INTERNATIONAL MATHEMATICAL OLYMPIAD
"FORMULO DE INTEGRECO"

Problems of the first round for senior participants

1. A boy multiplied a five-digit number by the sum of its digits. Next, he multiplied the result by the sum of its (result's) digits. Surprisingly, the result of the second multiplication was a five-digit number again. What number did the boy multiply the first time? (Find all possible answers.)

2. In the square 7 times 7 each cell (1 times 1) is colored red, yellow or green. Prove that there exist a row, a column and a color such that there are at least 3 cells of this color in this row and at least 3 cells of this color in this column.

3. How many ways are there to write down a string of n letters "A" and n letters "B" in such a way that it doesn't contain the fragment ABB?

4. In an acute triangle ABC the angle C is equal to 45 degrees; AA_1 and BB_1 are the heights. Prove that $A_1B_1 = \sqrt{(A_1B^2 + A_1C^2)/2}$.

5. 100 numbers are arranged around the circumference. Each of the numbers is equal to either 2, 5 or 9. If two numbers are adjacent, they are different. The numbers were grouped into 50 adjacent pairs. The numbers in pairs were multiplied and the 50 products were recorded on the first board. Then the same 100 numbers were grouped into 50 pairs of adjacent numbers in another way. Once more the numbers in pairs were multiplied, and the new 50 products were recorded on the second board. Prove that the sums of the numbers written on the first and the second boards are equal.

6. Positive integers n and k are such that $k^2 + n^2 - k$ is divisible by kn. Prove that k is a perfect square.