International Mathematics Olympiad
“Formulo de Integreco” / “The Third Millennium”

Year 2014/2015. Round 2. February 8, 2015

Problems for grade R5

Please do not forget to prove your answers

1. There is a language with 3 vowels and 5 consonants. A vowel and a consonant (in any order) make a syllable, and any two syllables make a word. How many words are there in this language?

2. Find an example of integers a and b such that $ab(2a + b) = 2015$.

3. Maria and Helen left their home at the same time and went to a store for an ice cream. Maria walked faster, so she reached the store in 12 minutes. She spent 2 minutes to buy an ice-cream and then left. 2 minutes later on her way back she met Helen. How long did it take for Helen to reach the store? Speeds of both girls are constant.

4. Six students planted five trees at the school yard. Can it happen that every student planted the same number of trees while every tree was planted by the different number of students?

5. In the Flatworld there is an ocean and two rectangular islands. *Coastal waters* is a part of the sea no more than 50 km away from the coastline. Can it happen that one island has a greater area while the other island has a greater area of the coastal waters? The shortest distance between islands exceeds 50 km.

6. Anna, Gala, Diana, Sophie and Liz met at the “Formulo de Integreco” camp. All of them came from different cities: A, B, C, D and E. During the introduction meeting girls told about themselves the following information. Neither Sophie nor Diana have ever been in A. Both Gala and Sophie met the girl from B at the camp last year. Anna and Sophie exchanged souvenirs with the girl from C. Gala and Sophie helped the girl from D to carry her suitcase. Gala, Liz and the girl from C sometime chat by Skype. The girl from B and Anna are penpals. Which city is every girl from?

International Mathematics Olympiad
"Formulo de Integreco" / "The Third Millennium"

Year 2014/2015. Round 2. February 8, 2015

Problems for grade R6

Please do not forget to prove your answers

1. There is a language with 5 vowels and 7 consonants. A vowel and a consonant (in any order) make a syllable, and any two syllables make a word. How many words are there in this language?

2. Find an example of integers a and b such that $ab(2a + b) = 2015$.

3. There are 3 candies on a table and a bag with 2012 candies. Anna and Helen play the following game. In turns each girl takes candies from the bag and places them on the table. It is not allowed to add more candies than there are already on the table. The girl who takes the last candy from the bag is a winner. Anna starts first. Which of the girls can win the game no matter how the other girl plays?

4. Six students planted five trees at the school yard. Can it happen that every student planted the same number of trees while every tree was planted by the different number of students?

5. In the Flatworld there is an ocean and two rectangular islands. *Coastal waters* is a part of the sea no more than 50 km away from the coastline. Can it happen that one island has a greater perimeter while the other island has a greater area of the coastal waters? The shortest distance between islands exceeds 50 km.

6. Dima cooks porridge. The recipe requires to cook it for 24 minutes. Dima has the 20-minute and the 7-minute hourglasses. How can he measure 24 minutes precisely?

International Mathematics Olympiad
“Formulo de Integreco” / “The Third Millennium”
Year 2014/2015. Round 2. February 8, 2015

Problems for grade R7

Please do not forget to prove your answers

1. There is a language with 3 vowels and 7 consonants letters. A vowel and a consonant (in any order) make a syllable, and any three syllables make a word. A word is called funny if it contains two consecutive identical letters. How many funny words are there in this language?

2. Find an example of integers a and b such that

$$(10a + b)(a + 10b)(a + b + 1) = 2015.$$

3. Suppose ABC is an isosceles triangle. Let O be a point of intersection of medians AA_1 and BB_1. Given that $\angle AOB = 120°$, find angles of the triangle ABC.

4. Organizers plan to arrange a math battle tournament according to the following rules: 1) In each game three teams meet; 2) Each two teams meet exactly once. Find the minimal number of teams required to organize the tournament consisting of more than one game.

5. In the Flatworld there is an ocean and two triangular islands. *Coastal waters* is a part of the sea no more than 50 km away from the coastline. Can it happen that the perimeters of these islands are the same while the areas of their coastal waters are different? The shortest distance between islands exceeds 50 km.

6. A convex 2015-gon and all its diagonals are drawn on a blackboard. Alex and Ben play the following game. In turns each boy erases either any number from 1 to 10 of adjacent sides, or any number from 1 to 9 of diagonals (not necessarily from the same vertex). The player who cannot make a move loses. Alex starts first. Which of the boys can win the game no matter how the other boy plays? What is the winning strategy?

International Mathematics Olympiad
“Formulo de Integreco” / “The Third Millennium”
Year 2014/2015. Round 2. February 8, 2015

Problems for grade R8

1. There is a language with 3 vowels and 8 consonants letters. A vowel and a consonant (in any order) make a syllable, and any three syllables make a word. A word is called funny if it contains two consecutive identical letters. How many funny words are there in this language?

2. One of the endpoints of a segment is blue, while the other is red. The segment is split by 2015 points, each being either red or blue, into 2016 parts. Can it happen that the number of parts with both ends blue equals the number of parts with both ends red?

3. Suppose ABC is an isosceles triangle. Let O be a point of intersection of medians AA_1 and BB_1. Given that $\angle AOB = 120°$, find angles of the triangle ABC.

4. Positive integers a, b, c and d satisfy the equality

   $$2015^a + 2015^b = 2015^c + 2015^d.$$

   Is it possible that the numbers $a^{2015} + b^{2015}$ and $c^{2015} + d^{2015}$ are different?

5. In the Flatworld there is an ocean and two triangular islands. *Coastal waters* is a part of the sea no more than 50 km away from the coastline. Can it happen that the first island has a greater perimeter while the second island has the greater area of the coastal waters? The shortest distance between islands exceeds 50 km.

6. Mark thought of a number m and counted the number of all diagonals of a convex m-gon. He obtained the number k. Then he counted the number of all diagonals of a convex k-gon and obtained the number 2015.

   Find the number m Mark thought of initially.

International Mathematics Olympiad
"Formulo de Integreco" / "The Third Millennium"

Year 2014/2015. Round 2. February 8, 2015

Problems for grade R9

1. Positive integers a, b, c and d satisfy the equality

$$2015^a + 2015^b = 2015^c + 2015^d.$$

   Is it possible that the numbers $a^{2015} + b^{2015}$ and $c^{2015} + d^{2015}$ are different?

2. One of the endpoints of a segment is blue, while the other is red. The segment is split by 2015 points, each being either red or blue, into 2016 parts. Can it happen that the number of parts with both ends blue equals the number of parts with both ends red?

3. Points M and P are chosen on sides of AB and BC respectively of a square $ABCD$, so that $AM = CP$. A circle with the diameter DP intersects the segment CM at the point K. Prove that $MK \perp BK$.

4. Each of 10 consecutive integers greater than 1 is decomposed into a product of prime factors. Let p be the greatest factor in all these decompositions. What is the minimal possible value of p?

5. In the Flatworld there is an ocean and two islands which are convex polygons. *Coastal waters* is a part of the sea no more than 50 km away from the coastline. Can it happen that the first island has a greater perimeter while the second island has the greater area of the coastal waters? The shortest distance between islands exceeds 50 km.

6. Mark thought of a number m and counted the number of all diagonals of a convex m-gon. He obtained the number k. Then he counted the number of all diagonals of a convex k-gon and obtained the number 2015.

   Find the number m Mark thought of initially.

International Mathematics Olympiad
"Formulo de Integreco" / "The Third Millennium"
Year 2014/2015. Round 2. February 8, 2015

Problems for grade R10

1. Positive integers a, b, c and d satisfy the equality

$$2015^a + 2015^b = 2015^c + 2015^d.$$

   Is it possible that the numbers $a^{2015} + b^{2015}$ and $c^{2015} + d^{2015}$ are different?

2. How many 5-digit numbers are divisible by their last digit?

3. Points H, K and M are marked respectively on the sides BC, AC and AB of a triangle ABC. Let AH be an altitude. Prove that HA is the bisector of $\angle KHM$ if and only if AH, BK and CM intersect at the same point.

4. Each of 10 consecutive integers greater than 1 is decomposed into a product of prime factors. Let p be the greatest factor in all these decompositions. What is the minimal possible value of p?

5. In the Flatworld there is an ocean and two islands which are convex polygons. *Coastal waters* is a part of the sea no more than 50 km away from the coastline. Can it happen that the first island has a greater perimeter while the second island has the greater area of the coastal waters? The shortest distance between islands exceeds 50 km.

6. Mark thought of a number m and counted the number of all diagonals of a convex m-gon. He obtained the number k. Then he counted the number of all diagonals of a convex k-gon and obtained the number 2015.

   Find the number m Mark thought of initially.

International Mathematics Olympiad
“Formulo de Integreco” / “The Third Millennium”
Year 2014/2015. Round 2. February 8, 2015

Problems for grade R11

1. Positive integers a, b, c and d satisfy the equality

$$2015^a + 2015^b = 2015^c + 2015^d.$$

   Is it possible that the numbers $a^{2015} + b^{2015}$ and $c^{2015} + d^{2015}$ are different?

2. How many 5-digit numbers are divisible by their last digit?

3. Points H, K and M are marked respectively on the sides BC, AC and AB of a triangle ABC. Let AH be an altitude. Prove that HA is the bisector of $\angle KHM$ if and only if AH, BK and CM intersect at the same point.

4. Each of 10 consecutive integers greater than 1 is decomposed into a product of prime factors. Let p be the greatest factor in all these decompositions. What is the minimal possible value of p?

5. Suppose $ABCD$ is a regular tetrahedron with edge of the length 1. Through an interior point of face ABC three planes are drawn parallel to faces ABD, ACD and BCD respectively. These planes split the tetrahedron in several parts. Find the total sum of the lengths of the edges of the part that contains vertex D.

6. Mark thought of a number m and counted the number of all diagonals of a convex m-gon. He obtained the number k. Then he counted the number of all diagonals of a convex k-gon and obtained the number 2015.

   Find the number m Mark thought of initially.