STUDENT 5

4) Triangles ADC, BAD and AED are isosceles, and thanks to this, we can obtain different angles.

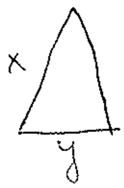
As ΔAED is isosceles, E is on the median of $\overline{AD}$, which also goes through C.

Using that the sum of the three angles of a triangle is 180, we obtain that $\angle BEC = 60^\circ$

Now we just have to show that
$\overline{EC} = \overline{AB}$.

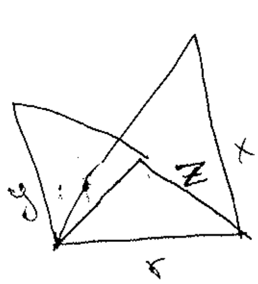
And this is enough to prove that ΔECB is equilatera

In an isosceles triangle:



we have $y = 2\sqrt{x\,(x - \text{sen}\,\alpha)}$

If we simplify, what we have is:



$z = 2y\sqrt{\text{sen}^2 50 - 1}$

badly used formula

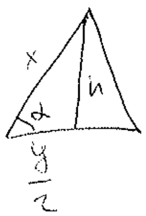
$r^2 = 2z\sqrt{\text{sen}^2 50 - 1}$

$r^2 = 4y\,(\text{sen}^2 50 - 1)$

$r^2 = 2x\sqrt{\text{sen}^2 80 - 1}$

$2x\sqrt{\text{sen}^2 80 - 1} = 4y\,(\text{sen}^2 50 - 1)$

Prove of the formula



$$x^2 - \left(\frac{y}{2}\right)^2 = h^2 \ , \quad x^2 - \frac{y^2}{4} = h^2 \ ;$$

$$h = x \cdot \text{sen}\,\alpha \Rightarrow$$

$$\Rightarrow x^2 - \frac{y^2}{4} = x \cdot \text{sen}\,\alpha \ ;$$

$$y^2 = 4x^2 - 4x \cdot \text{sen}\,\alpha$$

$$y = \sqrt{4x^2 - 4x \cdot \text{sen}\,\alpha}$$

$$y = 2\sqrt{x \cdot (x - \text{sen}\,\alpha)}$$

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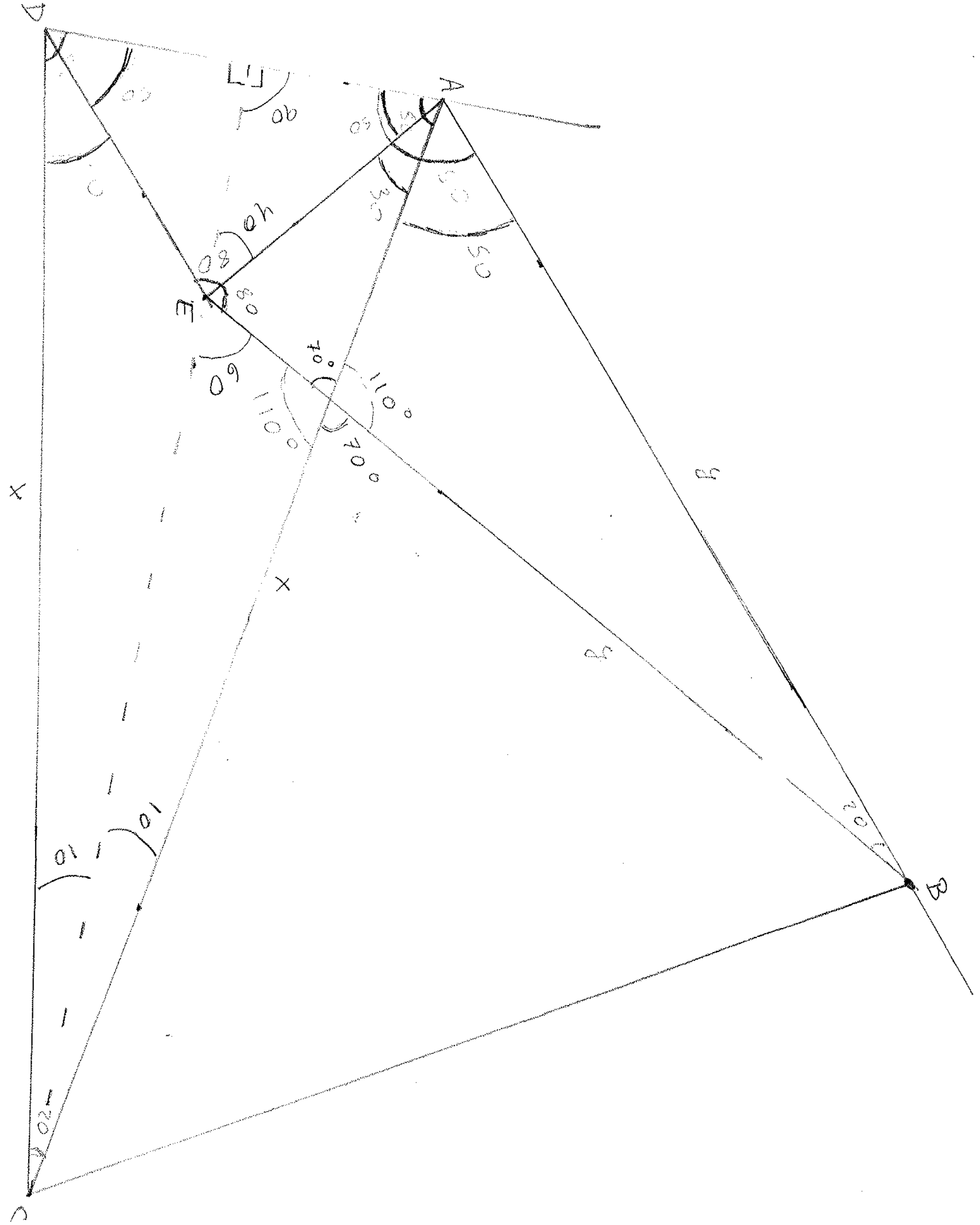


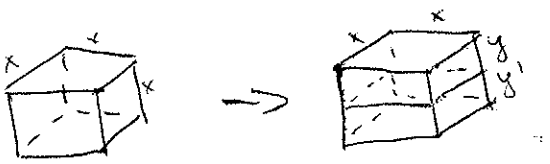
STUDENT 5

2) Let's call n the number of parallelepiped

If $n=1$, the parallelepiped is not typical, because it has to be the cube itself.

If $n=2$, we divide the cube in two parallelepiped.

This case is equivalent to "cutting" the cube with a plane, perpendicular to one of its sides. It is obvious that, among the six faces, we divide 4 ~~an~~ into two parts, which can be different, but we still have two faces which have not been divided, and so, they have two equal dimensions.

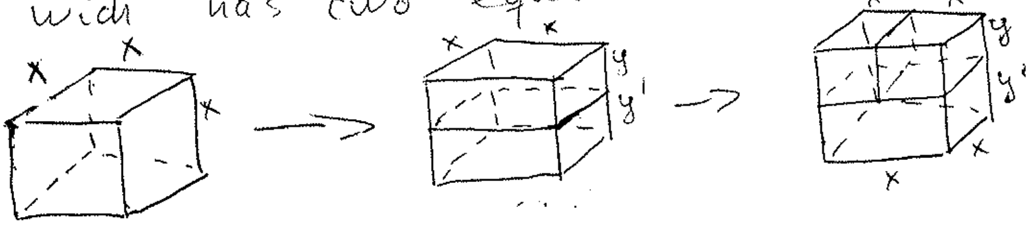


$x = x$

If $n=3$, we divide the cube in three parallelepiped

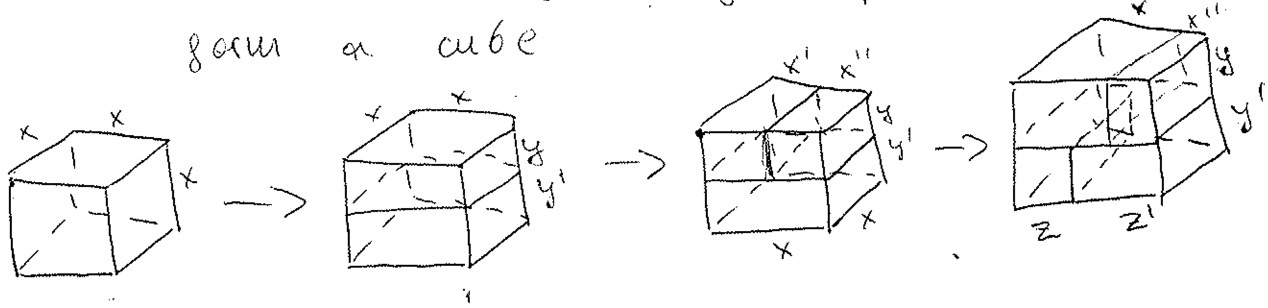
This case is equivalent to "cutting" the cube with a plane, perpendicular to one of its sides, and then choosing one of the two formed parallelepiped and also "cutting" it with a perpendicular plane.

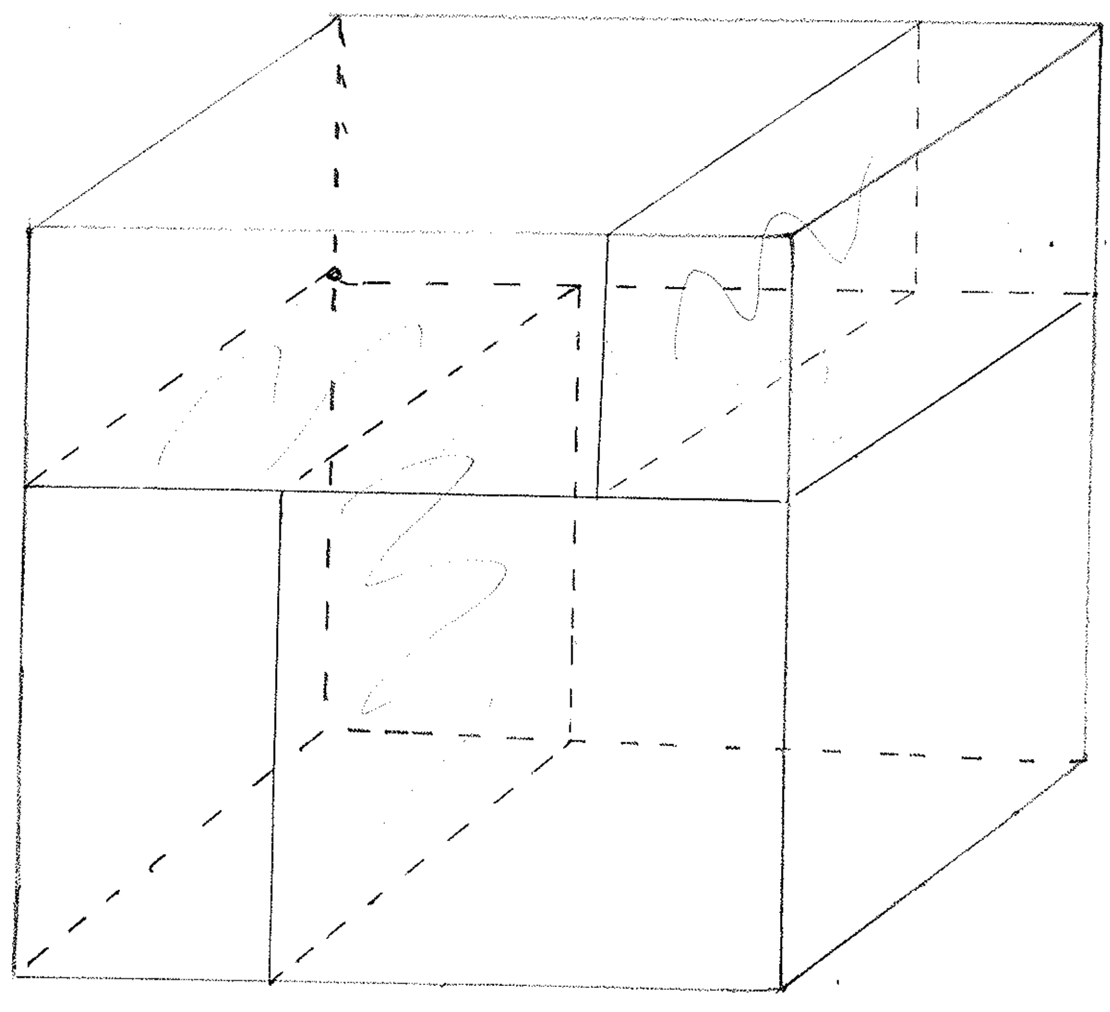
If we do this, there still remains a parallelepiped wich has two equal dimensions



$x = x$

If $n=4$ we divide the cube in four parallelepiped, which is equivalent to divide it with three planes. With two planes, we divide the cube, just as we did in the case $n=3$.
Now, with the third plane, we divide the parallelepiped which had not been divided, and we divide it. By doing this, we can obtain four typical parallelepipad which can form a cube





STUDENT 5

5) ~~xxx~~ To solve this problem, we'll just calculate the number of "sets" for the three complexities.

Complexity 1

First number	Second number	Third number
$3 \cdot 3 \cdot 3 \cdot 3$	$\underbrace{1 \cdot 1 \cdot 1 \cdot 2}$	$1 \cdot 1 \cdot 1 \cdot 1$

As this 2 can be in various positions, we take them into account by multiplying it by $\binom{4}{1}$ (number of different positions)

Sum: $3 \cdot 3 \cdot 3 \cdot 3 \cdot 2 \cdot \binom{4}{1} \cdot 1 = 3^4 \cdot 2^3$ sets

Complexity 2

First number	Second number	Third number
$3 \cdot 3 \cdot 3 \cdot 3$	$1 \cdot 1 \cdot 2 \cdot 2$	$1 \cdot 1 \cdot 1 \cdot 1$

The same as before, but multiplying by $\binom{4}{2}$

Sum: $3^4 \cdot 2\binom{4}{2} \cdot 1 = 3^4 \cdot 24 = 3^5 \cdot 2^3$ sets

Complexity 3

First number	second number	Third number
$3 \cdot 3 \cdot 3 \cdot 3$	$2 \cdot 2 \cdot 2 \cdot 1$ Same as before, but with $\binom{3}{4}$	$1 \cdot 1 \cdot 1 \cdot 1$

Sum $3^4 \cdot 2^3 \cdot \binom{4}{3} \cdot 1 = 3^4 \cdot 2^5$ sets

Then we obtain that in complexity 3 we have more sets. We might think that, to take into account all the positions, we need to multiply the sum by the three positions. This is not necessary, but even if it were, the result would not change, becaus we multiply the three amounts of sets by the same number.

STUDENT 5

3) If we express $2^n + n^{2066}$ in mod 2 we obtain:

$$2^n + n^{2016} \equiv_2 0 + n^{2016} \equiv_2 n$$

we can simplify ~~to~~ n^{2016}, given the fact that:

$$0^n \equiv_2 0 \quad \text{and} \quad 1^n \equiv_2 1$$

So we obtain that, when n is even, we obtain an even number, which is divisible by 2 except in the case $n = 0$, in which the number is 0 (prime)

Now we express the number in mod 3:

n can be 0, 1 or 2

2^n can have two results:

- 1 if n is even
- 2 if n is odd

As we've proved that n can't be even, we say it's odd,

so $2^n \equiv_3 2$

n^{2016} can have three posibilities:

$$0^{2016} \equiv_3 0$$
$$1^{2016} \equiv_3 1$$
$$2^{2016} \equiv_3 1$$

if $n \equiv_3 1$ or 2, we obtain:

$$2^n + n^{2016} \equiv_3 2+1 \equiv_3 0$$

so it would not be prime. Then, $n \equiv_3 0$, and it's not even, so it can be expressed as $n = 3 \cdot k$, being k an odd number we also include the cases for which $n=0$ and $n=1$, whose results are 0 and 3 respectively.

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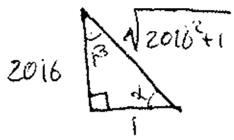
1) we can say that the triangle is rectangle, and the sides have this lenght :



$$Tg\ \alpha = \frac{2016}{1} = 2016$$
$$Tg\ \beta = \frac{1}{2016} \simeq \frac{1}{2^{10}} \simeq 0,0007$$
$$Tg\ 90 = \text{not defined}$$

In this case, the sum of tangents is close to 2016, and the biggest angle is 90°