\dfrac{AD}{AE} = \dfrac{2\cos 50°}{1}$ and $\dfrac{AC}{AB} = \dfrac{\cos 50°}{\cos 80°} \Big/ \dfrac{1}{2\cos 80°} = 2\cos 50°$,

so $\dfrac{AD}{AE} = \dfrac{AC}{AB}$. Also, $\angle BAC = \angle BAE - \angle CAE = 80° - \angle CAE = \angle CAD - \angle CAE = \angle EAD$. Thus, by SAS similarity, $\triangle ABC \sim \triangle AED$. Thus, $\angle ABC = \angle AED$, so $\angle EBC = \angle ABC - \angle ABE = \angle AED - 20° = 80° - 20° = 60°$.

Also, $\angle BCA = \angle EDA = 50^\circ = \angle EAD = \angle BAC$, so $\triangle ABC$ is isoceles. $\angle BAE = \angle BEA = 80^\circ$, so $\triangle ABE$ is also isoceles. $\triangle ABC$ isoceles $\Rightarrow AB = BC$ and $\triangle ABE$ isoceles $\Rightarrow AB = BE$. Thus, $BC = BE$, so $\triangle BCE$ is isoceles. $\angle EBC = 60^\circ$, so $\angle BEC = \angle BCE = \frac{180-60}{2} = 60$, so $\angle EBC = \angle BCE = \angle CEB = 60^\circ$, so $\triangle BEC$ is an equilateral triangle.

QED

R10
Problem: 5

We consider each complexity (from 0 to 4).
Let a position be <u>special</u> if all digits are different at that position (for example, in the set $\{1111, 1112, 1113\}$, the fourth position is special).

<u>Case 1 (Complexity = 0):</u> In this case, since the complexity is 0, we must have **all** digits the same for each position; in other words, all three numbers in the set are the same. There are 3 choices for each position, for a total of $3 \cdot 3 \cdot 3 \cdot 3 = \underline{81}$ sets of complexity 0.

<u>Case 2 (Complexity = 1):</u> In this case, we have exactly one special position. There are 4 ways to pick this position and $3!$ ways to place 1, 2, 3 into the chosen special position of the three numbers in the set. For the remaining positions, we have 3 choices (1, 2, or 3) for each one, for a total of

$4 \cdot 3! \cdot 3 \cdot 3 \cdot 3$ choices, except we overcounted because the set $\{a, b, c\}$ is the same as $\{a, c, b\}$, ... There are $3!$ ways to arrange a, b, c (note that a, b, c are pairwise distinct), so we actually have $4 \cdot 3! \cdot 3 \cdot 3 \cdot 3 / 3!$
$= 4 \cdot 3 \cdot 3 \cdot 3 = \underline{108}$ sets of complexity 1.

Case 3 (Complexity = 2): We have exactly 2 special positions. There are $\binom{4}{2}$ ways to pick them and 3! ways to place 1, 2, 3 into the two special positions of each of the three numbers in the set (so 3!·3!, one for each special position). The remaining positions each have 3 choices (1, 2, or 3). The three numbers in the set are pairwise distinct and so we must divide by 3! to account for the overcount (same reasoning as in case 2). We thus have $\binom{4}{2}\cdot 3!\cdot 3!\cdot 3\cdot 3/3!$ $= 6\cdot 6\cdot 3\cdot 3 =$ $\underline{324}$ sets of complexity 2.

Case 4 (complexity = 3): We have exactly 3 special positions. There are $\binom{4}{3}$ ways to pick them and 3! ways to place 1, 2, 3 into the 3 special positions of each of the three numbers in the set (so 3!·3!·3!). The remaining position has 3 choices (1, 2, or 3). The 3 numbers in a set are pairwise distinct, so we must divide by 3! for the same reasoning as in case 2, 3. We thus have $\binom{4}{3}\cdot \cancel{3!}\cdot 3!\cdot 3!\cdot 3 / \cancel{3!} = 4\cdot 6\cdot 6\cdot 3$ $= \underline{432}$ sets of complexity 3.

R10
Problem: 5

Case 5 (complexity = 4): All positions are special. There are $3!$ ways to distribute 1, 2, 3 to each of the 4 special positions of each of the three numbers in the set (so $3! \cdot 3! \cdot 3! \cdot 3!$). The 3 numbers in the set are clearly pairwise distinct, so we divide by $3!$ for the same reasoning as in Case 2, 3, 4. Thus, we have $3! \cdot 3! \cdot 3! \cdot 3! / 3!$ $= 6 \cdot 6 \cdot 6 = \underline{216}$ sets of complexity 4.

Out of these cases, Case 4 is the most common, with $\underline{432}$ sets. Case 4 corresponds to when the complexity is 3, so the sets of complexity $\boxed{3}$ are the most numerous in the game.