3. The only such value is $n=1$. Let $S_n = 2^n + n^{2016}$ $\forall$ non-negative integers n. Note $S_0 = 2^0 + 0^{2016} = 1$, which is not prime. Also, $S_1 = 2^1 + 1^{2016} = 3$ which is prime. If $n > 1$ and $2 \mid n$, $S_n > 2$ and $2 \mid 2^n + n^{2016} = S_n$. Thus, S_n is not prime when n is an even positive integer. For the remaining odd integers $n > 1$, we consider two cases: whether or not it is divisible by 3.

<u>Case 1:</u> $3 \mid n$

Let $n = 3k$ for some positive integer k. Then,

$$S_n = 2^{3k} + (3k)^{2016}$$
$$= (2^k)^3 + ((3k)^{672})^3$$

As S_n is a sum of cubes of 2^k and $(3k)^{672}$, we know $2^k + (3k)^{672} \mid S_n$. Also, $2^k + (3k)^{672} > 1$. Hence, S_n is not prime for any odd integer n with $3 \mid n$.

<u>Case 2:</u> $3 \nmid n$

Then $n \equiv 1 \pmod 3$ or $n \equiv 2 \pmod 3$. We also know $n \equiv 1 \pmod 2$. Thus, $n \equiv 1 \pmod 6$ or $n \equiv 5 \pmod 6$.

Problem 3 continued on next page $\longrightarrow$

Case 2: $3 \nmid n$ (continued from last page)

In other words, $n \equiv \pm 1 \pmod 6$. Note if $n \in \mathbb{N}$:

$$2^n \equiv \begin{cases} 2 \pmod 6, & \text{if } n \text{ is odd} \\ 4 \pmod 6, & \text{if } n \text{ is even} \end{cases}$$

Since $n \equiv 1 \pmod 2$, $2^n \equiv 2 \pmod 6$.

Furthermore, if $n \equiv \pm 1 \pmod 6$, then $n^{2016} \equiv 1 \pmod 6$.

Thus,

$$S_n = 2^n + n^{2016} \equiv 2 + 1 \equiv 3 \pmod 6$$

This means $3 \mid S_n$. As $2^n + n^{2016} > 3$ when $n > 1$, it follows S_n is not prime for any odd integer $n > 1$ with $3 \nmid n$.

Therefore, $n = 1$ is the only non-negative integer such that $2^n + n^{2016}$ is prime.

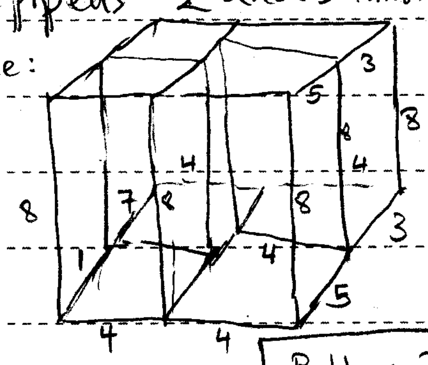
We claim the minimum number of typical rectangular parallelepipeds needed to form a cube is 4.
Let l_i, w_i, h_i be the length, width and height respectively of the i^{th} rectangular parallepiped, where $1 \leq i \leq 4$.
We will construct a working example with 4 below:
Let:

$$w_1 = w_2 = w_3 = w_4 = 4$$

$$l_1 = l_2 = l_3 = l_4 = 8$$

$$h_1 = 1, \quad h_2 = 5, \quad h_3 = 7, \quad h_4 = 3$$

Now, place parallelepiped 1 next to parallelepiped 2 such that side w_1 and w_2 are adjacent and l_1 and l_2 share an edge. Next, place parallelepiped 1 next to parallelepiped 3 such that l_1, l_3 share an edge, h_1, h_3 are adjacent and w_1, w_3 share an edge. Finally, place parallelepiped 4 so that it is adjacent to parallelepipeds 2 and 3 and h_4 and h_2 are adjacent.
It looks like:

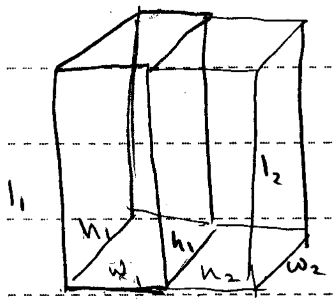


which is a cube of side length 8. Hence, 4 parallelepipeds that are typical can form a cube.

Problem 2 continued on next page ⟶

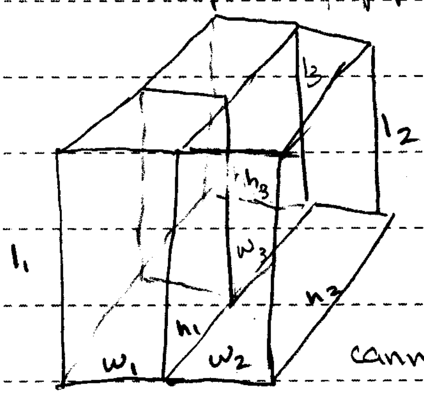
Now, we will show 4 typical rectangular parallelepipeds are indeed the minimum amount required to form a cube. Clearly, we cannot use 1 as all three dimensions must be different.

If there are 2 parallelepipeds, the configuration would have to look like:



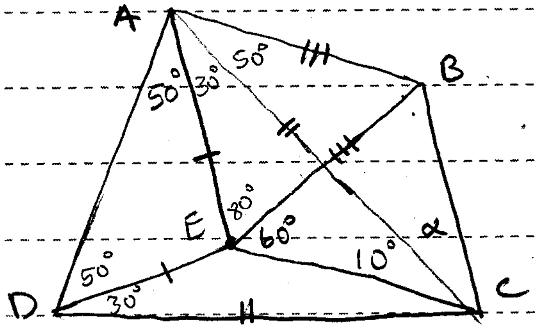
This implies one of the cube's sides is l_1 and another side is h_1. However, this is clearly false as $l_1 \neq h_1$. Hence, 2 typical parallelepipeds cannot form a cube.

If there are 3 parallelepipeds, it would have to look like:



Here, we can notice one of the cube's sides is l_2 while another side is h_2. However, this cannot be as $l_2 \neq h_2$. Hence, 3 typical parallelepipeds cannot form a cube.

Therefore, we must use a minimum of 4 typical parallelepipeds to form a cube.



Since $\angle EDA = 50°$ and $\angle CDA = 80°$, it follows $\angle EDC = 30°$. Likewise, since $\angle CAD = 80°$ and $\angle EAD = 50°$, it follows $\angle EAC = 30°$. Since $\angle BAE = 80°$, it follows $\angle CAB = 50°$. As $\angle EAD = \angle EDA$, $AE = DE$ and since $\angle CDA = \angle CAD$, $AC = CD$. Consider $\triangle ACE$ and $\triangle CED$:

- $\angle CAE = \angle CDE = 30°$
- $AE = DE$
- $AC = CD$

Thus, $\triangle EDC \cong \triangle EAC$ so $\angle ECD = \angle ECA$. In $\triangle ACD$,

$\angle ACD = 180° - (\angle CAD + \angle CDA) = 180° - (80° + 80°) = 180° - 160° = 20°$.

As $\angle ECD = \angle ECA$, it follows $\angle ECA = \frac{1}{2}\angle DCA = \frac{1}{2}(20°) = 10°$.

In $\triangle AEC$, $\angle BEC = 180° - (\angle BEA + \angle EAC + \angle ECA) = 180° - (80° + 30° + 10°) = 180° - 120° = 60°$.

Let $\alpha = \angle ACB$. Since $\angle BAE = \angle BEA$, $AB = BE$. Therefore,

$$\frac{\sin(\alpha + 10°)}{\sin 60°} = \frac{BE}{BC} = \frac{AB}{BC} = \frac{\sin\alpha}{\sin 50°}$$

by Sine Laws on $\triangle BEC$ and $\triangle ABC$ respectively. Thus,

$$\frac{\sin(\alpha + 10°)}{\sin\alpha} = \frac{\sin 60°}{\sin 50°} = \frac{\sin(50° + 10°)}{\sin 50°}$$

Problem 4 continued on next page →

Note:

$$\sin(\alpha+10^\circ) = \sin\alpha\cos10^\circ + \cos\alpha\sin10^\circ$$

$$\sin(50^\circ+10^\circ) = \sin50^\circ\cos10^\circ + \cos50^\circ\sin10^\circ$$

Note $\sin\alpha$, $\sin50^\circ \neq 0$. Therefore,

$$\frac{\sin\alpha\cos10^\circ+\cos\alpha\sin10^\circ}{\sin\alpha} = \frac{\sin50^\circ\cos10^\circ+\cos50^\circ\sin10^\circ}{\sin50^\circ}$$

$$\cos10^\circ + \cot\alpha\sin10^\circ = \cos10^\circ + \cot50^\circ\sin10^\circ$$

$$\cot\alpha\sin10^\circ = \cot50^\circ\sin10^\circ$$

$$\therefore \quad \cot\alpha = \cot50^\circ \quad (\because \sin10^\circ \neq 0)$$

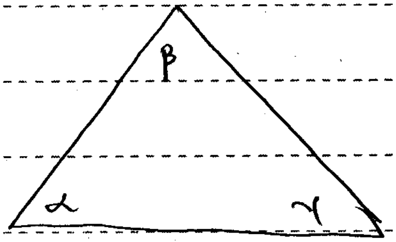
Since $0^\circ < \alpha < 180^\circ$ and $\cot\alpha = \cot50^\circ$, it follows $\alpha = 50^\circ$.

Thus, $\angle ACB = 50^\circ$ so $\angle ECB = 60^\circ$. Since $\angle BEC = \angle ECB = 60^\circ$,

$\angle EBC = 180^\circ - (\angle ECB + \angle BEC) = 180^\circ - (60^\circ+60^\circ) = 180^\circ - 120^\circ = 60^\circ$.

Hence, $\triangle BEC$ is equilateral.

Q.E.D.



Let α, β, γ be the angles of a triangle such that $\tan\alpha + \tan\beta + \tan\gamma = 2016$.

Without loss of generality, assume $\tan\alpha \geq \tan\beta \geq \tan\gamma$.

Thus, $\tan\alpha \geq \dfrac{\tan\alpha + \tan\beta + \tan\gamma}{3} = \dfrac{2016}{3} = 672$.

Note when $90° < \alpha < 180°$, $\tan\alpha < 0$. Thus, $0° < \alpha \leq 90°$.

Since $f(x) = \tan x$ is an increasing function on $(0, 90°]$, we know $\alpha \geq \tan^{-1}(672) \geq 89.9°$. Thus, $89.9° \leq \alpha \leq 90°$.

If $\beta, \gamma < 90.5°$, then the value of the biggest angle, estimated to 1 degree, is $90°$.

Suppose however, one of β or γ is at least $90.5°$, making the estimated value of the biggest angle at least $91°$. Without loss of generality, assume $\beta \geq 90.5°$.

Then, $\alpha + \beta \geq 89.9° + 90.5° = 180.4° > 180°$ – a contradiction!

Therefore, it follows $\beta, \gamma < 90.5°$ and $89.9° \leq \alpha \leq 90°$.

Hence, the value of the biggest angle, estimated to 1 degree, is $90°$.

Sets of complexity 4 are most numerous.

First, we count the number of numbers that can be formed. Note 3 of the digits are 1, 2, 3 so the 4^{th} digit has 3 choices (1, 2, or 3). Furthermore, there are $\frac{4!}{2!} = 12$ arrangements of these numbers. Hence, there are $3 \times 12 = 36$ possible numbers.

When determining a set, once the first two numbers are chosen, the third number is fixed. However, the first 2 numbers of a given set could've been selected in $\binom{3}{2} = 3$ ways. Thus, the total number of sets is $\frac{\binom{36}{2}}{3} = \frac{630}{3} = 210$.

Note a complexity of 0 is impossible as that would imply all three numbers of the set are the same.

If the complexity of a set is 3, 1 digit of the three numbers are different. Thus, the numbers in such a set are of the form abcd, abce and abcf.

Continued on Next Page →

Case 1: a, b, c are distinct

Then, it does not matter which of the two numbers that is not d is assigned to e or f. Thus, there are two sets in Case 1.

Case 2: Two numbers of a, b, c are equal

Without loss of generality, assume $a=b$. Then, the numbers are $aacd$, $aace$ and $aacf$. Note $e, f \neq c$ and $e, f \neq d$. Furthermore $c \neq d$ and $e \neq f$. That means c, d, e, f are 4 distinct numbers – a contradiction. Thus, there are 0 sets in Case 2.

When the complexity of a set is 3, there are 4 possible positions for the number that is different. Therefore, there are $2 \times 4 = 8$ sets with complexity 3.

When the complexity of a set is 2, two digits of the three numbers are different. Thus, the numbers in such a set are of the form $abcd$, $abef$, $abgh$.

Problem 5 continued on Next Page →

Case 1: $a=b$

Then, the numbers are of the form aacd, aaef, aagh. Then, as $c \neq a$, that implies one of e or g must be a - a contradiction! Hence, there are 0 sets of complexity 2 in Case 1.

Case 2: $a \neq b$

Then, the numbers are abcd, abef and abgh. Therefore, note $c=d$, $e=f$ and $g=h$. However, we can see then that $c=e=g$ - a contradiction! Hence, there are 0 sets of complexity 2 in Case 2.

Therefore, there are $0+0=0$ sets of complexity 2.

If the complexity of a set is 1, 3 digits of the 3 numbers are different. Thus, the numbers in such a set are of the form abcd, aefg, ahij. However, there are clearly less than 101 sets of complexity 1, making complexity 4 the most numerous.

Hence, sets of complexity 4 are most numerous in the game.