1. 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12

All possible perfect cubes we can make with these numbers are: $1 \cdot 2 \cdot 4 = 8 = 2^3$

$2 \cdot 4 \cdot 8 = 64 = 4^3$

$1 \cdot 3 \cdot 9 = 27 = 3^3$

$3 \cdot 8 \cdot 9 = 216 = 6^3$

$4 \cdot 6 \cdot 9 = 216 = 6^3$

$3 \cdot 6 \cdot 12 = 216 = 6^3$

$2 \cdot 9 \cdot 12 = 216 = 6^3$

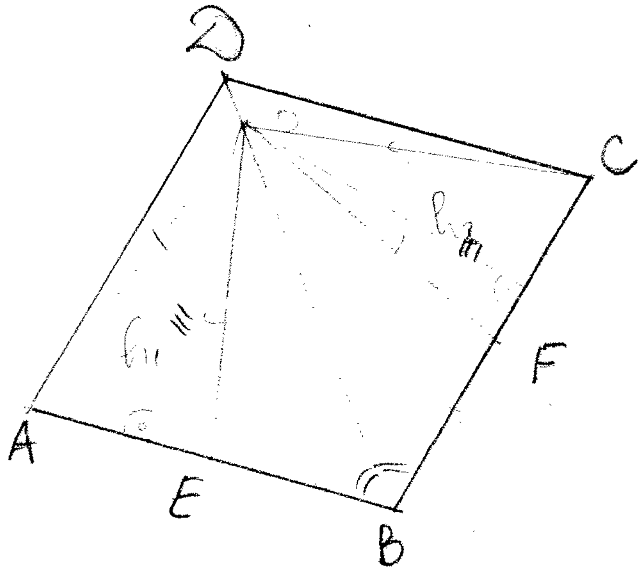
These are 7 multiplications and each factor is in no more than 3 multiplications.

$\Rightarrow$ if there are $12-2=10$ elements in the subset A, there will be one possible perfect cube.

So the maximum number of elements of the subset A is $12-3=9$

On example for the subset is $\{1,3,4,5,7,8,10,11,12\}$

2.



ABCD- rhombus

$\Rightarrow AB = BC = CD = DA$

E - center of AB

F - center of BC

$\Rightarrow AE = EB = \frac{1}{2} AB$

$BF = FC = \frac{1}{2} BC$

$\Rightarrow$ 1) $AE = EB = BF = FC$

2) $PA = PF$

3) $PE = PC$

$\Rightarrow \triangle AEP \cong \triangle FCP$

2. Let's h_1 is ~~the~~ a height in $\triangle AEP$ and h_2 is ~~the~~ a height in $\triangle FCP$

But $\triangle AEP \cong \triangle FCP$

$\Rightarrow h_1 = h_2$

ABCD is rhombus so BD is a bisector of $\sphericalangle ABC$ and each point from line BD is at same distance from AB and BC.

$\Rightarrow$ P lies on the line BD.

3.

$$\left(\frac{xy}{z} + \frac{zx}{y} + \frac{yz}{x}\right)\left(\frac{x}{yz} + \frac{y}{zx} + \frac{z}{xy}\right)$$

After uncovering the brackets the expression becomes

$$\frac{x^2\cancel{y}}{\cancel{y}z^2} + \frac{\cancel{x}y^2}{z^2\cancel{x}} + \frac{\cancel{xyz}}{\cancel{xyz}} + \frac{\cancel{z}x^2}{y^2\cancel{z}} + \frac{\cancel{xyz}}{\cancel{xyz}} + \frac{z^2\cancel{x}}{\cancel{x}y^2} + \frac{\cancel{xyz}}{\cancel{xyz}} +$$

$$+ \frac{y^2\cancel{z}}{x^2\cancel{z}} + \frac{\cancel{y}z^2}{x^2\cancel{y}} =$$

$$= \frac{x^2}{z^2} + \frac{y^2}{z^2} + 1 + \frac{x^2}{y^2} + 1 + \frac{z^2}{y^2} + 1 + \frac{y^2}{x^2} + \frac{z^2}{x^2} =$$

$$= 3 + \frac{x^2}{z^2} + \frac{z^2}{x^2} + \frac{y^2}{z^2} + \frac{z^2}{y^2} + \frac{y^2}{z^2} + \frac{z^2}{y^2}$$

$$\left(\frac{x}{z} - \frac{z}{x}\right)^2 = \frac{x^2}{z^2} - 2\frac{x}{z}\cdot\frac{z}{x} + \frac{z^2}{x^2} = \frac{x^2}{z^2} - 2 + \frac{z^2}{x^2} \geq 0$$

$$\Rightarrow \frac{x^2}{z^2} + \frac{z^2}{x^2} \geq 2$$

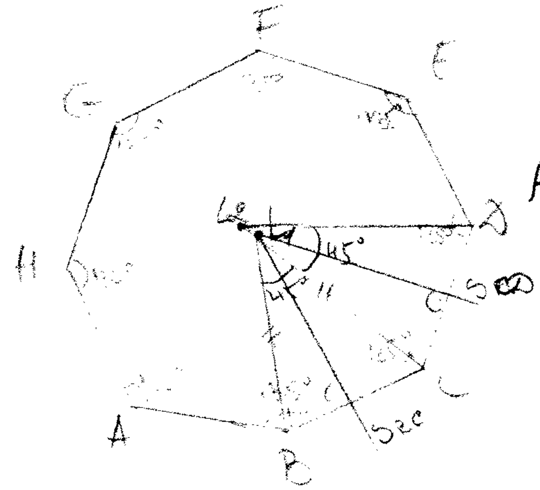
$\Rightarrow$ on this way $\frac{x^2}{y^2} + \frac{y^2}{x^2} \geq 2$ and $\frac{y^2}{z^2} + \frac{z^2}{y^2} \geq 2$

3.

$$\Rightarrow 3 + \frac{x^2}{z^2} + \frac{z^2}{x^2} + \frac{x^2}{y^2} + \frac{y^2}{x^2} + \frac{y^2}{z^2} + \frac{z^2}{y^2} \geq 3+2+2+2=9$$

$\Rightarrow$ the minimal positive value of the expression is 9 and it's possible when $x=y=z=1$

4.



The sum of all angles in the octagon is 1080°

All angles are equal $\Rightarrow$ each angle is $\frac{1080}{8} = 135°$

Let's L_1 is a transversal of the bisectors of $\angle ABC$ and $\angle BCD$

$\angle L_1BC = \frac{1}{2} \cdot 135° = \angle BCL_1$

$\Rightarrow \triangle BCL$ is isosceles and $L_1B = L_1C$

$\Rightarrow L_1 \in S_{BC}$ (~~and $L_1 \in S_{CD}$~~)

Let's L_2 is a transversal of the bisectors of $\angle BCD$ and $\angle CDE$

(~~But $CL_1 \in \ell \angle BCL$~~) $\Rightarrow \angle DCL_2 = \frac{1}{2} \cdot 135° = \angle CDL_2$

$\Rightarrow \triangle CDL_2$ is isosceles and $CL_2 = DL_2$

$\Rightarrow L_2 \in S_{CD}$ (~~and $L_2 \in S_{DE}$~~)

Let's see the pentagon $BCDL_2L_1$

$\angle BL_1C = 180° - (\angle CBL_1 + \angle \cancel{BCL_1}) = 180° - (\frac{1}{2} \cdot 135 + \frac{1}{2} \cdot 135°) =$

$= 180° - 135° = 45°$

$\angle CL_2D = 180° - (\angle L_2CD + \angle L_2DC) = 180° - (\frac{1}{2} \cdot 135° + \frac{1}{2} \cdot 135°) =$

$= 180° - 135° = 45°$

~~In the~~ rectangle $BCDL_2$: $\angle DL_2C = \angle$

$\sphericalangle (S_{BC} \cap S_{CD}) = 45^\circ$

But $\sphericalangle BL_2C = \sphericalangle CL_2D = 45^\circ$

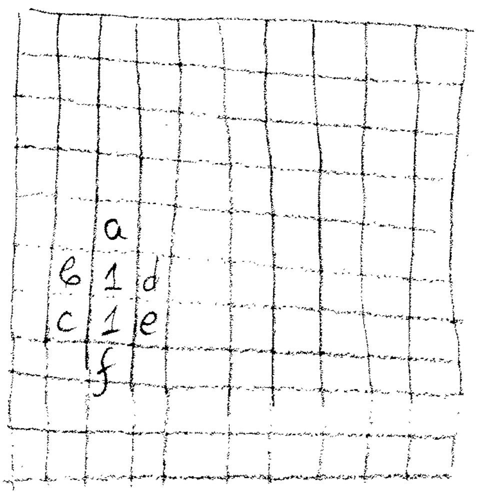
$\Rightarrow$ this can be impossible if $L_1 \equiv L_2$

$\Rightarrow BL_1 = CL_1 = DL_1$

On this way L_1 is at same distance from all vertices in the octagon.

$\Rightarrow L_1$ is a center of symmetry

5.



The smallest possible sum of two positive integers is $1+1=2$

(~~But this c~~)

If a cell with 1 isn't coloured next to it there must be one more cell with one. The second cell with isn't coloured, too

d is coloured $\Rightarrow 1<d$ and $e<d$

But e is coloured, too. $1<e$ ~~and~~

$\Rightarrow d<e$ but $e<d$ $\Rightarrow$ this is impossible

$\Rightarrow$ the smallest possible sum can't be 2.

If there is a uncoloured cell in the table, next to it there will be another cell with 1 in it, which is also uncoloured

$\Rightarrow$ in the table there aren't uncoloured cells with 1 in them.

5.

$\Rightarrow$ the smallest possible sum in this case is $2+2=4$