Cucăzunbea Ion Emanuel, grade 9

Subject I

We note the speed of Jerry v_J and speed of Tom v_T and l and L length of the 2 parts of road (we cut the roads in to parts: part 1 (of length l)  and part 2. 

Let's note the initial position I; the position after 20 min. II and final position III (we note time t between position II and III). Between position I and II Jerry run l and Tom L and between II and III Jerry run L and Tom $L+l$ $\Rightarrow$

$v_J = \frac{l}{20} = \frac{L}{t}$ and $v_T = \frac{L}{20} = \frac{L+l}{t}$ (from speed formula speed $= \frac{\text{distance}}{\text{time}}$)

$\Rightarrow \frac{v_J}{v_T} = \frac{l}{L} = \frac{L}{L+l} \Rightarrow Ll + l^2 = L^2$. We note $K = \frac{L}{l}$

$\Rightarrow L\cdot l + l^2 = L^2 \,|\, : l^2 \quad K+1 = K^2 \Rightarrow K^2 - K - 1 = 0 \Rightarrow \Delta = 1+4 = 5$

$\Rightarrow K = \frac{1 \pm \sqrt{5}}{2}$ but $K > 0 \Rightarrow K = \frac{1+\sqrt{5}}{2}$ ($\frac{1-\sqrt{5}}{2} < 0$ because $1 < \sqrt{5}$)

$v_J = \frac{l}{20} = \frac{L}{t} \Rightarrow \frac{t}{20} = \frac{L}{l} = \frac{1+\sqrt{5}}{2} \Rightarrow t = 10(1+\sqrt{5}) \Rightarrow$

$\Rightarrow$ total time is $t + 20 = 10(3+\sqrt{5}) = 30 + 10\sqrt{5}$ minutes

Subject II

If a player name a number with last digit $x \Rightarrow$ The next player name a number $\overline{xabc}$, a can take 9 values b 9 values and c 1 ($x+a+b+c \vdots 9 \Rightarrow c \equiv 9 \pmod 9$ s fixed $c \neq 0$ $\Rightarrow \exists$ only one number c) $\Rightarrow$ the next player can choose 81 numbers minus the numbers from previous steps who have the first digit x.

We note $M = \{ \overline{9abc} \mid \overline{9abc} \vdots 9 \}$ and $N = \{ \overline{abc9} \mid \overline{abc9} \vdots 9 \}$

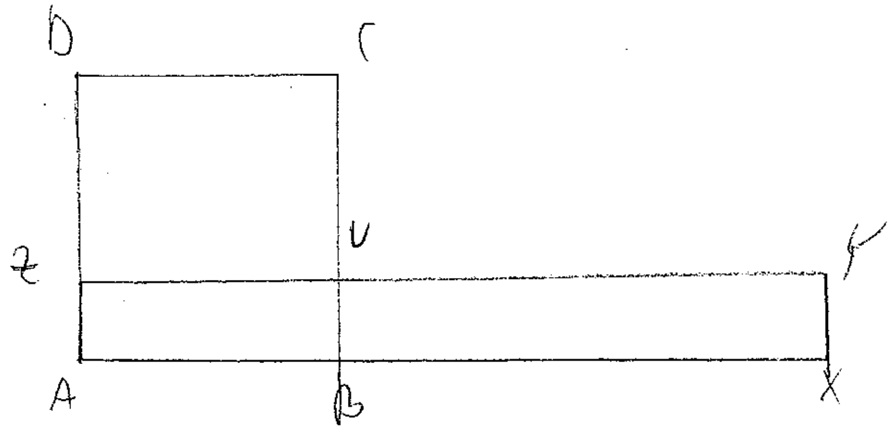
We proved the cardinal of N is 81, and by analogy cardinal of M is 81.

We will name strategy 1: if a player name $\overline{abcd}$ the next player (the player who aply this strategy) will name a number $\overline{dbc9}$ and force the next player to take a number from $M \Rightarrow$ the forced player can choose numbers from M.

We will show that the player 2 can win with this strategy. If player 1 don't use this strategy $\Rightarrow$ player 2 can choose numbers from a larger multitude of numbers than player 1 and he can win. If player 1 aply this strategy so he will loose because he must choose from a smaller multitude of numbers than player 2.

Subject III

Because the rectangle of side 1 and 10 and the square of side l have the same area $\Rightarrow 1 \cdot 10 = l^2 \Rightarrow l = \sqrt{10}$



$AZ = 1$

$AY = 10$ $\quad AB = AD = \sqrt{10}$

We will show that we can cut DZVC in 4 parts to make the rectangle BVYX

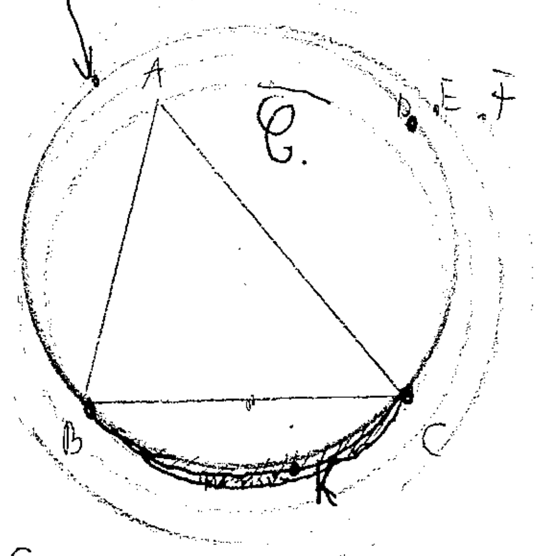
Subject IV

We use mathematical induction (for n)

First step $n=1$ $\Rightarrow$ The circumcircle of the triangle formed with the 3 points has the property from enunciation.

Second step For a "n" fixed the property is true. We will show is true for $n+1$.

We note A, B, C the points from the circle with the property $A_1, A_2 \dots A_{n-1}$ the points outside of it and $B_1, B_2 \dots B_{n-1}$ the points inside of it. We note $\mathcal{C}$ the circumcircle of ABC and increase its ray until he touch one point D and note it $\mathcal{C}_2$. Analogous we construct $\mathcal{C}_2$ and $\mathcal{C}_3$ and E, F. Let X, Y be the points added $\mathcal{C}_4$



If X is inside $\mathcal{C}$ and Y outside $\Rightarrow$ $\mathcal{C}$ has the property for $n+1$ too. $\Rightarrow$ we consider the case where X, Y is outside (by analogy the case when X, Y is inside is solved)

Two circles can have maximum 2 common points $\Rightarrow$ The circumcircle of DBC and ABC have only B, C commons. We note that circle $\mathcal{C}_4$. If in the shading part is no point of $B_1, \dots B_{n-1}$ $\Rightarrow$ C_4 has the property because it has inside $n-1+1$ (point A) $= n$ points and outside of it $n-1+2$ (X, Y) -1 (Y) $= n$ points outside

If in shade part is a point K we consider the C_5 circumcircle of ABC and by analogy like C_4 $\Rightarrow$ it has the property as in the blue part is a point K_2. We apply this reasoning until we won't have any other points (we can make this because inside the circle is a finitely numbers of points.)

Cerveinnerau Ion Emanuel, grade 9

Subject 5

If we make a layouts with 32 tiles we cover all the surface of board and when we make a layouts with 16 tiles we only cover half of the board $\Rightarrow S > N$ (because we have more ways to place the 16 tiles than 18 tiles)

6/6