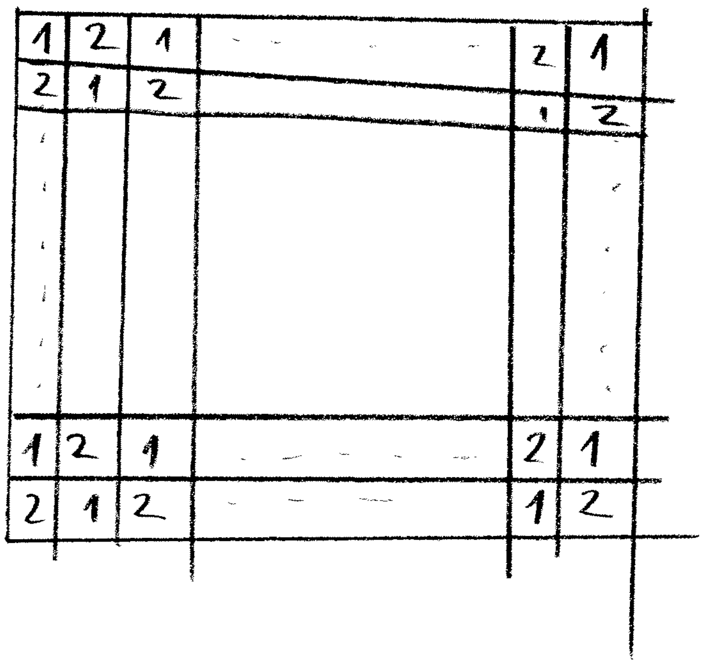
Question 1.

Solution:

Lets notice that the two smallest possible values of integers that can be placed into the cells of the given table are 1 and 2.

Something else to notice is that there are at most $\frac{100 \cdot 100}{2} = 5000$ equal numbers in the cells of the table - if there are more than 5000 equal numbers in the table, we will have 2 equal numbers in adjecent cells which is with contradiction with ~~the~~ what is said in the question.

That's why the smallest possible sum of the numbers in the table we will get when we have 5000 1s and 5000 2s. So the answer is $5000 \cdot 1 + 5000 \cdot 2 = 15\,000$. This is possible like this: (like a chess table)



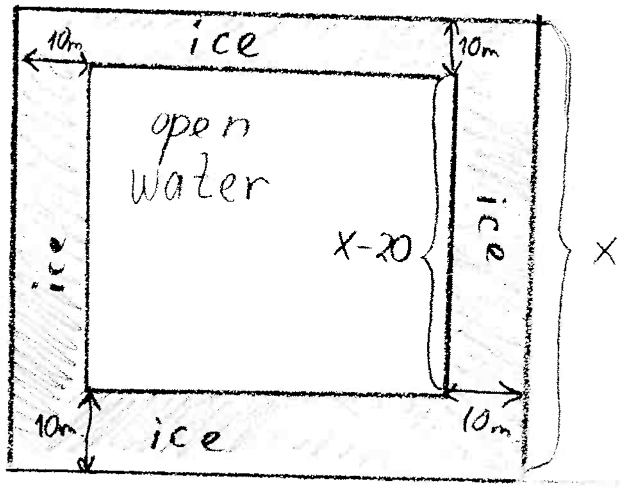
Question 2.
Solution:

41 → 123 → 369 → 1107 → 107 → 321 → 963 →
→ 2889 → 289 → 867 → 2601 → 7803 →
→ 23409 → 71127 → 7127 → 127 → 381 →
→ 1143 → 143 → 429 → 1487 → 4461 → 461 →
→ 1383 → 138 → 414 → 41

Question 3.

Solution:

Let's make a diagram of the pond after the first frosty day:



Let the lenght of the side of the square pond be x.

So we have

$$x^2-(x-20)^2=\frac{35}{100}\cdot x^2 \Rightarrow$$

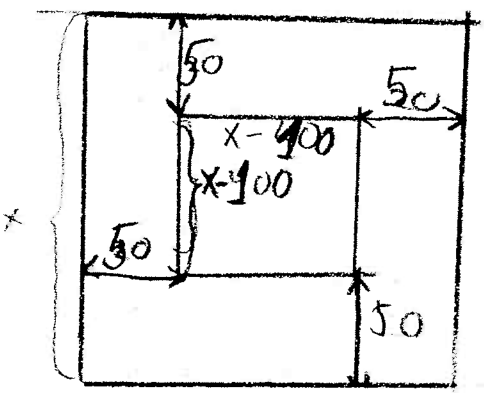
$$\frac{65}{100}\cdot x^2=(x-20)^2 \Rightarrow$$

$$\frac{65}{100}\cdot x^2=x^2-40x+400$$

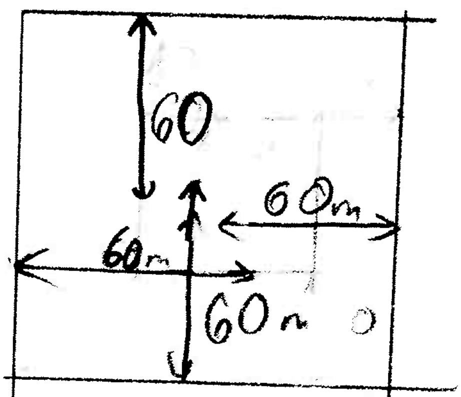
$$40x=\frac{35}{100}\cdot x^2+400$$, from where we have $x \leq 120$ and $x \geq 100$.

Now let's make a diagram of the pond after the ~~second~~ fifth frosty day and after the ~~third~~ sixth frosty day:

After the ~~second~~ fifth:



After the ~~third~~ sixth:



So after the sixth frosty day the whole pond will be covered with ice.

Question 4

Solution:

Let x be the number of stripes 1×6 that are cut and let y be the number of stripes 1×7 that are cut.

So we have

$$6x + 7y = 11.12$$

and as $6/6.x$ and $6/11.12$ we have

$6/7.y$, which means that $6/y$.

To have the smallest number of stripes cut we should have the most stripes 1×7 and the least possible number of stripes 1×6.

So the first possibility is $x = 1$ and $y = 18$.

We will prove that it is not possible to cut the rectangle 11×12 into 18 stripes 1×7 and 1 stripe 1×6:

Lets colour the big rectangle in 7 colours - 1,2,3,4,5,6 and 7 as shown:

1	2	3	4	5	6	7	1	2	3	4	5
7	1	2	3	4	5	6	7	1	2	3	4
6	7	1	2	3	4	5	6	7	1	2	3
5	6	7	1	2	3	4	5	6	7	1	2
4	5	6	7	1	2	3	4	5	6	7	1
3	4	5	6	7	1	2	3	4	5	6	7
2	3	4	5	6	7	1	2	3	4	5	6
1	2	3	4	5	6	7	1	2	3	4	5
7	1	2	3	4	5	6	7	1	2	3	4
6	7	1	2	2	4	5	6	7	1	2	3
5	6	7	1	2	3	4	5	6	7	1	2

So we will have 20 1's; 20 2's; 19 3's; 18 4's; 18 5's; 18 6's; 19 7's

If it is possible to cut it into 1 stripe 1×6 and 18 stripes 1×7 After cutting every stripe 1×7 will contain one square of each colour, so for the 1 stripe 1×6 are left 2 1's, 2 2's; 1 3's and 1 square with colour 6 which is impossible.

So the next possibility is cutting the rectangle into 12 stripes 1×7 and 8 stripes 1×6 which is possible as shown on the next page.

Question 4. (continued here)

Solution:

1	1	1	1	1	1	1	8	9	10	11	12
2	2	2	2	2	2	2	8	9	10	11	12
3	3	3	3	3	3	3	8	9	10	11	12
4	4	4	4	4	4	4	8	9	10	11	12
5	5	5	5	5	5	5	8	9	10	11	12
6	6	6	6	6	6	6	8	9	10	11	12
7	7	7	7	7	7	7	8	9	10	11	12
13	13	13	13	13	13	17	17	17	17	17	17
14	14	14	14	14	14	18	18	18	18	18	18
15	15	15	15	15	15	19	19	19	19	19	19
16	16	16	16	16	16	20	20	20	20	20	20

- The way the stripes are cut is shown with numbers.

Question 5.

Solution:

Lets have the number of candies in each bag be $x_1, x_2, \ldots, x_{100}$ and let $S = x_1 + x_2 + \ldots + x_{100} \Rightarrow$

$S \geq 1+2+\ldots+100 = 5050$

Now lets see how many candies we should count twice in the sum $x_1 + x_2 + \ldots + x_{100}$ - those are

$S - 2018 \geq 5050 - 2018 = 3032$.

But as the total number of candies is 2018 we can't count more than once 3032 without having to count the number of candies in one of the bags ~~at them~~ more than twice. (we count the number of candies in a bag more than twice if if it is in a bag, which is also in a bag).

That's how we proved that we should have a bag that contains another bag, which also contains a bag.