

International mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
2017/2018 year, final round

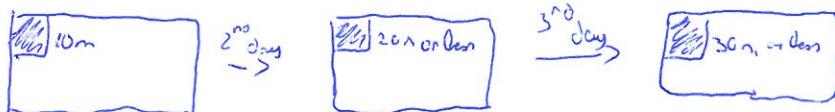
SOLUTIONS TO THE SECOND ROUND FOR PARTICIPANTS.

GRATE: ...8.... NUMBER: ...16....

(1)

Handwritten calculations and diagrams for problem 1:

- Top right: A circle with radius $R = \sqrt{314 \cdot 400} = 125$. Below it is a grid of numbers: 21, 5, 52, 65, 77, 78, 89.
- Middle left: A sketch of a trapezoid divided into two triangles. Labels include 36860, 120², 1666, 500, 950, 18424, 1674, 3, 388, 95, 1940, 3492, 36860, 400, 10, 95, and 10.
- Middle center: A diagram showing a large rectangle divided into smaller rectangles. Labels include 36860, 120², 1666, 500, 950, 18424, 1674, 3, 388, 95, 1940, 3492, 36860, 400, 10, 95, and 10.
- Middle right: A diagram showing a large rectangle divided into smaller rectangles. Labels include 36860, 120², 1666, 500, 950, 18424, 1674, 3, 388, 95, 1940, 3492, 36860, 400, 10, 95, and 10.
- Bottom right: A fraction $\frac{20^2}{388} = \frac{95}{182^4}$.



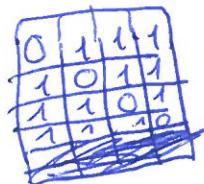
After 6 days

After 7 days

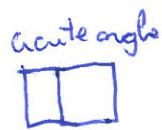
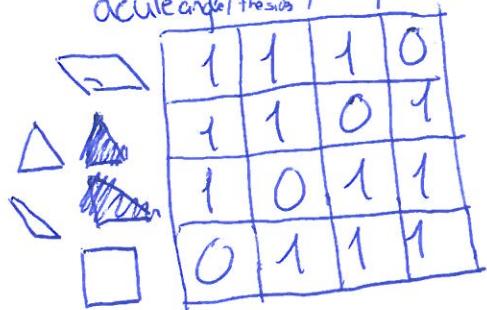
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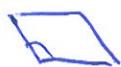


acute angle/some sides / it hasn't an obtuse angle like this ↗

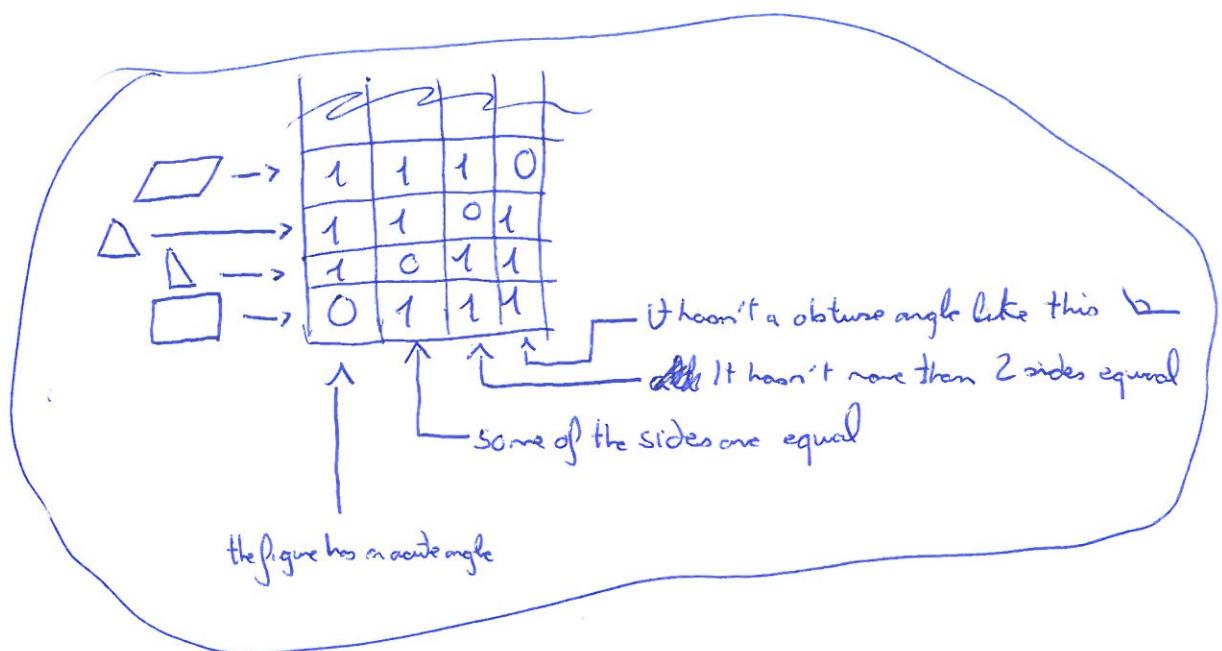


some of the sides are equal

the figure hasn't an obtuse angle like this ↗



it hasn't more all the sides than are equal



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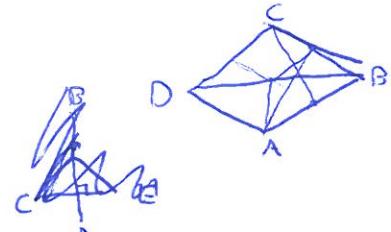
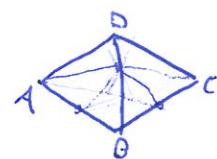
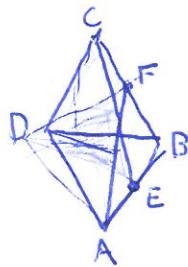
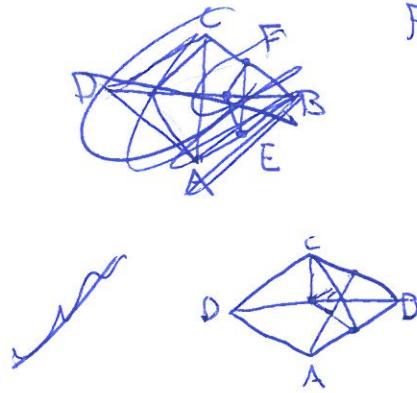
SOLUTIONS TO THE SECOND ROUND FOR PARTICIPANTS.

GRADE: 8 NUMBER: 16

(3)

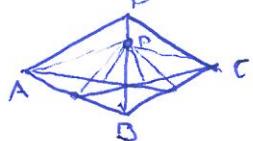
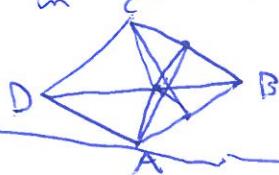
$$PA = PF$$

$$PE = PC$$



~~P can't be on the line DB because if $PA = PF$ and $PC = PE$ it forms a triangle and the line BD will pass on the center of it and form a red angle $\angle B$ with the line AF or CE but if we draw the line in the other side it doesn't form a red angle.~~

P is on the line BD because if we draw ~~to~~ the line AF and CE, and we draw a red angle line in the middle of it we can see that the lines cross in the line BD. It's because if we put a point X in the red angle line always $\angle AX = \angle AF$ and if the two lines cross in the line BD it's because P is in BD. If it crosses in other place it isn't in BD line



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(4)



① (1)
 (11) 5
 ① (11) 4
 3 2 (13) 10
 4 6 5 (10) 10

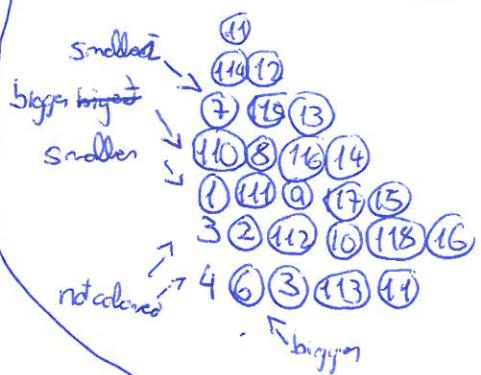
1
 2 (3)
 (7) 4

① (3) 2 (6) 7
 4 (5) 6 8

4
 5 6

① (1)
 3 2 (4)
 5 6 (7) 8
 ① (11)
 3 2 (12)
 4 (6) (5) 10 4

The smallest possible sum of these two numbers is $3+4=7$ because if we want to put just two numbers in the table we put it in the corners (1) and (1 and 2) (3 and 2) can't because it will be 3 or more numbers not colored. And 3 and 4 can be not colored only 2



(Example)

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(5)



$$\begin{array}{r} 16 \\ \times 9 \\ \hline 144 \\ 144 \\ \hline 144 \end{array}$$

$$\begin{array}{r} 16 \\ \times 17 \\ \hline 112 \\ 16 \\ \hline 284 \end{array}$$

$$18 \times 18 = 324$$

$$\begin{array}{r} 21 \\ \times 21 \\ \hline 42 \\ 21 \\ \hline 441 \end{array}$$

$$\begin{array}{r} 25 \\ \times 25 \\ \hline 125 \\ 25 \\ \hline 625 \end{array}$$

$$\begin{array}{r} 30 \\ \times 30 \\ \hline 900 \\ 900 \\ \hline 900 \end{array}$$

$$\begin{array}{r} 28 \\ \times 28 \\ \hline 224 \\ 56 \\ \hline 784 \\ 784 \\ \hline 784 \end{array}$$

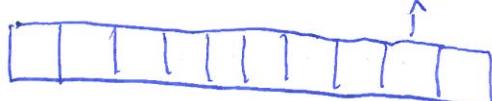
$$\begin{array}{r} 29 \\ \times 29 \\ \hline 201 \\ 58 \\ \hline 841 \end{array}$$

Alex

Alex - 1 to 9 moves

Ben 1 move

Alex - the remaining moves



~~Ben~~
Alex

Alex - 4, 5, 6, 9

Ben - 1, 2, 3, 7, 8

$$2 \cdot 2 = 4$$

$$3 \cdot 3 = 9$$

$$4 \cdot 4 = 16$$

$$5 \cdot 5 = 25$$

$$6 \cdot 6 = 36$$

$$7 \cdot 7 = 49$$

$$8 \cdot 8 = 64$$

- All the perfect squares ends in 4, 5, 6 and 9
- If Alex wants to win he will play do less than 9 moves because if he put the first 9 moves and Ben do the last move, Ben will put 12345678 by the final number
- Alex shouldn't put 8 zeroes in the first move because Ben put numbers like 5 or 7 and he can't have a perfect square

Alex is the player with the winning strategy, he need to put 7 zeroes in the first move and then, if Ben put 1 put 21, if Ben put 2 put 25, if Ben put 3 put 24, if Ben put 4 put 00, if Ben put 5 put 29 ...

(if Ben put 1)
→
Ben put 2

0	0	0	0	0	0	0	1	2	1
0	0	0	0	0	0	0	2	2	5
0	0	0	0	0	0	0	3	2	9
0	0	0	0	0	0	0	4	6	0
0	0	0	0	0	0	0	5	2	4
0	0	0	0	0	0	0	6	1	3
0	0	0	0	0	0	0	7	2	9
0	0	0	0	0	0	0	8	4	1
0	0	0	0	0	0	0	9	0	0
0	0	0	0	0	0	0	0	0	0

