Since $(\frac{1}{2}\cdot k)!$ is defined, $k \equiv 0 \pmod 2$.

Let a be the integer such that $2a = k$.

If $k=0$, then $(\frac{1}{2}\cdot k)!(\frac{1}{4}\cdot k) \neq 2016+k^2$.

$\therefore$ $k \neq 0$, so $a \geq 1$, so $(a-1)!$ is well-defined.

$\therefore$ $(\frac{1}{2}\cdot k)!(\frac{1}{4}\cdot k) = 2016+k^2$

$\Leftrightarrow$ $a!(\frac{1}{2}\cdot a) = 2016+(2a)^2$

$\Leftrightarrow$ $a!(a) = 8a^2+4032$

$\Leftrightarrow$ $a^2((a-1)!-8) = 4032$.

Let $f(x) = x^2((x-1)!-8)$ for all integers $x \geq 1$.

$f(1) = -7$ and $f(2) = -28$ and $f(3) = -54$ and $f(4) = -32$ and $f(5) = 400$ and $f(6) = 4032$ and $f(x)$ is increasing for $x \geq 5$.

$\therefore$ $4032 = a^2((a-1)!-8) = f(a) \Leftrightarrow a=6 \Leftrightarrow 12 = 2a = k$.

$\therefore$ the only such k is $k=12$. ■

Jury notes:

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Here, I assume that the 5-digit numbers are positive.

The possibilities for each digit are 1, 2, 4, 5, 7, 8.

These are 6 possibilites, so the # of numbers that Lydia likes is 6^5.

For each 5-digit number that Lydia likes, consider the 5-digit number that replaces each digit x with $-x+9$. This new number is a 5-digit number that Lydia likes, and when this is done to the set of all 5-digit numbers that Lydia likes, the same set is obtained.

For each pair, the sum of their 10 digits is $9+9+9+9+9=45$.

$\therefore$ the total sum of all digits of all 5-digit numbers Lydia likes is $\frac{1}{2}\cdot(6^5)(45) = 174960$.

$\therefore$ the answer is 174960. ■

Jury notes:

It was given to me that A, B, and C are nodes.

From the Pythagorean Thm., the distance from C to $\overleftrightarrow{AB}$ is $\sqrt{1533^2 - (\frac{1}{2}\cdot 2016)^2} = \sqrt{1533^2 - 1008^2} = 1155$.

W.L.O.G., assume that $A = (0,0)$ and $B = (2016, 0)$ and $C = (1008, 1155)$.

Let a be the # of nodes in the interior of $\triangle ABC$ but not on the boundary.

Let b be the # of nodes on the boundary of $\triangle ABC$.

The # of nodes on $\overline{AB}$ is 2017.

$\frac{1008}{21} = 48$ and $\frac{1155}{21} = 55$ and $GCD(48, 55) = 1$, so the set of nodes on $\overline{AC}$ is $\{(48j, 55j) : 0 \le j \le 21 \text{ is an integer}\}$.

$\therefore$ the # of nodes on $\overline{AC}$ is 22, and the # of nodes on $\overline{BC}$ is 22.

$\therefore \quad b = 2017 + 22 + 22 - 3 = 2058$.

From Pick's Thm., $\therefore \; a + \frac{1}{2}\cdot 2058 - 1 = a + \frac{1}{2}\cdot b - 1$

$$= [ABC] = \frac{1}{2}\cdot AB \cdot 1155 = \frac{1}{2}(2016)(1155)$$

$$\Rightarrow \quad a = 1163212.$$

$\therefore \quad a + b = 1163212 + 2058 = 1165270$.

$\therefore$ the # of nodes which lie in $\triangle ABC$ including the nodes on the sides of the triangle is 1165270. ∎

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Jury notes:

Barycentric coordinates are used.

Let $(1,0,0)=A$ and $(0,1,0)=B$ and $(0,0,1)=C$.

Let $a=BC$ and $b=AC$ and $c=AB$ and $d=AM=AN$.

$\therefore$ $M=(c-d:d:0)$ and $N=(b-d:0:d)$.

$\left[\ (x:y:z)=\left(\frac{x}{x+y+z}, \frac{y}{x+y+z}, \frac{z}{x+y+z}\right).\ \right]$

$\therefore$ the equation of $\overleftrightarrow{BN}$ is $dx-(b-d)z=0$, and the equation of $\overleftrightarrow{CM}$ is $dx-(c-d)y=0$.

Since $O=\overleftrightarrow{BN}\cap\overleftrightarrow{CM}$, $\therefore$ $O=((b-d)(c-d):(b-d)d:(c-d)d)$.

Since $BO=CO$, O is on the perpendicular bisector of $\overline{BC}$.

The midpoint of $\overline{BC}$ is $(0,\frac{1}{2},\frac{1}{2})$ and the circumcentre of $\triangle ABC$ is $(a^2:b^2:c^2)$, so the equation of the perpendicular bisector of $\overline{BC}$ is $0=\begin{vmatrix} x & y & z \\ 0 & 1 & 1 \\ a^2 & b^2 & c^2 \end{vmatrix}$.

$$\therefore\ 0=\begin{vmatrix} (b-d)(c-d) & (b-d)d & (c-d)d \\ 0 & 1 & 1 \\ a^2 & b^2 & c^2 \end{vmatrix}$$

$$=(b-d)(c-d)(-b^2+c^2)+a^2((b-d)d-(c-d)d), \quad [\text{from column } 1]$$

$$=-(b-c)(b+c)(b-d)(c-d)+a^2(b-c)d$$

$$=(b-c)(a^2d-(b+c)(b-d)(c-d)).$$

~~Assume for the sake of contradiction that~~ $\cancel{0=a^2d-(b+c)(b-d)(c-d)}$.

~~→ see next page~~

Jury notes:

~~Then $0 = a^2d - (b+c)(bc - (b+c)d + d^2)$~~
~~$= -(b+c)d^2 + (a^2+(b+c)^2)d - bc(b+c).$~~
~~This is a quadratic in d with leading coefficient $-(b+c)<0$, so its graph in an xy-plane is a concave down parabola. The y-coordinate of the vertex of the parabola is~~

~~$$\frac{-(a^2+(b+c)^2)^2}{4(-(b+c))} + (-bc(b+c))$$~~
~~$$= \frac{(a^2+(b+c)^2)^2}{4(b+c)} - bc(b+c)$$~~

Strangely, if $a=5$ and $b=3$ and $c=4$, then from the above quadratic in d, there is a seemingly valid solution.

Jury notes:

If it is proved that in a graph with 100 vertices such that the degree of each vertex is at least 90, then there is a vertex with degree 99, then the problem is solved.

Jury notes: