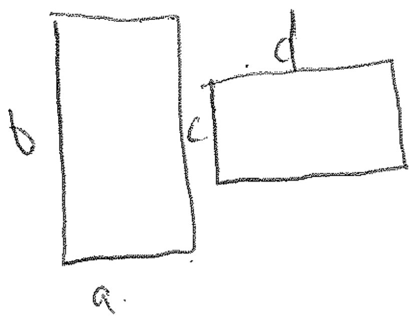
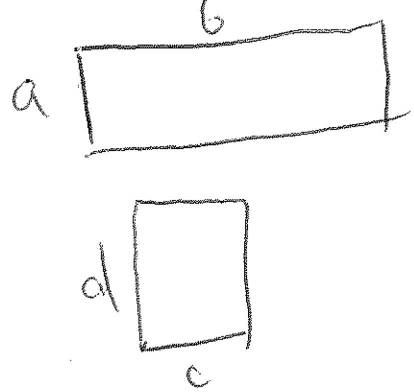
2. Let one rectangle be $a \times b$, where $a < b$, and the other rectangle be $c \times d$, where $c < d$. Then we have the rectangles positioned like this:

Either

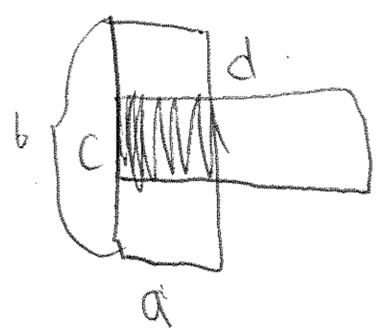


Or

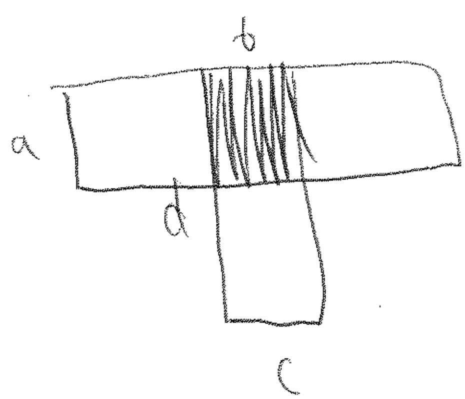


Then the maximum possible intersection would look like this:

Either



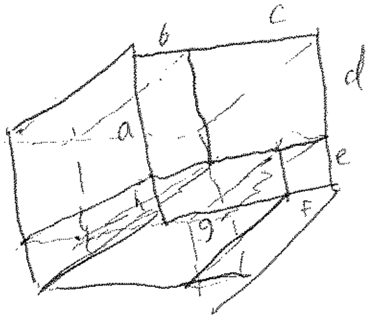
Or



Where the shaded part is the intersection. We see that the area of the intersection is always ac.

2. Therefore, we want to maximize the smaller side lengths in both of the rectangles. For the rectangle with area 2016, we see that this is achieved with side lengths 42×48. In the rectangle with area 2015, this is achieved with side lengths 31×65. Thus, the ~~area~~ maximal possible common part of the area is $\boxed{42 \cdot 31 = 1302}$.

3. The minimal number is 4, for example,



where a, b, c, d, e, f, g, and h are all different.

Each of the corners must be touched by a parallelepiped.
Therefore, 2 is not possible because by the pigeonhole
principle one of the parallelepipeds will be touching 4 corners or
more. This means that two of the parallelepiped's sides are
sides of the cube, and thus are equal; not generic.
Almost the same applies to 3 parallelepipeds. By the pigeonhole
principle, one of the parallelepipeds contains at least three, and thus
four of the cube's vertices, and then the same ~~[illegible]~~ happens as with
two parallelepipeds.

Thus, $\boxed{4}$ is the minimum number of generic parallelepipeds
required to form a cube.

4. Let us first find all the digits not divisible by 3: 1, 2, 4, 5, 7, 8.

First, we look at the places where each digit can be. It can be in any of 5 places. For each of these 5 places, there are $6 \cdot 6 \cdot 6 \cdot 6$ ways to determine the other four numbers. To determine the sum of all the digits, we take each of these digits, multiply it by $5 \cdot 6 \cdot 6 \cdot 6 \cdot 6$, and then add the results together. This technique accounts for everything. We see then that the sum of all the digits is

$$(1+2+4+5+7+8)5 \cdot 6 \cdot 6 \cdot 6 \cdot 6 = 135 \cdot 1296 = \boxed{174960.}$$

1. We have that $1 \cdot 2 \cdot 4 \cdot 5 \cdot 25 = 1000$, and that all five of these numbers are different. Therefore, these 5 numbers work.

5. Let this be the board, and let the diagonal determine the right side and left side of the board.

	0	1	2	3	4	5	6	7	8	9
0	00	10	20	30	40	50	60	70	80	90
1	01	11	21	31	41	51	61	71	81	91
2	02	12	22	32	42	52	62	72	82	92
3	03	13	23	33	43	53	63	73	83	93
4	04	14	24	34	44	54	64	74	84	94
5	05	15	25	35	45	55	65	75	85	95
6	06	16	26	36	46	56	66	76	86	96
7	07	17	27	37	47	57	67	77	87	97
8	08	18	28	38	48	58	68	78	88	98
9	09	19	29	39	49	59	69	79	89	99

Each square is uniquely determined by a two digit number ranging from 00 to 99, where the column ~~number~~ digit goes first, and the row digit goes second.

We see that the edge squares have three squares bordering them, and thus cannot be balanced.

Now we go on to the problem. If we let 01 (or 10) be white and the rest of the numbers to be blue, we have 1 balanced square.

If we let both 01 and 08 (or 19) be white and the rest blue, we have two balanced squares.

Now we will make a table which shows which squares are white and which are blue for each number of balanced squares.

5.

The number with opposite digits is added as blue.

#	White
1	None
2	01
3	add 12
4	add 23
5	add 34
6	add 45
7	add 56
8	add 67
9	add 78
10	add 89
11	
12	
13	
14	
15	
16	
17	
18	
19	
20	

We continue filling in this chart with opposite diagonals until we fill in 08, 19, 80, and 91. We can also make all the even numbers white, giving us exactly 68 squares balanced. This leaves us with a total of

$\boxed{\text{68 different values of } n}$