

Put 1 in the top-left corner of the table.

Then, make the first row alternate 1, 2, 1, 2, 1, 2, etc. until the end.

Make the second row say 2, 1, 2, 1, 2, 1, etc.

Make the third row say 1, 2, 1, 2, 1, 2, etc.

Follow this pattern for 100 rows (each row with 100 digits)

1	2	1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1	2	1
1	2	1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1	2	1
1	2	1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1	2	1
1	2	1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1	2	1
1	2	1	2	1	2	1	2	1	2
2	1	2	1	2	1	2	1	2	1

Figure 1: An example with a 10x10 table.

Figure 1 shows such a table.

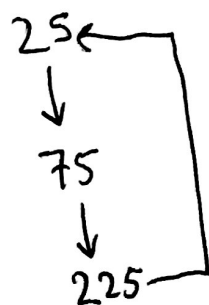
In this manner, every 1 will be completely surrounded by 2s and every 2 will be completely surrounded by 1s.

In the table, there will be an equal number of 1s and 2s (as each row has half ones and half 2s).

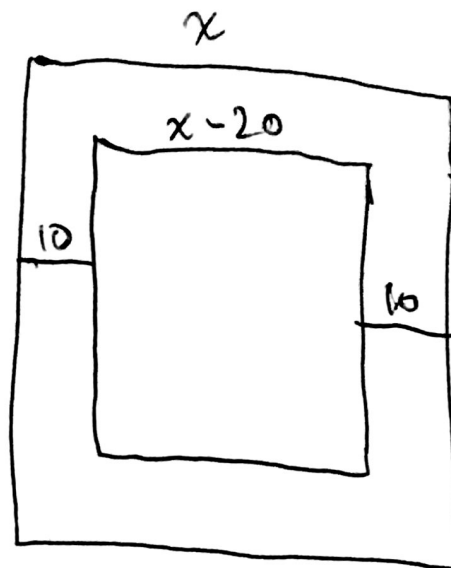
Thus, the sum of all the digits is  $\frac{100 \times 100}{2} (2) + \frac{100 \times 100}{2} (1) =$

**15,000**

25 times 3 is 75. 75 times 3 is 225. Take out  
one of the twos to get 25 again.  $\square$



Let the side-length of the square be  $x$ . Then, the diagram below follows:



where the inner square represents the water area left and the border represents the amount of decrease. The border area is

$$x^2 - (x-20)^2$$

or

$$x^2 - (x^2 - 40x + 400) = 40x - 400.$$

Thus, we get:

$$\frac{40x - 400}{x^2} = \frac{35}{100}.$$

Solving,  $x = \frac{400 + 10\sqrt{1046}}{7}$  or around  $\frac{720}{7}$ .

The entire pond is frozen when, on the  $d^{\text{th}}$  day,

$$d^2 \geq 720/7.$$

The smallest such  $d$  is  $\boxed{11}$ .  $\square$

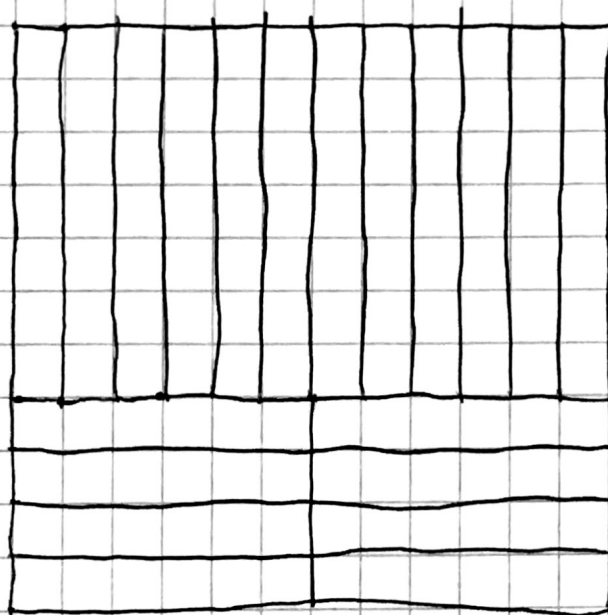
It is possible to split 132 (11x12) into groups of 6 and 7 as shown:

$$6 \times 22 + 7 \times 0 \quad \left\{ \right. \quad 6 \times 15 + 7 \times 6 \quad \left\{ \right. \quad 6 \times 8 + 7 \times 12 \quad \left\{ \right. \quad 6 \times 1 + 7 \times 18$$

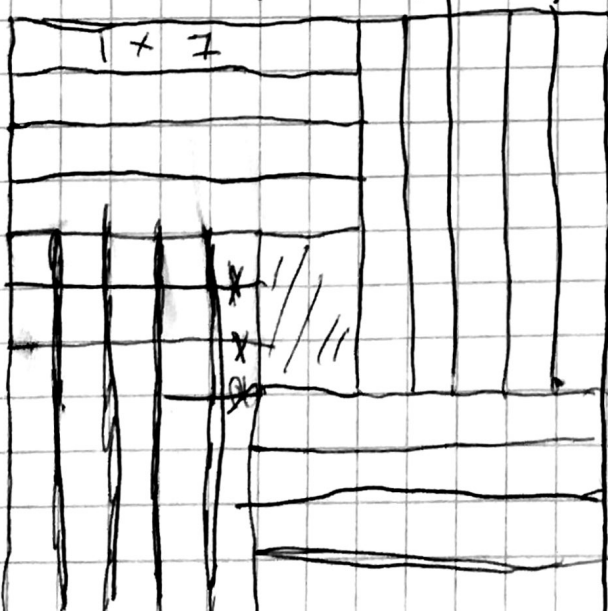
This is done by noticing that  $132 = 6 \times 22$ , and then noticing that 7 groups of 6 can be replaced by 6 groups of 7, thus making the total number of groups smaller.

Each of the above corresponds to a layout with 22, 21, 20, and 19 strips, respectively. Ideally, we want 19 strips.

Proof that 20 strips work:



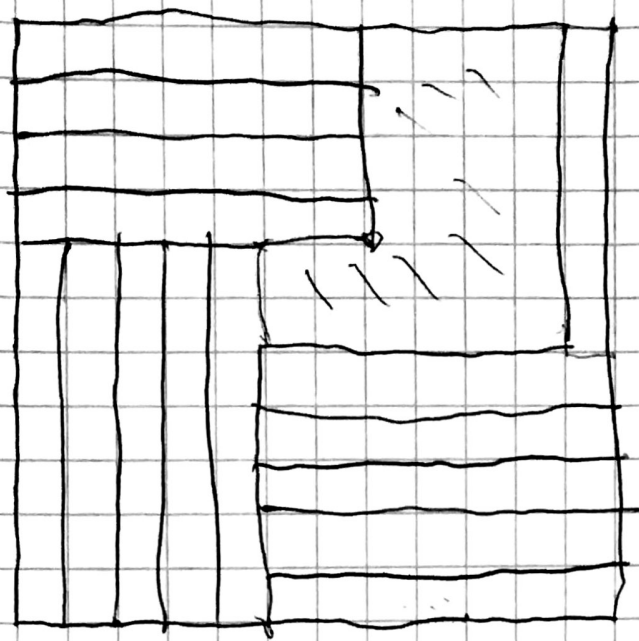
Let us try to fit 19 strips (18 1x7s, 1 1x6). WLOG let us put a 1x7 in the corner.



in the corner.

Then, we are left with a 2x3.

Now, we try ~~it~~ the  $1 \times 6$  in the corner.



Then, we are left with a shaded region.

Since either the  $1 \times 6$  or  $1 \times 7$  must have been in a corner, but neither works, you cannot fit 18  $1 \times 7$ s and 1  $1 \times 6$ .

Thus, you need a minimum of 20 strips.  $\square$

Assume that no two bags were inside each other. Then, there would be a minimum of  $1+2+3+\dots+98+99+100=5050$  candies. However, we have fewer candies than that! Thus, our original assumption was wrong, and thus, there is a bag with another bag inside of it.  $\square$