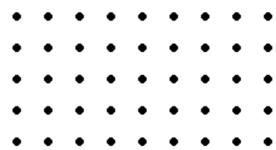
International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2018/2019. Qualifying round

Problems for grade R5

1. James multiplied several one-digit numbers among which there were no twos or fives. Could he have obtained a number written only with digits two and five?

2. There is a rectangular 5×9 grid (picture). Draw 15 non-intersecting triangles (each two of them have no common points on their perimeters) with vertices at this grid points.



3. There were twice as many cones on a fir as on a pine. Peter knocked down several cones, and now there are three times as many cones on the pine as on the fir. Will Peter be able to knock down as many cones from these trees as he has already knocked down?

4. Hobbits and Vikings live in the Wonderland. Someday 27 inhabitants sat around the round table so that the distances between the neighbors were the same. It turned out that between every two hobbits sat at least two Vikings. Prove that there were three Vikings sitting at equal distances from each other.

5. There are numbers from 1 to 49 in the cells of a square 7×7. The amount of odd numbers in each two rows is different. Is it possible that the amount of odd numbers in any two columns is also different?

6. John suggests Anna to play a game. At the beginning of the game, Anna chooses who is to make the first move. Then they take turns writing one number from 1 to 9 at a time. The numbers cannot be repeated. The person, whose move results in such three digits that sum of two of them is equal to the third one, loses. What should Anna do to win the game?

7. There are 100, 200, 500, 1000, 2000 and 5000 dollar banknotes in the ATM. Alex has 10000 dollars on his bank card. Alex wants to take some cash from the ATM and then buy a train ticket at the ticket machine. Alex knows that price of a ticket is divisible by 100 and that it is not greater than 10000 dollars. The ticket machine does not give change. Can Alex take money at the ATM in no more than two operations so that he would be able to buy a ticket without change for sure? (Withdrawing money from an ATM, Alex can choose the total sum, but cannot choose the banknotes that ATM will give him.)

• The qualifying round of the Olympiad takes place **from October 15 to November 12 inclusively**. Winners of the qualifying round will be invited to the final round which will be held in January–February 2019.
• Remember that the majority of the problems require not only an answer but also its full proof.
• Please send your papers in electronic form (both text files and scanned copies of paper sheets are allowed). If you have not such possibility, you can send your paper via "usual" mail instead. Detailed instructions for submitting solutions can be found at `formulo.org`.
• The paper should not contain personal data of the participant, that is, **you should not sign your paper** (the personal data are specified separately when the paper is uploaded to the site).
• Please solve the problems by yourself. Solving together or cheating is not allowed.

International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2018/2019. Qualifying round

Problems for grade R6

1. James multiplied several one-digit numbers among which there were no twos or fives. Could he have obtained a number written only with digits two and five?

2. There were twice as many cones on a fir as on a pine. Peter knocked down several cones, and now there are three times as many cones on the pine as on the fir. Will Peter be able to knock down as many cones from these trees as he has already knocked down?

3. Hobbits and Vikings live in the Wonderland. Someday 27 inhabitants sat around the round table so that the distances between the neighbors were the same. It turned out that between every two hobbits sat at least two Vikings. Prove that there were three Vikings sitting at equal distances from each other.

4. There are numbers from 1 to 49 in the cells of a square 7×7. The amount of odd numbers in each two rows is different. Is it possible that the amount of odd numbers in any two columns is also different?

5. For any 4-digit number, it is possible to list all possible rearrangements of its digits (including the number itself) in ascending order. For example, for the number 3433 we receive this list: 3334, 3343, 3433, 4333. Let us call a number *miserable* if it appears on the 13th place in such row. How many miserable numbers are there?

6. There are 100, 200, 500, 1000, 2000 and 5000 dollar banknotes in the ATM. Alex has 10000 dollars on his bank card. Alex wants to take some cash from the ATM and then buy a train ticket at the ticket machine. Alex knows that price of a ticket is divisible by 100 and that it is not greater than 10000 dollars. The ticket machine does not give change. Can Alex take money at the ATM in no more than two operations so that he would be able to buy a ticket without change for sure? (Withdrawing money from an ATM, Alex can choose the total sum, but cannot choose the banknotes that ATM will give him.)

7. Is it possible to cut a square into 144 equal parts in such a way that three different squares (each two of them are different) can be composed from all those parts?

• The qualifying round of the Olympiad takes place **from October 15 to November 12 inclusively**. Winners of the qualifying round will be invited to the final round which will be held in January–February 2019.
• Remember that the majority of the problems require not only an answer but also its full proof.
• Please send your papers in electronic form (both text files and scanned copies of paper sheets are allowed). If you have not such possibility, you can send your paper via "usual" mail instead. Detailed instructions for submitting solutions can be found at `formulo.org`.
• The paper should not contain personal data of the participant, that is, **you should not sign your paper** (the personal data are specified separately when the paper is uploaded to the site).
• Please solve the problems by yourself. Solving together or cheating is not allowed.

International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2018/2019. Qualifying round

Problems for grade R7

1. After a guy erased the first digit (the leftmost one) in a five-digit number without zeroes, it became n times smaller where n is an integer number. After that he erased the first digit (the leftmost one) again and the number became m times smaller than the previous one where m is also an integer number. Then he repeated this process twice and on each step obtained a result that was some integer number times smaller than the previous one. Find an example of such five-digit number.

2. Hobbits and Vikings live in the Wonderland. Someday 27 inhabitants sat around the round table so that the distances between the neighbors were the same. It turned out that between every two hobbits sat at least two Vikings. Prove that there were three Vikings sitting at equal distances from each other.

3. A number can be written as a sum of 8 prime numbers but cannot be written as a sum of 8 composite numbers. Is it possible to represent this number as a product of multiplication of a prime and a composite number?

4. There are numbers from 1 to 49 in the cells of a square 7×7. The amount of odd numbers in each two rows is different. Is it possible that the amount of odd numbers in any two columns is also different?

5. For any 4-digit number, it is possible to list all possible rearrangements of its digits (including the number itself) in ascending order. For example, for the number 3433 we receive this list: 3334, 3343, 3433, 4333. Let us call a number *brilliant* if it appears on the 5th place in such row. How many brilliant numbers are there?

6. Three painting collectors A, B, and C were selling some of their paintings at an auction. A sold 3% of his paintings, B sold 7%, and C sold 20%. B bought all the paintings of A, C — all the paintings of B, and A — all the paintings of C. Which minimal amount of paintings could have been sold on the auction if the amount of paintings of each collector did not change?

7. Is it possible to cut a square into 144 equal parts in such a way that three different squares (each two of them are different) can be composed from all those parts?

- The qualifying round of the Olympiad takes place **from October 15 to November 12 inclusively**. Winners of the qualifying round will be invited to the final round which will be held in January–February 2019.
- Remember that the majority of the problems require not only an answer but also its full proof.
- Please send your papers in electronic form (both text files and scanned copies of paper sheets are allowed). If you have not such possibility, you can send your paper via "usual" mail instead. Detailed instructions for submitting solutions can be found at `formulo.org`.
- The paper should not contain personal data of the participant, that is, **you should not sign your paper** (the personal data are specified separately when the paper is uploaded to the site).
- Please solve the problems by yourself. Solving together or cheating is not allowed.

International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2018/2019. Qualifying round

Problems for grade R8

1. After a guy erased the first digit (the leftmost one) in a five-digit number without zeroes, it became n times smaller where n is an integer number. After that he erased the first digit (the leftmost one) again and the number became m times smaller than the previous one where m is also an integer number. Then he repeated this process twice and on each step obtained a result that was some integer number times smaller than the previous one. Find an example of such five-digit number.

2. Hobbits and Vikings live in the Wonderland. Someday 27 inhabitants sat around the round table so that the distances between the neighbors were the same. It turned out that between every two hobbits sat at least two Vikings. Prove that there were three Vikings sitting at equal distances from each other.

3. 100 sheep are running in a row at a distance 6 meters from each other. Their speed is 5 km/h. The shepherd is walking towards them at a speed of 1 km/h. When a sheep meets the shepherd, it instantly turns around and runs in the opposite direction with the same speed. Find distances between the sheep running backwards.

4. Three painting collectors A, B, and C were selling some of their paintings at an auction. A sold 3% of his paintings, B sold 7%, and C sold 20%. B bought all the paintings of A, C — all the paintings of B, and A — all the paintings of C. Which minimal amount of paintings could have been sold on the auction if the amount of paintings of each collector did not change?

5. On the sides AB and BC of a square $ABCD$, two equilateral triangles ABK and BCE are built, such that K is inside the square but E is outside of it. Prove that K lies on the segment DE.

6. Is it possible to cut a square into 144 equal parts in such a way that three different squares (each two of them are different) can be composed from all those parts?

7. n players took part in a tennis tournament (each one played 1 game with each other). For which minimal n, there definitely should be 4 players X, Y, Z, and T such that X beat Y, Z, and T; Y beat Z and T; Z beat T?

• The qualifying round of the Olympiad takes place **from October 15 to November 12 inclusively**. Winners of the qualifying round will be invited to the final round which will be held in January–February 2019.
• Remember that the majority of the problems require not only an answer but also its full proof.
• Please send your papers in electronic form (both text files and scanned copies of paper sheets are allowed). If you have not such possibility, you can send your paper via "usual" mail instead. Detailed instructions for submitting solutions can be found at `formulo.org`.
• The paper should not contain personal data of the participant, that is, **you should not sign your paper** (the personal data are specified separately when the paper is uploaded to the site).
• Please solve the problems by yourself. Solving together or cheating is not allowed.

International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2018/2019. Qualifying round

Problems for grade R9

1. After a guy erased the first digit (the leftmost one) in a five-digit number without zeroes, it became n times smaller where n is an integer number. After that he erased the first digit (the leftmost one) again and the number became m times smaller than the previous one where m is also an integer number. Then he repeated this process twice and on each step obtained a result that was some integer number times smaller than the previous one. Find an example of such five-digit number.

2. Mark found two quadratic trinomials with positive integer roots. Then he noticed that trinomial that is the sum of these two also has positive integer roots. Is it possible for all six roots to be different?

3. 100 sheep are running in a row at a distance 6 meters from each other. Their speed is 5 km/h. The shepherd is walking towards them at a speed of 1 km/h. When a sheep meets the shepherd, it instantly turns around and runs in the opposite direction with the same speed. Find distances between the sheep running backwards.

4. On the sides AB and BC of a square $ABCD$, two equilateral triangles ABK and BCE are built, such that K is inside the square but E is outside of it. Prove that K lies on the segment DE.

5. Let us denote by *popularity* of a digit the amount of numbers from set $2^0, 2^1, 2^2, \ldots, 2^{999999}$ with this digit on the first place (the leftmost one). Prove that there are two non-zero digits whose popularity is at least 5 times different.

6. In a convex quadrilateral diagonals are perpendicular. Can the lengths of its sides be equal to four consecutive integers?

7. n players took part in a tennis tournament (each one played 1 game with each other). For which minimal n, there definitely should be 4 players X, Y, Z, and T such that X beat Y, Z, and T; Y beat Z and T; Z beat T?

• The qualifying round of the Olympiad takes place **from October 15 to November 12 inclusively**. Winners of the qualifying round will be invited to the final round which will be held in January–February 2019.
• Remember that the majority of the problems require not only an answer but also its full proof.
• Please send your papers in electronic form (both text files and scanned copies of paper sheets are allowed). If you have not such possibility, you can send your paper via "usual" mail instead. Detailed instructions for submitting solutions can be found at `formulo.org`.
• The paper should not contain personal data of the participant, that is, **you should not sign your paper** (the personal data are specified separately when the paper is uploaded to the site).
• Please solve the problems by yourself. Solving together or cheating is not allowed.

International Mathematical Olympiad
“Formula of Unity” / “The Third Millennium”
Year 2018/2019. Qualifying round

Problems for grade R10

1. Are there three different quadratic trinomials such that the product of each two of them is divisible by the third one?

2. There are numbers from 1 to 49 in the cells of a square 7×7. The amount of odd numbers in each two rows is different. Is it possible that the amount of odd numbers in any two columns is also different?

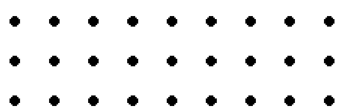
3. Draw the set of all such points on the coordinate plane such that the expression

$$\left(x^2 + y^2 - 4y + 3\right)^2 \left(3 - \sqrt{x^2 + y^2} - \sqrt{x^2 + (y-3)^2}\,\right)$$

takes maximal possible value.

4. Let us denote by *popularity* of a digit the amount of numbers from set $2^0, 2^1, 2^2, \ldots, 2^{999999}$ with this digit on the first place (the leftmost one). Prove that there are two non-zero digits whose popularity is at least 5 times different.

5. There is a rectangular $m \times n$ grid with the total amount of grid points divisible by 3 (e. g. a 3×9 grid is demonstrated below). Find all m and n such that it is not possible to draw $\frac{mn}{3}$ non-intersecting triangles (each two of them have no common points) with vertices at these grid points.



6. Angle bisectors AK, BL, and CM of a triangle ABC intersect in the point I (the points K, L, M lie at the sides of the triangle). Prove that $\frac{IK}{IA} + \frac{IL}{IB} + \frac{IM}{IC} \geqslant \frac{3}{2}$.

7. There are a hundred numbers, each of them initially equals to 0. In one move, it is allowed to choose 9 numbers and decrease the first one of them by 1, the second one by 2, the third by 3, … the eighth by 8, but increase the ninth number by 9. What maximal amount of numbers can become positive with such operations?

• The qualifying round of the Olympiad takes place **from October 15 to November 12 inclusively**. Winners of the qualifying round will be invited to the final round which will be held in January–February 2019.
• Remember that the majority of the problems require not only an answer but also its full proof.
• Please send your papers in electronic form (both text files and scanned copies of paper sheets are allowed). If you have not such possibility, you can send your paper via “usual” mail instead. Detailed instructions for submitting solutions can be found at `formulo.org`.
• The paper should not contain personal data of the participant, that is, **you should not sign your paper** (the personal data are specified separately when the paper is uploaded to the site).
• Please solve the problems by yourself. Solving together or cheating is not allowed.

International Mathematical Olympiad
“Formula of Unity” / “The Third Millennium”
Year 2018/2019. Qualifying round

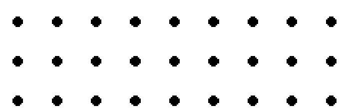
Problems for grade R11

1. Find two positive numbers if it is known that the square of the first one is 16 times greater than the cube of the second one, and the square of the second one is 2 times smaller than the cube of the first one.

2. There are numbers from 1 to 49 in the cells of a square 7×7. The amount of odd numbers in each two rows is different. Is it possible that the amount of odd numbers in any two columns is also different?

3. Let us denote by *popularity* of a digit the amount of numbers from set $2^0, 2^1, 2^2, \ldots, 2^{999999}$ with this digit on the first place (the leftmost one). Prove that there are two non-zero digits whose popularity is at least 5 times different.

4. There is a rectangular $m \times n$ grid with the total amount of grid points divisible by 3 (e. g. a 3×9 grid is demonstrated below). Find all m and n such that it is not possible to draw $\frac{mn}{3}$ non-intersecting triangles (each two of them have no common points) with vertices at these grid points.



5. Angle bisectors AK, BL, and CM of a triangle ABC intersect in the point I (the points K, L, M lie at the sides of the triangle). Prove that $\frac{IK}{IA} + \frac{IL}{IB} + \frac{IM}{IC} \geqslant \frac{3}{2}$.

6. There are a hundred numbers, each of them initially equals to 0. In one move, it is allowed to choose 9 numbers and decrease the first one of them by 1, the second one by 2, the third by 3, … the eighth by 8, but increase the ninth number by 9. What maximal amount of numbers can become positive with such operations?

7. Find an example of a polyhedron whose projections onto three coordinate planes are a regular triangle, a regular quadrilateral, and a regular hexagon. Specify the coordinates of each vertex of the polyhedron, provide a list of its edges and faces.

• The qualifying round of the Olympiad takes place **from October 15 to November 12 inclusively**. Winners of the qualifying round will be invited to the final round which will be held in January–February 2019.
• Remember that the majority of the problems require not only an answer but also its full proof.
• Please send your papers in electronic form (both text files and scanned copies of paper sheets are allowed). If you have not such possibility, you can send your paper via “usual” mail instead. Detailed instructions for submitting solutions can be found at `formulo.org`.
• The paper should not contain personal data of the participant, that is, **you should not sign your paper** (the personal data are specified separately when the paper is uploaded to the site).
• Please solve the problems by yourself. Solving together or cheating is not allowed.