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Problem no. 1.

The road is made of 3 parts : - the small loop
- the big loop
- the intersection.

From the hypothesis ⇒ Tom and Jerry start from the intersection (Jerry-up; Tom-down)

⇒ Tom runs though the big loop in 20 minuts and though the small loop in 15 minuts

⇒ Jerry runs though the small loop in 20 minuts.

We notate with d_1 = the distance of the first loop. (the small one)
d_2 = the distance of the big loop.

$d_1 = v_{Tom} \cdot 15 min = v_{Jerry} \cdot 20 min$

$\Rightarrow v_{Tom} \cdot 15 = v_{Jerry} \cdot 20 \Rightarrow v_{Tom} \cdot 3 = v_{Jerry} \cdot 4$

$\Rightarrow v_{Tom} = \frac{v_{Jerry} \cdot 4}{3} \Rightarrow v_{Tom} = v_{Jerry} \cdot \frac{4}{3}.$

In the moment Tom catches up Jerry we had the relation:

~~$d_{Tom} = d_{Jerry} + d_2 \Rightarrow v_{Tom} \cdot t = v_{Tom} \frac{3}{4} t + v_{Tom} \cdot 20$~~

~~$\Rightarrow \frac{1}{4} t = 20 \Rightarrow t = 80$~~

~~$d_{Tom} = d_{Jerry} + d_1 \Rightarrow v_{Tom} \cdot t = \frac{3}{4} v_{Tom} \cdot t + v_{Tom} \cdot 15$~~

~~$\Rightarrow \frac{1}{4} t = 15 \Rightarrow t = 60$~~

$d_{Tom} = d_{Jerry} + d_2 \Rightarrow v_{Tom} \cdot t = v_{Jerry} \cdot t + v_{Tom} \cdot 20$

$\Rightarrow \cancel{v_{Jerry}}\ v_{Tom} \cdot t = v_{Tom} \cdot \frac{3}{4} \cdot t + v_{Tom} \cdot 20$

$\Rightarrow t = 20 \cdot 4 = 80.$

(because at the beggining the diference beetwen the ~~at~~ two was d_2).

$\Rightarrow$ They will meet at the half of the second loop.

Problem 2

Because MASS is the sum of the cubes of all the divisors of our number (we will call it „m")

$\Rightarrow$ Every cube is lower than MASS; MASS ≥ 1000
but one of the divisors in m

$\Rightarrow$ $m^3 < MASS < 10000$

We observe that $21^3 < 10.000 < 22^3$

$\Rightarrow m^3 < 22^3 \Rightarrow m < 22 \Rightarrow m \in \{1, 2, 3, \ldots, 21\}$.

I. $m = 1 \Rightarrow MASS = 1^3 < 1000 \Rightarrow$ impossible

II. $m = 2 \Rightarrow MASS = 1^3 + 2^3 = 1 + 8 = 9 < 1000 \Rightarrow$ impossible

III. $m = 3 \Rightarrow MASS = 1^3 + 3^3 = 1 + 27 = 28 < 1000 \Rightarrow$ impossible

IV. $m = 4 \Rightarrow MASS = 1^3 + 2^3 + 4^3 = 1 + 8 + 64 = 73 < 1000 \Rightarrow$ impossible

V. $m = 5 \Rightarrow MASS = 1^3 + 5^3 = 1 + 125 = 126 < 1000 \Rightarrow$ impossible.

VI. $m = 6 \Rightarrow MASS = 1^3 + 2^3 + 3^3 + 6^3 = 1 + 8 + 27 + 216 = 252 < 1000 \Rightarrow$ imp.

VII. $m = 7 \Rightarrow MASS = 1^3 + 7^3 = 1 + 343 = 344 < 1000 \Rightarrow$ imp.

VIII. $m = 8 \Rightarrow MASS = 1^3 + 2^3 + 4^3 + 8^3 = 1 + 8 + 64 + 512 = 585 \Rightarrow$ imp (< 1000)

~~VIII.~~ IX. $m = 9 \Rightarrow MASS = 1^3 + 3^3 + 9^3 = 1 + 27 + 729 = 757 \Rightarrow$ imp (< 1000)

X. $m = 10 \Rightarrow MASS = 1^3 + 2^3 + 5^3 + 10^3 = 1 + 8 + 125 + 1000 = 1134 \Rightarrow$ impossible, because the last 2 digits are not the same.

XI. $m = 11 \Rightarrow MASS = 1^3 + 11^3 = 1 + 1331 = 1331 \Rightarrow$ impossible, because the last 2 digits are not the same.

XII. $m = 12 \Rightarrow MASS = 1^3 + 2^3 + 3^3 + 4^3 + 6^3 + 12^3 = 1 + 8 + 27 + 64 + 216 + 1728$
$= 2144 \Rightarrow$ The least possible value of MASS is 2144.

$MASS > MATH \Rightarrow MASS - MATH > 0 \Rightarrow SS - TH > 0$.

The least value of MATH is for the least value of MASS.

$\Rightarrow$ The least value of MATH is for $MASS = 2144$.

$SS - TH > 0 \Rightarrow SS > TH \Rightarrow 44 > TH$

The difference between SS and TH is one of the cubes of the divisors of $n (=12)$.

$\Rightarrow SS - TH \in \{1, 8, 27, 64, 216, 1728\}$.

$SS - TH < SS = 44$

$\Rightarrow SS - TH \in \{1, 8, 27\}$.

① $SS - TH = 1 \Rightarrow TH = 44 - 1 = 43 \Rightarrow T = S \Rightarrow$ impossible

②. $SS - TH = 8 \Rightarrow TH = 44 - 8 = 36 \Rightarrow \left.\begin{array}{l} MATH = 2136 \\ MASS = \cancel{20}2144 \end{array}\right\} \Rightarrow$ this value is possible

③ $SS - TH = \cancel{24}27 \Rightarrow TH = 44 - 27 = \cancel{07}\ 17 \Rightarrow MATH = 2117 \Rightarrow A = T$
$\Rightarrow$ impossible

$\Rightarrow$ The least value of $MATH = 2136$.

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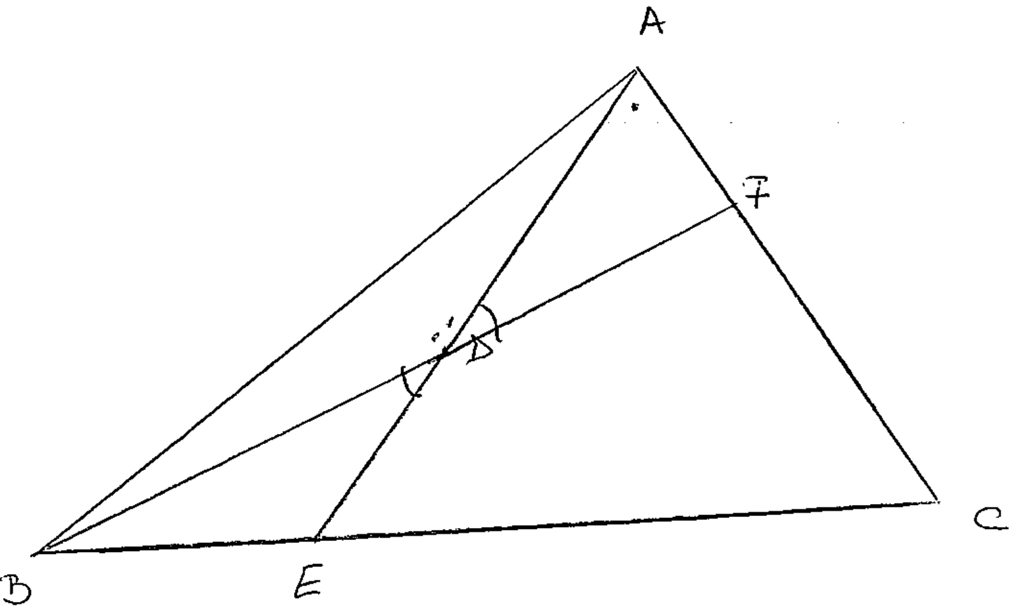
Problem no. 3



$E = AD \cap BC$.

$\cancel{m(\widehat{BED}) = 180^\circ - m(\widehat{AD}}$

$m(\widehat{BDE}) = m(\widehat{ABD}) + m(\widehat{BAD})$.

$\underline{m(\widehat{ABD})} + m(\widehat{DBC}) + m(\widehat{BCA}) + m(\widehat{CAD}) + \underline{m(\widehat{DAB})}$

$= 180$

$\Rightarrow m(\widehat{BDE}) = 180^\circ - m(\widehat{DBC}) - m(\widehat{BCA}) - m(\widehat{CAD})$.

We need to demonstrate that $m(\widehat{BDE}) < m(\widehat{DAC})$

$\Rightarrow \cancel{m}(180 - \widehat{DBC} - \widehat{BCA} - \widehat{CAD} < \widehat{CAD}$

$\Rightarrow 180 - \widehat{DBC} - \widehat{BCA} < 2 \cdot \widehat{CAD}$

$\Rightarrow \widehat{BAC} + \widehat{ACB} - \widehat{DBC} < 2 \cdot \widehat{CAD}$.

$\Rightarrow \widehat{BAD} + \widehat{ACB} - \widehat{DBC} < \widehat{CAB}$

$\widehat{DAC} + \widehat{ADB} > 180 \Leftrightarrow \widehat{DBE} < \widehat{DAF} \Leftrightarrow \widehat{ADF} < \widehat{DAF}$

$\Leftrightarrow AF < DF$. True (we can see it).

$BD + AC < BC \Rightarrow BD + AC < A\cancel{B} + AB$

$\Rightarrow BD < AB$

$\Rightarrow m(\widehat{BAD}) < \widehat{BDA}$

$\Rightarrow \widehat{BAD} + \widehat{DAC} < \widehat{BDA} + \widehat{DAC}$

$\Rightarrow \widehat{BAC} < \widehat{BDA} + \widehat{DAC}$

$BD < AB$

$\Rightarrow BE < AB \Rightarrow \widehat{BAE} < \widehat{AEB}$

$\Rightarrow \widehat{BAE} + EAC < \widehat{AEB} + \widehat{EAC}$

$\Rightarrow \widehat{BAC} < \widehat{AEB} + \widehat{EAC}$

II. $\widehat{DAC} + \widehat{ADB} > 180 \Leftrightarrow \cancel{180} - \widehat{DBE} + \widehat{DEB} + \widehat{CAD} - 180$

$\Leftrightarrow \widehat{DBE} + \widehat{DEB} + \widehat{CAD} > \widehat{DBE} + \widehat{DEB} + \widehat{BDE}$

$\Leftrightarrow \widehat{CAD} > \widehat{BDE} \Leftrightarrow \widehat{CAD} > \widehat{ADF} \Leftrightarrow AF < DF$

$\Leftrightarrow \cancel{AF + AF < DF + AF}$ $\cancel{AF + AD < AD + DF}$

$\Leftrightarrow AF < BF \cancel{+} - BD \Leftrightarrow \cancel{AF} - AC - FC < BF - BD$

$\Leftrightarrow AC + BD - FC < BF \Rightarrow BC - FC < BF$ ~~adevă~~ true because BFC is an triangle.

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Problem no. 4

We will call the person who starts - player A
the person's who starts rival - player B.

If player A writes the number $\overline{x_1 x_2 x_3 x_4 x_5 x_6 x_7 x_8}$, then the other person, the player B will write the number $x_8 x_7 x_6 x_5 x_4 x_3 x_2 x_1$. This number fulfils all the conditions, because the sum of the digits is the same with the number before and it starts with the ~~same~~ last digit of it.
Doing this every time, he will always have what number to write.

But there is a special case, where the number ~~has i~~ of player A has the form $\overline{abcddcba}$, then the number written by player B will be the same.

~~We observe that the numbers of player A al~~

$\Rightarrow 2(a+b+c+d) = 0 \pmod 9$, $\Rightarrow (a+b+c+d) = 0 \pmod 9$

~~Every time this happends player B will write the~~ number ~~a(b±3)(c±3)~~ $a(b+3)(c+3)(d+3)(d+3)(c+3)(b+3)$

~~for the digits higher than 6, the number plus 3 will be~~

~~7 -> 1~~
~~8 -> 2~~
~~9 => 3.~~

If player A writes this tipe of number, player be will write the number $adcbbcda$. It will work for all the cases except the one when $d=b$.

When $d=b$ we can take the number $a(d+3)(c+3)(d+3)(d+3)(c+3)(d+3)a$

Those numbers couldn't be written before, because the number written by A would be written twice. (for 7, 8, 9 we replace with 1, 2 or 3 respectively)

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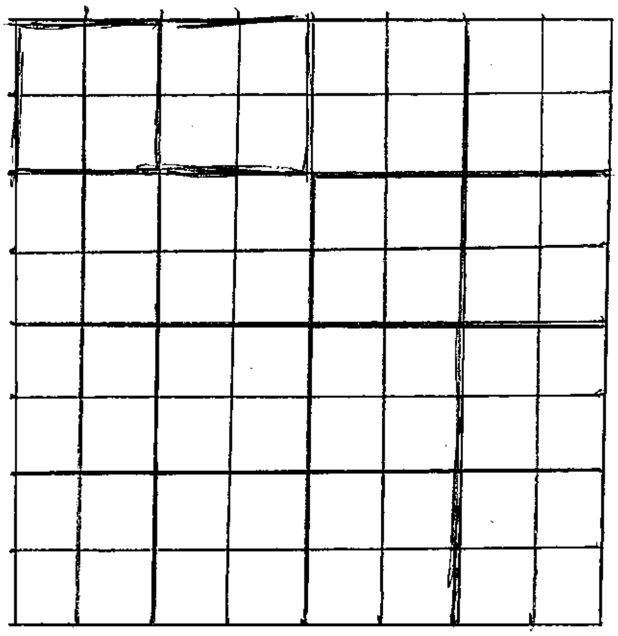
Problem 5

For the domino tiles we are divising the board in 16 big squares (2x2).



In each square we have 2 domino tiles. Those can be placed in 2 different ways and we have 16 squares. (1), =)

$\Rightarrow N = \cancel{8^{16}} 2^{16}$

For the tokens, we keep the division.

If we leave only one token in the 2x2 square, Than we have 4 ways of putting ~~them~~ one token and 4^{16} for all the tokens.

$\Rightarrow F > 4^{16}$ (because we have more cases if there are 2, 3 or 4 tokens in one square) $> 2^{16} = N$

$\Rightarrow$ F is greater than N.

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