

**International mathematical Olympiad  
 "Formula of Unity" / "The Third Millennium"  
 2017/2018 year, final round**

SOLUTIONS TO THE SECOND ROUND FOR PARTICIPANTS.

GRATE: 8..... NUMBER: 16.....

①

$36860 \cdot 120\%$   
 $1666 \cdot 100\%$   
 $1500 \cdot 18424$   
 $980$   
 $3$   
 $1674$

$1147$   
 $13 \cdot 23$

$1373$   
 $5 \cdot 2$

$117$   
 $4$

$100 = 11R^2 = 314 \cdot 100 = 31400 \cdot 10 \cdot 53$

$20m \cdot 11R^2 = 314 \cdot 400 \cdot 125$

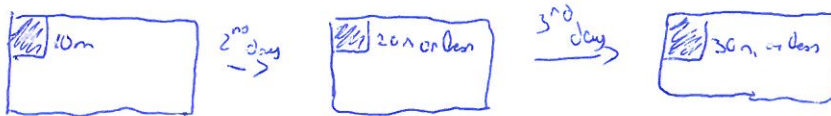
$52 \quad 65 \quad 77 \quad 89$

$10 \cdot 95$   
 $10$   
 $400$   
 $95 m^2$

$80 = \frac{10}{x} \Rightarrow x = \frac{1000}{80} = 125$

$4500 \cdot 120\%$   
 $1000 \cdot 47$   
 $18000 \cdot 20\%$

$20 \cdot 2 = \frac{95}{182 \cdot 4}$



~~After 6 days~~

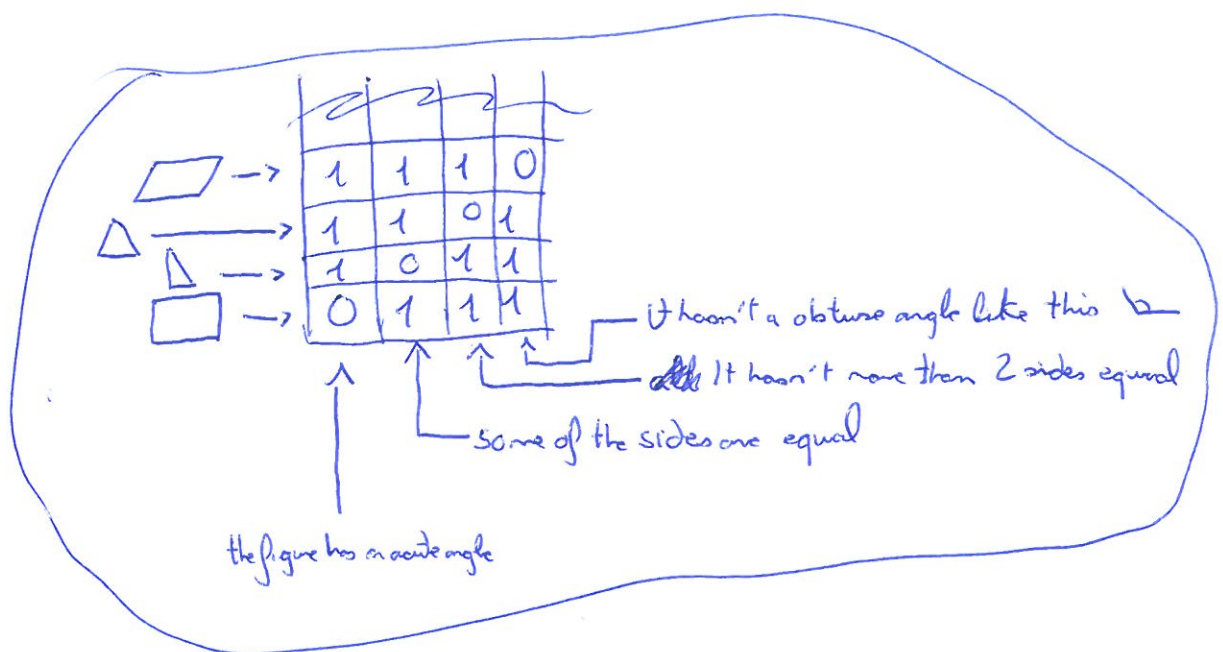
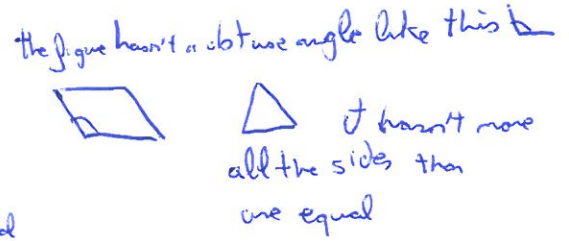
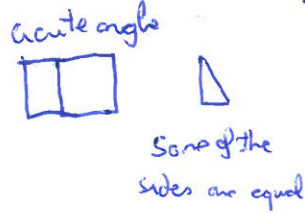
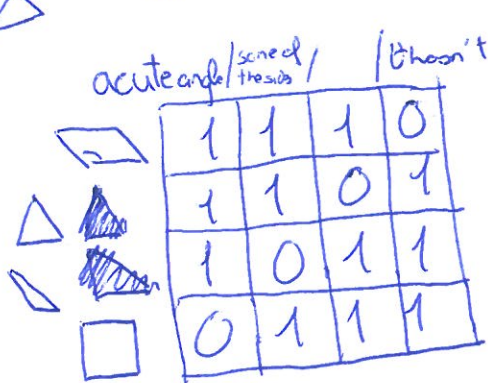
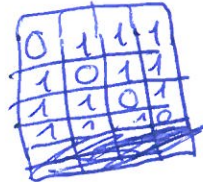
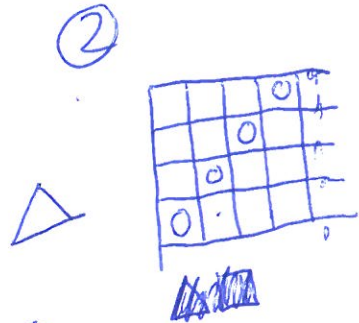
After 7 days



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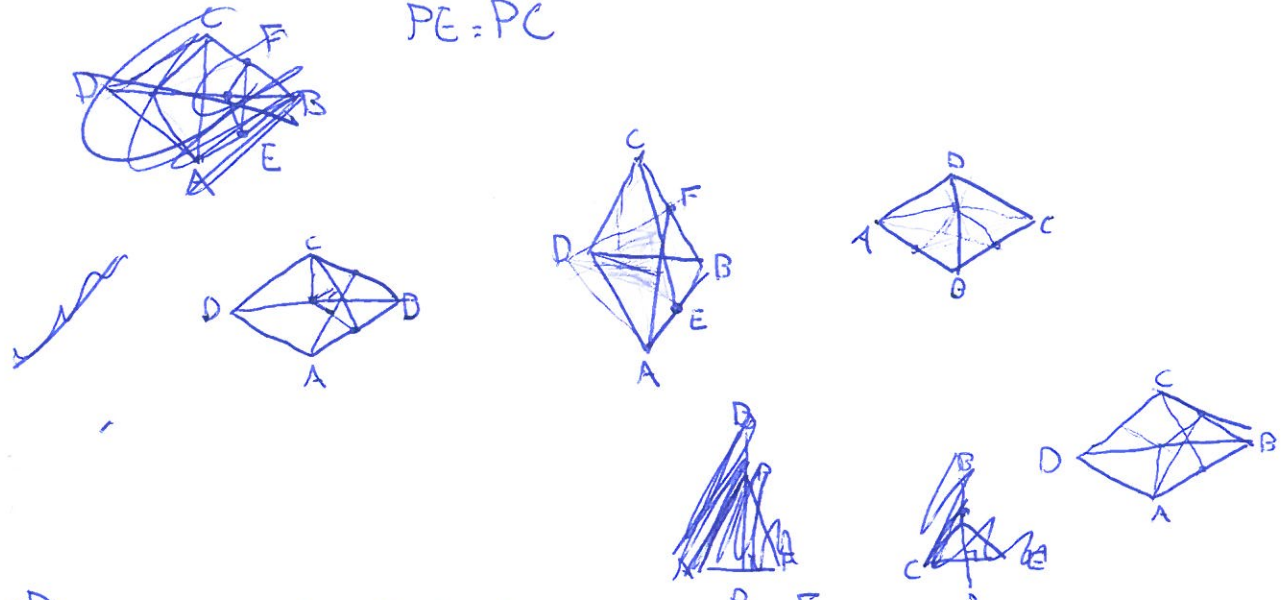
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GRATE: 8 NUMBER: 16

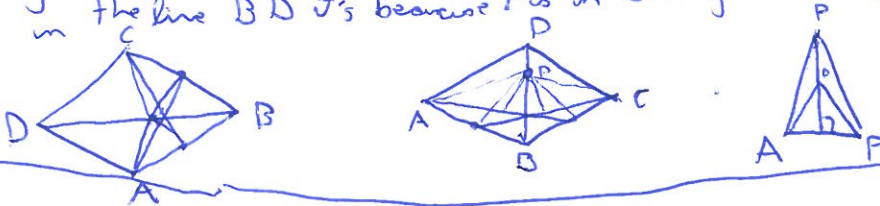
③

$PA = PF$   
 $PE = PC$



~~P can't be on the line DB because of  $PA = PF$  and  $PC = PE$  it forms a triangle and the line BD will pass on the center of it and form a right angle with the line AF or CE but if we draw the line on the rhombus it doesn't form a right angle.~~

P is on the line BD because if we draw the line AF and CE, and we draw a right angle line in the middle of it we can see that the lines cross on the line BD, it's because if we put a point X in the right angle line always  $AX = AF$  and if the two lines cross in the line BD it's because P is in BD if it crosses in other place it isn't in BD line





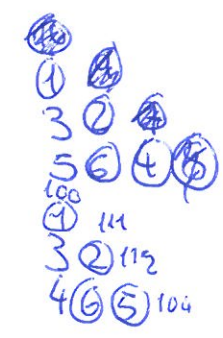
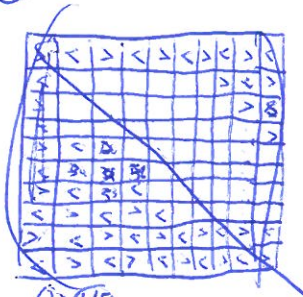


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④



The smallest possible sum of these two numbers is  $3+4=7$  because if we want to put just two numbers in the table we put it in the corners  $\square$  and  $\square$  and  $1$  and  $2$  /  $3$  and  $2$  can't because it will be  $3$  <sup>on more</sup> ~~color~~ numbers not colored. And  $3$  and  $4$  can be not colored only  $2$

Example



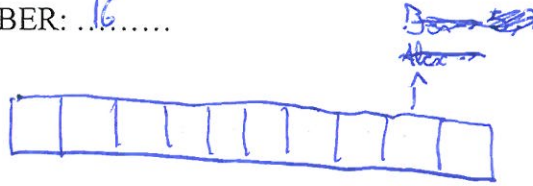


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5



Alex ~ 1-9 moves  
 Ben 1 move  
 Alex the remaining moves

Alex - 4, 5, 6, 9  
 Ben - 1, 2, 3, 7, 8

- 2 · 2 = 4
- 3 · 3 = 9
- 4 · 4 = 16
- 5 · 5 = 25
- 6 · 6 = 36
- 7 · 7 = 49
- 8 · 8 = 64

- All the perfect squares ends in 4, 5, 6 and 9
- If Alex want to win he will put do less than 9 moves because if he put the first 9 moves and Ben do the last move, Ben will put 123... by the final number
- Alex shouldn't put 8 zeros in the first move because Ben put numbers like 5 or 7 and he can't have a perfect square

144  
144  
144  
56  
15  
010

15  
15  
75

15  
025

16    17  
16    17  
46    14

16    17  
256    384

18    20  
18    20  
144    600

21    23  
21    23  
21    64

42    46  
441    1924

22    27  
22    27  
44    184

25    27  
25    27  
125    34

30    29  
30    29  
900    201

224    38  
56    841  
784

255  
255  
1430  
230  
2520  
1225

Alex is the player with the winning strategy, he need to put 7 zeros in the first move and then, if Ben put 1 put 21, if Ben put 2 put 25, if Ben put 3 put 24, if Ben put 4 put 00, if Ben put 5 put 29...

0	0	0	0	0	0	1	2	1
0	0	0	0	0	0	2	2	5
0	0	0	0	0	0	3	2	9
0	0	0	0	0	0	4	0	0
0	0	0	0	0	0	5	2	4
0	0	0	0	0	0	6	2	3
0	0	0	0	0	0	7	2	9
0	0	0	0	0	0	8	4	1
0	0	0	0	0	0	9	0	0

