

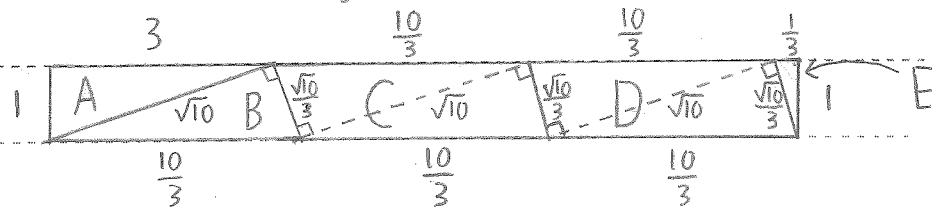
The first person has a winning strategy.

He starts with the number 9999. His opponent must use a number that starts with 9. Let this number be $9abc$, where a, b, c are non-zero digits and $9+at+bt+c$ is divisible by 9. He can respond by using the number $cba9$, because its sum of digits is divisible by 9. Now, his opponent has to use another number starting with 9, so he can repeat this strategy.

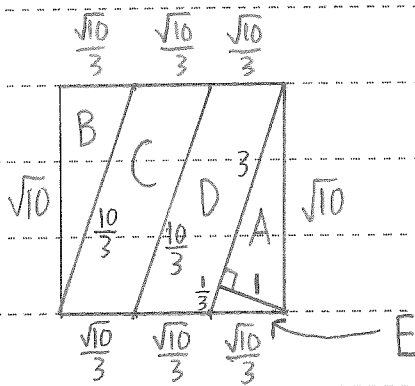
The only time when he cannot use $cba9$ is when his opponent writes a palindrome, $9aa9$. However, the sum of digits, $2a+18$, must be divisible by 9, so a is divisible by 9. Then, $9aa9$ can only be 9009 or 9999. 9009 has a 0, so it cannot be used. 9999 was already used on the first move. Therefore, his opponent cannot use a palindrome, so the first person always has an available number to use. Eventually, his opponent will run out of numbers that start with 9, and will lose the game.

Jury notes:

We can cut a 1×10 rectangle like this:

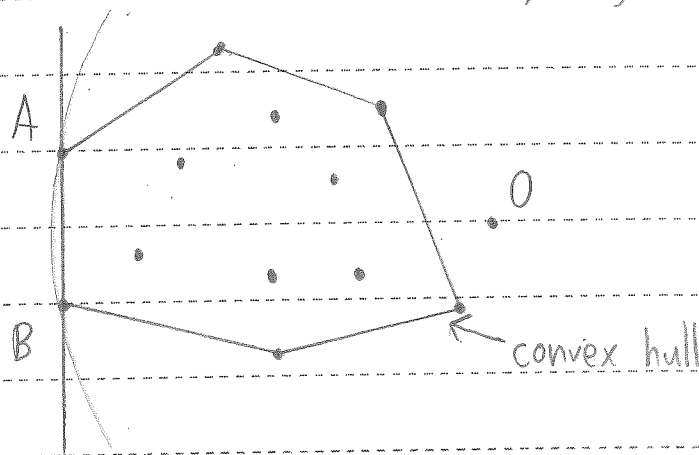


The 5 pieces can be arranged into a $\sqrt{10} \times \sqrt{10}$ square:



Jury notes:

Let A and B be two adjacent vertices on the convex hull of the $2n+1$ points. This means that the rest of the points are on the same side of line AB . Let O be a point on the perpendicular bisector of AB , on the same side of line AB as the other points, such that the circle with center O and radius $OA=OB$ contains all the remaining $2n-1$ points. A point O that satisfies these conditions will always exist, because as O becomes infinitely far away from line AB , the boundary of the circle approaches the line AB , and the area inside the circle will cover everything on that side of line AB .



As O moves left on the perpendicular bisector, the circle shrinks, then grows after O passes AB . One by one, the points inside the circle will be outside the circle after the boundary of the circle passes them. Only one point passes the boundary at a time, because if there were two (or more) points on the boundary at the same time, the two points, along with A and B , would be concyclic, which is a contradiction.

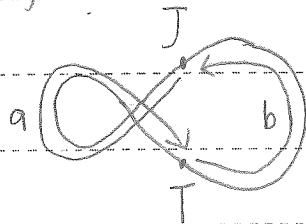
Jury notes:

As O becomes infinitely far on the left, the boundary of the circle approaches AB again, but the area inside the circle will be everything on the left of AB . All the points are now outside the circle. Let C be the n^{th} point to cross the boundary. The $n-1$ points that crossed before it must be outside the circumcircle of ABC , so there are $n-1$ points inside it. Therefore, the circumcircle of ABC is a valid circle satisfying the conditions.

Olympiad "Formula of Unity" / "The Third Millennium", February 25, 2017, Toronto, Ontario, Canada

Jury notes:

Let T and J be the starting points of Tom and Jerry, and t and j be their speeds. Let a be the distance from J to T in the direction of the road, and let b be the distance from T to J ,



In 20 minutes, Jerry ran from J to T and Tom ran from T to J , so $20 = \frac{a}{j} = \frac{b}{t}$, and $\frac{j}{t} = \frac{a}{b}$. Let x be the time it took Tom to catch up with Jerry. In the last $x-20$ minutes, Tom ran from J to J (a full loop) and Jerry ran from T to J , so $x-20 = \frac{a+b}{t} = \frac{b}{j}$, and $\frac{j}{t} = \frac{b}{a+b}$. Then $\frac{b}{a+b} = \frac{j}{t} = \frac{a}{b}$, so $b^2 = a(a+b) \rightarrow a^2 + ab - b^2 = 0$.

Solving this quadratic, we get $a = b\left(\frac{\sqrt{5}-1}{2}\right)$, so $\frac{a}{b} = \frac{\sqrt{5}-1}{2}$. Since Jerry ran from J to J in time x , and Tom ran from J to J in time $x-20$, $\frac{j}{t} = \frac{x-20}{x}$. Because $\frac{j}{t} = \frac{a}{b}$, $\frac{x-20}{x} = \frac{\sqrt{5}-1}{2} \rightarrow x = \frac{40}{3-\sqrt{5}} = 30+10\sqrt{5}$. Therefore, it took Tom $30+10\sqrt{5}$ seconds to catch Jerry.

Jury notes: