International mathematical olympiad
"Formula of Unity" / "The Third Millenium"
2013/2014 year

2nd round, grade R5*

1. Divide the chessboard into two connected parts so that the first part is 4 squares more than the second one but the second part contains 4 black squares more than the fist one.

   Part is *connected* if it stays in one piece; connection must be by a segment.

2. From Monday to Friday a worker painted a fence in 8 hours' shifts. On Monday he worked twice as slow as during midweek (Tuesday, Wednesday, Thursday).

   On Friday he worked twice as fast as during midweek and finished his job after 6 hours. In the result he painted 300 meters more on Friday than on Monday. How long was the fence?

3. Determine the number of 4-digit numbers each composed of distinct digits with the first digit divisible by 2 and the sum of the first and the last digits divisible by 3.

4. Simpsons family celebrates only those birthdays when one's age equals to the sum of the digits of his/her birth year. Adam's celebration was in 2013 and Betty's celebration was in 2014. Who is older and by how many years?

5. Karlsson bought in cafeteria several crepes (25 rubles per piece) and several jars with honey (340 rubles per jar). When he told Smidge the total amount he spent Smidge was able to determine the number of crepes and the number of jars with honey. Can it happen that Karlsson spent more than 2000 rubles?

6. Brothers found a treasure of gold and silver. They divided it so that each share was 100 kg. The oldest brother got 30 kg (more than anyone else) of gold and one fifth of all the silver. How much gold was there in the treasure?

Note. In each problem it is required not only to submit an answer but also provide an explanation. In particular, if it is required to find some variable, one should find all possible values and prove that the variable cannot take any other values.

---

* Corresponds to grade 6 in USA, Canada.

International mathematical olympiad
"Formula of Unity" / "The Third Millenium"
2013/2014 year

2nd round, grade R6*

1. Divide the chessboard into two connected parts so that the first part is 6 squares more than the second one but the second part contains 6 black squares more than the fist one.

   Part is *connected* if it stays in one piece; connection must be by a segment.

2. Simpsons family celebrates only those birthdays when one's age equals to the sum of the digits of his/her birth year. Adam's celebration was in 2013 and Betty's celebration was in 2014. Who is older and by how many years?

3. Determine the number of 5-digit numbers each composed of distinct digits with the first digit divisible by 2 and the sum of the first and the last digits divisible by 3.

4. At the beginning of year an exchange rate of US dollar to euro was 0.8. An expert predicted that during this year an exchange rate euro to ruble would increase by 8% while a rate US dollar to ruble would drop by 10%. If his prediction is correct what would be an exchange rate of US dollar to euro by the end of the year?

5. Karlsson bought in cafeteria several crepes (25 rubles per piece) and several jars with honey (340 rubles per jar). When he told Smidge the total amount he spent Smidge was able to determine the number of crepes and the number of jars with honey. Can it happen that Karlsson spent more than 2000 rubles?

6. Brothers found a treasure of gold and silver. They divided it so that each share was 100 kg. The oldest brother got 30 kg (more than anyone else) of gold and one fifth of all the silver. How much gold was there in the treasure?

Note. In each problem it is required not only to submit an answer but also provide an explanation. In particular, if it is required to find some variable, one should find all possible values and prove that the variable cannot take any other values.

---

* Corresponds to grade 7 in USA, Canada.

International mathematical olympiad
"Formula of Unity" / "The Third Millenium"
2013/2014 year

2nd round, grade R7*

1. Two children, Ann and Betty, thought of a number (each of her own). Then each girl wrote all the divisors of her number, Ann wrote 10 numbers and Betty wrote 9 numbers. How many distinct numbers were written on the board if the greatest number written twice was 50?

2. A closed broken line is constructed along the lines of a grid, with its total length equal to 36 cell sides. What is the maximal area bounded by this line?

3. Consider a circle and three equal chords passing through one point. Prove that each chord is a diameter.

4. Brothers found a treasure of gold and silver. They divided it so that each share was 100 kg. The oldest brother got 25 kg (more than anyone else) of gold and one eighth of all the silver. How much gold was there in the treasure?

5. The distance between two villages A and B is 45 km. Three friends have two bicycles, the speed of a cyclist is 15 km/h and the speed of a hiker is 5 km/h. What the minimal time is needed for them to go from A to B? Two people cannot ride the same bike simultaneously and they cannot leave the bike on the road unattended.

6. Lev took two natural numbers and added their sum to their product, getting 1000. What numbers might that be? Find all possible pairs.

Note. In each problem it is required not only to submit an answer but also provide an explanation. In particular, if it is required to find some variable, one should find all possible values and prove that the variable cannot take any other values.

* Corresponds to grade 8 in USA, Canada.

International mathematical olympiad
“Formula of Unity” / “The Third Millenium”
2013/2014 year

2nd round, grade R8*

1. Two children, Ann and Betty, thought of a number (each of her own). Then each girl wrote all the divisors of her number, Ann wrote 10 numbers and Betty wrote 9 numbers.How many distinct numbers were written if both students wrote number 6?

2. Consider a circle and three equal chords passing through one point. Prove that each chord is a diameter.

3. Brothers found treasure of gold and silver. They split it so that each weighed 100kg. The oldest brother got 1/5 of all gold, and 1/7 of all silver. The youngest brother got 1/7 of all gold. What part of all silver did the youngest brother get?

4. The distance between two villages A and B is 45 km. Three friends have two bicycles, the speed of a cyclist is 15 km/h and the speed of a hiker is 5 km/h. What the minimal time is needed for them to go from A to B? Two people cannot ride the same bike simultaneously but they can leave the bike on the road unattended.

5. Karlsson bought in cafeteria several crepes (25 rubles per piece) and several jars with honey (340 rubles per jar). When he told Smidge the total amount he spent Smidge was able to determine the number of crepes and the number of jars with honey. Can it happen that Karlsson spent more than 2000 rubles?

6. A closed broken line is constructed along the lines of a grid, with its total length equal to 2014 cell sides. What is the maximal area bounded by this line?

---

* Corresponds to grade 9 in USA, Canada.

International mathematical olympiad
“Formula of Unity” / “The Third Millenium”
2013/2014 year

2nd round, grade R9*

1. Given a convex pentagon. For each pair of its diagonals intersecting inside consider the smallest angle between them. Find all possible values of the sum of all these five angles.

2. Two children, Ann and Betty, thought of a number (each of her own). Then each girl wrote all the divisors of her number, Ann wrote 10 numbers and Betty wrote 9 numbers.How many distinct numbers were written if both students wrote number 6?

3. Brothers found treasure of gold and silver. They split it so that each weighed 100kg. The oldest brother got 1/5 of all gold, and 1/7 of all silver. The youngest brother got 1/7 of all gold. What part of all silver did the youngest brother get?

4. A disc of radius 1 is given. Prove that one can find 3 non-overlapping pieces of the disc which could be rearranged into 1×2.4 rectangle. One can rotate and turn over pieces.

5. Let a and n be natural numbers. Given that a^n is 2014-digit number find the smallest k such that a cannot be a k-digit number.

6. Pavel invented a new way to add numbers. For two numbers a and b the *pavelsum* is defined as $a\#b = (a + b)/(1 - ab)$ (if it is defined). He gave three numbers a, b and c to Boris and Michael and asked Boris to *paveladd* a and b and then *paveladd* c to the result while Michael was asked to *paveladd* b and c, and then *paveladd* a to the result. Could Boris and Michael get different results?

---

* Corresponds to grade 10 in USA, Canada.

International mathematical olympiad
"Formula of Unity" / "The Third Millenium"
2013/2014 year

2nd round, grade R10*

1. Given a convex pentagon. For each pair of its diagonals intersecting inside consider the smallest angle between them. Find all possible values of the sum of all these five angles.

2. Let $f(x) = x^3 + 9x^2 + 27x + 24$. Solve equation $f(f(f(f(x)))) = 0$.

3. A disc of radius 1 is given. Prove that one can find 4 non-overlapping pieces of the disc which could be rearranged into 1×2.5 rectangle. One can rotate and turn over pieces.

4. $100,000$ squares were drawn inside a given square with the side 100. Diagonals of distinct inner squares do not intersect. Prove that at least one of the inner squares has a side length less than 1.

5. Let a and n be natural numbers. Given that a^n is 2014-digit number find the smallest k such that a cannot be a k-digit number.

6. Pavel invented a new way to add numbers. For two numbers a and b the *pavelsum* is defined by $a\#b = (a+b)/(1-ab)$ (if it is defined). He gave four numbers a, b, c and d Boris and Michael and asked Boris to *paveladd* a and b, and then *paveladd* c, and finally *paveladd* d to the result while Michael was asked to *paveladd* c and d, and then *paveladd* b, and finally *paveladd* a. Could Boris and Michael get different results?

---

* Corresponds to grade 11 in USA, Canada.

International mathematical olympiad
"Formula of Unity" / "The Third Millenium"
2013/2014 year

2nd round, grade R11*

1. Given a convex pentagon. For each pair of its diagonals intersecting inside consider the smallest angle between them. Find all possible values of the sum of all these five angles.
2. Let $f(x) = x^3 + 9x^2 + 27x + 24$. Solve equation $f(f(f(f(x)))) = 0$.
3. A disc of radius 1 is given. Prove that one can find 5 non-overlapping pieces of the disc which could be rearranged into 1×2.7 rectangle. One can rotate and turn over pieces.
4. Does there exist a tetrahedron with height of 60 cm, based perimeter 62 cm and the height of any lateral face (drawn to a side of the base) 61 cm?
5. Let a and n be natural numbers. Given that a^n is 2014-digit number find the smallest k such that a cannot be a k-digit number.
6. Pavel invented a new way to add numbers. For two numbers a and b the *pavelsum* is defined by $a\#b = (a+b)/(1-ab)$ (if it is defined properly). As usual he defined a *pavelproduct* of a and a natural number n as the *pavelsum* of n equal terms: $a@n = ((a\#a)\#a)\ldots\#a$. Does there exist two natural numbers $x \neq y$ such that $x@y = y@x$?

* Corresponds to grade 12 in USA, Canada.