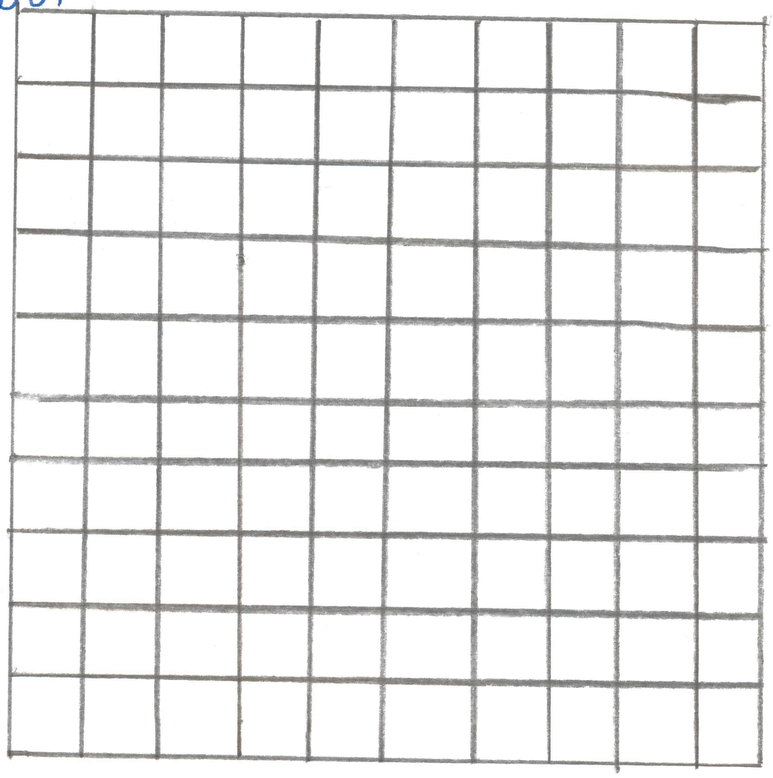
1) The maximum sum of the perimeters of the rectangles is obtined when the square is cuted in 100 squares 1X1 like in the figure.



The sum of the perimeters of the squares is equal with $4 \cdot 100 = 400$.

So the maximum sum of the perimeters of the rectangles is 400.

=) To obtain the sum equal with 398 we need to delete a line of lenght 1 who isn't from the original square 10X10 (every line segment who isn't from the original square is putted in the sum twice)

In total, are 180 line segments who aren't from the original square 10X10 and we can delete one of them to obtain the sum of perimeters of the rectangles 398, so we have 180 ways to cut a 10X10 square into several rectangles such that the sum of their perimeters is equal to 398.

② Ben has winning strategy.

His strategy is:

If Alex write a digit in the first, secont, thirth, fourth, fiveth, sixth, seventh or eighth square at the first move, Ben can write the digit 2 into the nineth square. The 9-digit number will have the last digit equal with 2 so it can't be a perfect square and Ben wins.

If Alex write a digit in the nineth square we have two cases:

Case one: If Alex writed an even digit.

If Alex writed a digit of the form $4k, k \in \mathbb{N}$, Ben can write an odd digit in the eighth square.

$\Rightarrow$ Because a number of two digits who have the first digit odd and the second of the form $4k, k \in \mathbb{N}$ isn't divisible by 4, and the 9-digit number has the last digit equal with a digit of the form $4k, k \in \mathbb{N}$, and the eighth digit equal with an odd digit, the 9-digit number isn't divisible by 4 but the 9-digit number is divisible by 2 so it isn't a perfect square and Ben wins

If Alex writed a digit of the form $4k+2, k \in \mathbb{N}$, Ben can write an even digit in the eighth square.

$\Rightarrow$ Because a number of two digits who have the first digit even and the second of the form $4k+2$, $k \in \mathbb{N}$ isn't divisible by 4 and the 8-digit number has the last digit equal with a digit of the form $4k+2$, $k \in \mathbb{N}$ and the eighth digit equal with an even digit, the 8-digit number isn't divisible by 4 but it is divisible by 2, so it isn't a perfect square and Ben wins

Case two: If ~~Alex~~ writes an odd digit:

Ben will write an odd digit in the eigth square.

I will prove that a number who has the last 2 digits odd isn't a perfect square:

Let be x a number an a, his last digit, a is odd.

I denote the number who is created by digits of x, excepting the last digit with n

$\Rightarrow x = 10n + a$

$x^2 = (10n+a)^2 = 100n^2 + 20n \cdot a + a^2 = 20(5n^2 + n \cdot a) + a^2$

a - odd digit $\Rightarrow a \in \{1, 3, 5, 7, 9\} \Rightarrow a^2 \in \{1, 9, 25, 49, 81\}$

$\Rightarrow$ Because a^2 has the first digit even, if we consider $1 = \overline{01}$ and $9 = \overline{09}$, $20(5n^2 + n \cdot a) + a^2$ has the second last digit even and the last digit odd $\Rightarrow$ A perfect square who has the last digit odd, has the second last digit even $\Rightarrow$ A number with the last two digits odd isn't a perfect square => the nine-digit number isn't a perfect square => Ben wins.

Because Ben wons in every case, he has a winning strategy.

③ Let be the table like in figure and the properties are:

0	1	1	1
1	0	1	1
1	1	0	1
1	1	1	0

The first column corresponds to the property: „the figure has an acute angle", the second one corresponds to the property: „some of the sides are equal", the thirth one corresponds to the property: „the figure has a side who has the lenght double than another side", the fourth one corresponds to the property: „the figure has a right angle".

The first row corresponds to a rectangle ABCD who has $AB = 2 \cdot BC$

The second row corresponds to a triangle ABC who has $AB = 2 \cdot BC$ and $m(\widehat{ABC}) = 90^\circ$

The thirth row corresponds to an isosceles triangle ABC who has $AB = BC$ and $m(\widehat{ABC}) = 90^\circ$

The fourth row corresponds to an isosceles trapezium ABCD who has $AB \parallel CD$, $AD = BC$, $AB = 2AD$, $m(\widehat{DAB}) = m(\widehat{CBA}) = 30^\circ$ and $m(\widehat{BCD}) = m(\widehat{ADC}) = 120^\circ$.

This table is valid.

④ We need to prove that there is a bag that contains a bag with another bag inside. Let assume the contrary:

I assume that there isn't a bag that contains a bag with another bag inside.

I will calculate the minimum number of candys who is necesary to can put the candies into 100 bags in such a way that no two bags contain the same number of candies and none of the bags is empty and doesn't exists a bag that contains a bag with another bag inside.

The minimum number of candies is obtined when in the first 50 bags is putted 1, 2, 3, ..., respectively 50 candies. In the next 50 bags is putted: bag_1 and 50 candies; bag_2 and 50 candies; ....., respectively, bag_{50} and 50 candies.

$\Rightarrow$ In bag_k are k candies, $k \in \{1, 2, 3, 4, 5, ..., 100\}$

In total, there are $1+2+3+...+50+50 \cdot 50$ candies

$1+2+3+....+50+50 \cdot 50 = \frac{50 \cdot 51}{2} + 50 \cdot 50 = 1275 + 2500 = 3775$

But $3775 > 2018 \Rightarrow$ We have contradiction $\Rightarrow$ There is a bag that contains a bag with another bag inside.

⑤ Let assume by contradiction that 1 can be uncolored:

Let consider the figure 1:

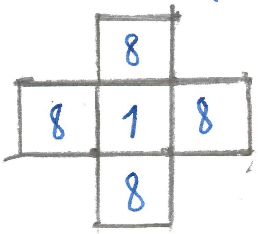


figure 1

If in the figure 1, 1 is the only number who is uncolored, his neighbors are colorated and because they are positive integers, they need to be 8 (because they can't be $\frac{1}{8}$).

Because all 1 neighbors's are egal with 1·8, 1 is colored but this contradict the assuming.

Let consider the figure 2

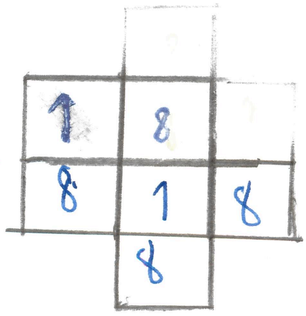


figure 2

If in the figure 2, 1 and his up neighbor are uncolorated, 1's down, left and right neighbors are colored and because they are positive integers, they are equal with 8 (they can't be $\frac{1}{8}$).

Because 1's left neighbour is colored, his up neighbor is 1 and his right neighbour is 8 so all, 1's neighbours are right => 1 is colored but this contradict the assuming.

Because every case contradict the assuming, we have contradiction => The smallest possible sum of the 2 uncolored number is $8+8=16$.

We have an example:

The sircled numbers are uncolored

8	1	8	1	8	1	8	1	8	1
1	8	1	8	1	8	1	8	1	8
8	1	8	1	8	1	8	1	8	1
1	8	1	8	1	8	1	8	1	8
8	1	8	64	8	1	8	64	8	1
1	8	64	8	64	8	64	8	64	8
8	1	8	1	8	1	8	1	8	1
1	8	1	8	1	8	1	8	1	8
8	1	8	1	8	1	8	1	8	1
1	8	1	8	1	8	1	8	1	8