International Mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
Year 2015/2016. Round 1

Problems for grade R5

Please do not forget to prove your answers.

1. Peter, Basil and Anatoly pooled their savings to buy a ball. It is known that each of them spent no more than a half of the money spent by two other boys together. The ball costs 9 dollars. How much money did Peter spend?

2. Pauline wrote down numbers A and B on a blackboard. Victoria erased them and wrote their sum C and their product D. After that Pauline erased those new numbers, replacing them with their sum E and their product F. One of the numbers E and F appeared to be odd. Which one and why?

3. Let's say that student A studies *better* than student B if his scores in the majority of the taken tests are higher. After 3 tests it turned out that student A studies better than student B, student B studies better than student C, and student C studies better that student A. Is such an outcome possible, if the grading system has grades 2, 3, 4 and 5 only?

4. If Leon gets a low grade at school, he spends the entire evening lying to his mother. Otherwise he tells her only the truth. Leon has a little sister who gets candies whenever she comes home without low grades. One evening Leon told his mom: "Today I got more low grades than my sister". Will his sister get candies or not?

5. A magic calendar shows the correct date on even days of the month and a wrong date on odd days. What is the maximum number of consecutive days when it could show the same date? What day of the month could it show during these days?

6. How many 10-digit numbers are there, such that all the digits are different, and the number contains the fragment 0123?

7. Alex has cut a 8×8 square (along the sides of the cells) into 7 parts with equal perimeter. Show the way he could have done it. (One example suffices.)

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Problems for grade R6

Please do not forget to prove your answers.

1. There are 14 people sitting in a circle. Peter, Victoria, Anatoly and Genghis are sitting in a row and each of them has a coin of value 1, 2, 5 and 10 respectively. Other children don't have any money. Any person sitting in the circle can pass a coin to a person to their left or their right if there are exactly 3 people between them. After a while it turned out that the coins returned to Peter, Victoria, Anatoly and Genghis. Which coin does each of them have now?

2. Pauline wrote down numbers A and B on a blackboard. Victoria erased them and wrote their sum C and their product D. After that Pauline erased those new numbers, replacing them with their sum E and their product F. One of the numbers E and F appeared to be odd. Which one and why?

3. Let's say that student A studies *better* than student B if his scores in the majority of the taken tests are higher. After 3 tests it turned out that student A studies better than student B, student B studies better than student C, and student C studies better that student A. Is such an outcome possible, if the grading system has grades 2, 3, 4 and 5 only?

4. If Leon gets a low grade at school, he spends the entire evening lying to his mother. Otherwise he tells her only the truth. Leon has a little sister who gets candies whenever she comes home without low grades. One evening Leon told his mom: "Today I got more low grades than my sister". Will his sister get candies or not?

5. A magic calendar shows the correct date on even days of the month and a wrong date on odd days. What is the maximum number of consecutive days when it could show the same date? What day of the month could it show during these days?

6. How many 10-digit numbers are there, such that all the digits are different, and the number contains the fragment 0123 or the fragment 3210?

7. Alex has cut a 8×8 square (along the sides of the cells) into 7 parts with equal perimeter. Show the way he could have done it. (One example suffices.)

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Problems for grade R7

Please do not forget to prove your answers.

1. A magic calendar shows the correct date on the even days of the month and a wrong date on the odd days. What is the maximum number of consecutive days when it could show the same date? What date could it be?

2. Fill the cells of a 5×5 square with different positive integers, in such a way that the sums in every row and every column are equal and (under this condition) least possible. One of the diagonsls is already filled with numbers 1, 2, 3, 4 and 2015 (you cannot use them again).

3. Alex has cut a 8×8 square (along the sides of the cells) into 7 parts with equal perimeter. Show the way he could have done it. (One example suffices.)

4. There are 27 cockroaches participating in cockroach racing. In each race, three cockroaches run. Each cockroach has his constant speed, not changing between the races. The speeds of all cockroaches are different. As a result of each race, we obtain only the order in which its participants have finished. We would like to know two fastest cockroaches (in the correct order). Would 14 races be sufficient?

5. Let's say that student A studies *better* than student B if his scores in the majority of the taken tests are higher. After more than 3 tests, it turned out that student A studies better than student B, student B studies better than student C, and student C studies better that student A. Is this situation possible?

6. Let us call a positive integer *beautiful*, if it is a product of primes' factorials (not necessarily distinct ones). Let us call a positive rational number *practical*, if it is a ratio of two beautiful numbers. Prove that all positive rational numbers are practical.

7. Let us call a positive integer *ascending*, if the sequence of its digits is strictly ascending (for example, 1589 is ascending, but 447 is not). What is the minimum number of ascending positive integers, the sum of which is 2015?

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Problems for grade R8

Please do not forget to prove your answers.

1. Fill the cells of a 5×5 square with different positive integers, in such a way that the sums in every row and every column are equal and (under this condition) least possible. One of the diagonsls is already filled with numbers 1, 2, 3, 4 and 2015 (you cannot use them again).

2. There are 27 cockroaches participating in cockroach racing. In each race, three cockroaches run. Each cockroach has his constant speed, not changing between the races. The speeds of all cockroaches are different. As a result of each race, we obtain only the order in which its participants have finished. We would like to know two fastest cockroaches (in the correct order). Would 14 races be sufficient?

3. Find a positive integer such that the product of its natural divisors is 10^{79}.

4. John has 12 sticks, the length of each stick is a positive integer not greater than 56. Prove that he has three sticks which could form a triangle.

5. Let us call a positive integer *beautiful*, if it is a product of primes' factorials (not necessarily distinct ones). Let us call a positive rational number *practical*, if it is a ratio of two beautiful numbers. Prove that all positive rational numbers are practical.

6. In triangle $\triangle ABC$, $\angle B = 30°$, $\angle C = 105°$, and D is the middle of BC. Find the measure of $\angle BAD$.

7. Let's say that student A studies *better* than student B if his scores in the majority of the taken tests are higher. After more than 3 tests, it turned out that student A studies better than student B, student B studies better than student C, and student C studies better that student A. Is this situation possible?

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Problems for grade R9

Please do not forget to prove your answers.

1. The vertices of a regular dodecagon are painted blue and red. We know that among every 3 vertices which form a regular triangle at least 2 are red. Prove that we may choose 4 vertices forming a square with at least 3 red vertices.

2. Let us call a positive integer *beautiful*, if it is a product of primes' factorials (not necessarily distinct ones). Let us call a positive rational number *practical*, if it is a ratio of two beautiful numbers. Prove that all positive rational numbers are practical.

3. There are 27 cockroaches participating in cockroach racing. In each race, three cockroaches run. Each cockroach has his constant speed, not changing between the races. The speeds of all cockroaches are different. As a result of each race, we obtain only the order in which its participants have finished. We would like to know two fastest cockroaches (in the correct order). Would 14 races be sufficient?

4. In triangle $\triangle ABC$, $\angle B = 30^\circ$, $\angle C = 105^\circ$, and D is the middle of BC. Find the measure of $\angle BAD$.

5. John has 12 sticks, the length of each stick is a positive integer not greater than 56. Prove that he has three sticks which could form a triangle.

6. Find a positive integer such that the product of its natural divisors is 10^{79}.

7. It is well known that $3^2 + 4^2 = 5^2$. It is less known that $10^2 + 11^2 + 12^2 = 13^2 + 14^2$. Does there exist 2015 consecutive positive integers such that the sum of the squares of the first 1008 of them is equal to the sum of the squares of the last 1007 ones?

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Problems for grade R10

Please do not forget to prove your answers.

1. Bugs Bunny and Roger Rabbit made a bet on who is faster. To determine the winner they decided to hold a competition. Each of them has to jump 50 meters in one direction, then turn around and jump back. It is known that Bugs's jump is of 50 cm length, when Roger's of 60 cm, but Bugs manages to make 6 jumps in time Roger makes only 5. Who is going to win?

2. For which n is it possible to divide the square into n similar rectangles, so that at least two of them are unequal?

3. Are there such positive integers a and b that $\operatorname{lcm}(a, b) = \operatorname{lcm}(a+2015, b+2016)$? Lcm stands for least common multiple.

4. In triangle $\triangle ABC$, $\angle B = 30°$, $\angle C = 105°$, and D is the middle of BC. Find the measure of $\angle BAD$.

5. Fill the cells of a 10×10 square with distinct integer numbers so that the sum in every row and every column is equal and (under this condition) least possible. One of the diagonals is already filled with numbers 1, 2, 3, 4, 5, 6, 7, 8, 9 and 2015 (you cannot use them again).

6. The inscribed circle of a triangle $\triangle ABC$ is tangent to AB, BC and AC at point C_1, A_1 and B_1 respectively. Prove the inequation:
$$\frac{AC}{AB_1} + \frac{CB}{CA_1} + \frac{BA}{BC_1} > 4.$$

7. It is well known that $3^2 + 4^2 = 5^2$. It is less known that $10^2 + 11^2 + 12^2 = 13^2 + 14^2$. Is it true that for any positive integer k there are $2k+1$ consecutive positive integers such that the sum of the squares of the first $k+1$ of them equals the sum of the squares of the rest k?

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Problems for grade R11

Please do not forget to prove your answers.

1. Bugs Bunny and Roger Rabbit made a bet on who is faster. To determine the winner they decided to hold a competition. Each of them has to jump 50 meters in one direction, then turn around and jump back. It is known that Bugs's jump is of 50 cm length, when Roger's of 60 cm, but Bugs manages to make 6 jumps in time Roger makes only 5. Who is going to win?

2. For which n is it possible to divide the square into n similar rectangles, so that at least two of them are unequal?

3. Are there such positive integers a and b that $\operatorname{lcm}(a, b) = \operatorname{lcm}(a + 2015, b + 2016)$? Lcm stands for least common multiple.

4. In triangle $\triangle ABC$, $\angle B = 30°$, $\angle C = 105°$, and D is the middle of BC. Find the measure of $\angle BAD$.

5. At each integral point of the Cartesian plane a tree of diameter 10^{-6} is growing. A woodcutter cut down the tree at point $(0, 0)$ and stood on the stump. Is the part of the plane visible to him limited? Treat each tree as an infinite cylindrical column with the axis containing an integral point of the plane.

6. Give an example of 4 positive numbers that cannot be radii of 4 pairwise tangent spheres.

7. It is well known that $3^2 + 4^2 = 5^2$. It is less known that $10^2 + 11^2 + 12^2 = 13^2 + 14^2$. Is it true that for any positive integer k there are $2k + 1$ consecutive positive integers such that the sum of the squares of the first $k + 1$ of them equals the sum of the squares of the rest k?