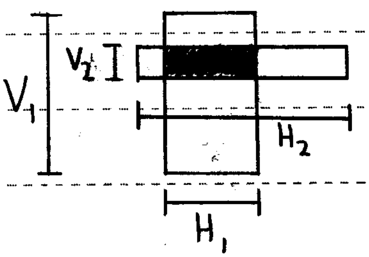
Yes. Let $A=9$, $B=6$, and $C=1$.

Then $\overline{ABC}=961=31^2$
$\overline{CBA}=169=13^2$
$\overline{CAB}=196=14^2$

$\overline{ABC}$, $\overline{CBA}$, and $\overline{CAB}$ are all perfect squares, so it is possible.

Let V_1 and V_2 be the vertical lengths of the first and second rectangles, respectively. Let H_1 and H_2 be their respective horizontal lengths, and let A_1 and A_2 be their respective areas. The question tells us that $V_1 > H_1$, $V_2 < H_2$, $2010 < A_1 < 2020$, and $2010 < A_2 < 2020$.

It is easy to show that the maximum intersection of the rectangles is a rectangle of dimensions $V_2 \times H_1$. Proof: Let V_3 and H_3 be the vertical and horizontal lengths of the intersection. This intersection is fully contained within both rectangles. Assume that $V_3 > V_2$. Then the intersection cannot be fully contained in the second rectangle, since it has a greater vertical length, contradiction. Therefore, $V_3 \leq V_2$. Similarly, $H_3 \leq H_1$. The area of the intersection is $V_3 H_3 \leq V_2 H_1$. Therefore, the maximum area is $V_2 H_1$.



$H_1^2 < V_1 H_1 = A_1$, so $H_1 < \sqrt{A_1}$. H_1 is also a factor of A_1. To maximize the area of the intersection, we maximize V_2 and H_1. To do this, we set H_1 to be the largest factor of A_1 that is less than $\sqrt{A_1}$. Then, we test values of A_1, between 2010 and 2020, and for each value, find the largest factor under $\sqrt{A_1}$.

(Continued on next page)

A_1	Largest factor less than $\sqrt{A_1}$
2011	1
2012	4
2013	33
2014	38
2015	31
2016	42
2017	1
2018	2
2019	3

The largest possible value of H_1, which is 42, can be obtained when $A_1 = 2016$

We can repeat the same steps to show that the largest possible value of V_2 is 42, when $A_2 = 2016$.

Then, the maximum area of the intersection of the rectangles is $42 \times 42 = 1764$.

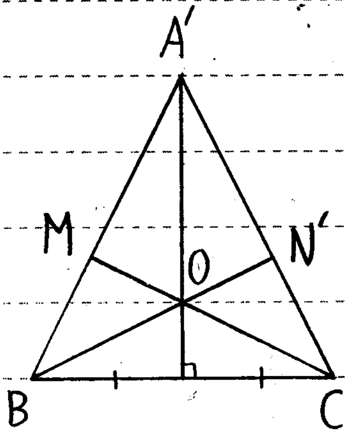


Figure 1

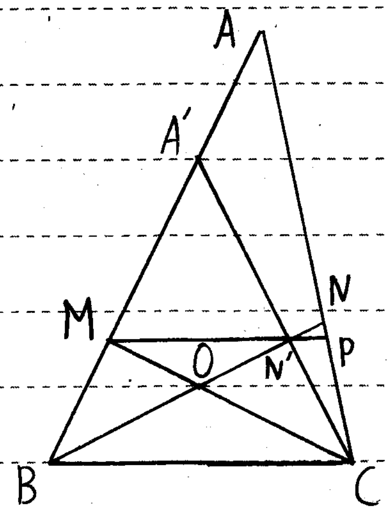


Figure 2

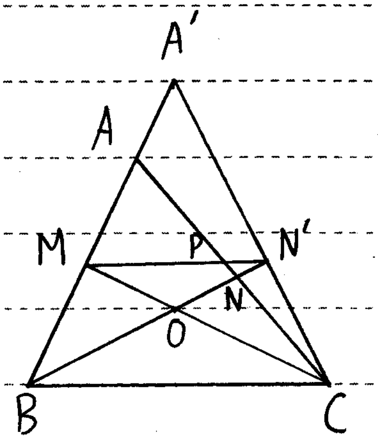


Figure 3

Let line ℓ be the perpendicular bisector of BC. Since $BO=CO$, O lies on ℓ. Fix point O on line ℓ. Fix point M on CO, extended past O. Let the intersection of BM and ℓ be A'. Let the intersection of BO and $A'C$ be N'. (See Figure 1).

Point A lies on the ray BM, extended past M. A can be past A' (Figure 2), between M and A' (Figure 3), or on A'. Define N as the intersection of BO and AC. I will prove that the condition $AM=AN$ cannot hold true in Figures 2 and 3, implying that A is on A', proving that $\triangle ABC$ is isoceles.

First we consider Figure 2. Since A is to the right of A', N is right of N'. (Continued on next page)

Let P be the intersection of AC and MN'. Note that since $MN' \parallel BC$, $MP \parallel BC$. Then $\angle ABC = \angle AMP$, and $\angle ACB = \angle APM$, so $\Delta AMP \approx \Delta ABC$. Since $AB > AC$ (because A is right of A'), $AM > AP$. However, since N is right of N', N is above line MP, so $AN < AP$. Combining $AM > AP$ and $AN < AP$, we get $AM > AN$, which contradicts $AM = AN$.

Similarly, in Figure 3, when A is left of A', $\Delta AMP \approx \Delta ABC$, so $AM < AP$. Since A is left of A', N is left of N', so N is under line MP, therefore $AN > AP$. Combining inequalities gives us $AM < AN$, contradiction

A cannot be to the right of A', and A cannot be to the left of A', so A coincides with A', therefore ABC is isoceles.

The maximum amount of balanced squares is 68.

Color the first row white, the second row blue, the third row white, the fourth row blue, and so on. Note that if a piece is on the edge, and is not on a corner, it cannot be balanced, since it is adjacent to 3 squares (impossible for half of them to be blue). There are 8 of these squares on each edge, and $8 \times 4 = 32$ in total. Therefore, the maximum possible balanced squares is $100 - 32 = 68$. This is achieved in the coloring described above, with rows alternating between white and blue. Each corner square has one adjacent square of the same color, beside it, and one adjacent square of the opposite color, vertical from it. Each square not touching an edge has two squares, left and right of it, with the same color, and two squares, above and below it, with the opposite color.

There are 4 corner squares, and $8 \times 8 = 64$ squares not touching the edge, which is $4 + 64 = 68$ squares in total. Therefore, the maximum number of balanced squares, 68, can be achieved.

There are 1080 sets in the game.
Let k be the number of positions of a set with all digits the same.
For example, in the set 1232, 2213, 3221, $k=1$.
Let S_k be the number of sets, including permutations (we will divide by $3!$ at the end), with k positions having the same digit.
k can be 0, 1, 2, or 3. If $k=4$, all positions are the same digit (for exampe, 1121, 1121, 1121), which is not a set, as stated in the problem statement: "each number is used once".
We want to find $\frac{S_0+S_1+S_2+S_3}{6}$.

First, we find S_k in terms of k. There are $\binom{4}{k}$ ways to choose k out of the 4 positions to have the same digit. For each of the k positions with the same digit, there are 3 choices for which digit it could be, so 3^k in total. For each of the $4-k$ positions, there are $3!=6$ ways to arrange the digits, so 6^{4-k} in total. Multiplying them together, we get $S_k=\binom{4}{k}3^k6^{4-k}$.

Using this formula, $S_0=1296$, $S_1=2592$, $S_2=1944$, and $S_3=648$. $S_0+S_1+S_2+S_3=6480$. Since each triple is counted $3!=6$ times (we included permutations) we must divide by 6, to obtain $\frac{6480}{6}=1080$.
Therefore, there are 1080 sets.