International mathematical Olympiad
"Formula of Unity" / "The Third Millennium"
2017/2018 year, final round

Participant form

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Problem 1) Let us have a (1×6) strips and b (1×7) $\Rightarrow$

$$\Rightarrow 6a+7b=11\cdot 12$$

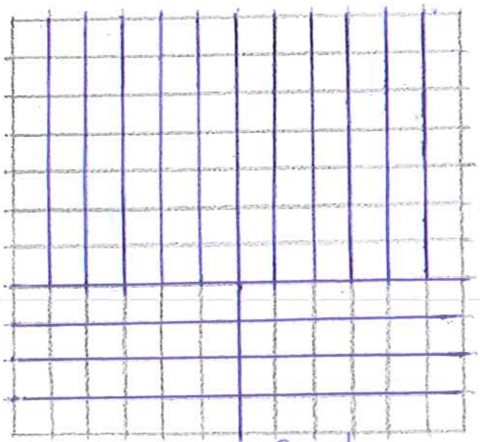
$$\Downarrow$$

$$6(a+b)+b=11\cdot 12 \Rightarrow a+b=22-\frac{b}{6} \Rightarrow 6 \mid b$$

$$22-\frac{b}{6}=a+b \geq b \Rightarrow 22 \geq \frac{7b}{6} \Rightarrow b \leq 18 \Rightarrow b \in \{0,6,12,18\}.$$

We see that we minimize our $(a+b)$ by maximizing b.

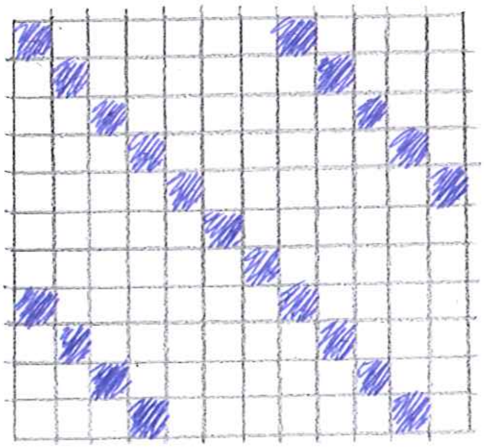
Let $b=12 \Rightarrow \underline{a+b=20}$



$\Rightarrow$ this is a possible solution

Suppose that we can construct a pattern with $b=18 \Rightarrow$

$$\Rightarrow a+b=22-\frac{18}{6}=19 \Rightarrow a=1$$



From this coloring we see that each (1×7) strip occupies exactly one colored square $\Rightarrow$
$\Rightarrow$ the one (1×6) must occupy $20-18=2$ colored squares. which is impossible.

$$\boxed{\min(a+b)=20}$$

Problem 2) Notations:

$$\sum = \sum_{cyc} \; ; \; \underline{a = x^2,\ b = y^2,\ c = z^2}$$

$$\Downarrow$$

$$a, b, c \geq 0$$

Our condition is rewritten as follows:

$$\left(\sum \frac{xy}{z}\right)\left(\sum \frac{x}{yz}\right) = \frac{\sum x^2y^2}{xyz} \cdot \frac{\sum x^2}{xyz} =$$

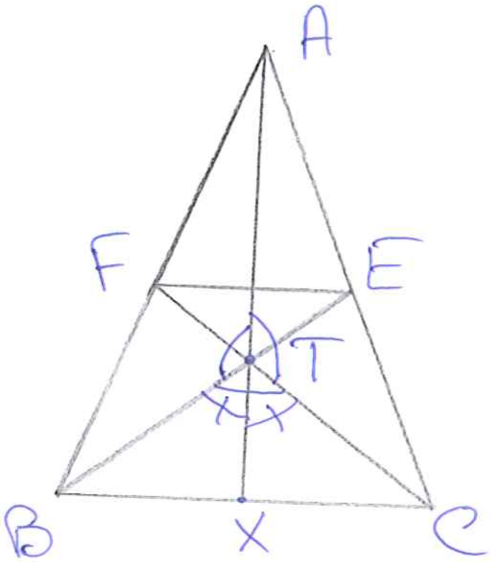
$$= \frac{\left(\sum x^2y^2\right)\left(\sum x^2\right)}{x^2y^2z^2} = \frac{\left(\sum ab\right)\left(\sum a\right)}{abc} = \frac{\sum a^2b + \sum ab^2 + 3abc}{abc}$$

By AM-GM inequality $\Rightarrow \begin{cases} \sum a^2b \geq 3abc \\ \sum ab^2 \geq 3abc \end{cases}$

So, $\dfrac{\sum a^2b + \sum ab^2 + 3abc}{abc} \geq \dfrac{9abc}{abc} = 9$

Equality holds for $a = b = c \Rightarrow$
$\Rightarrow x = y = z$ is one of equality cases.

Problem 4)

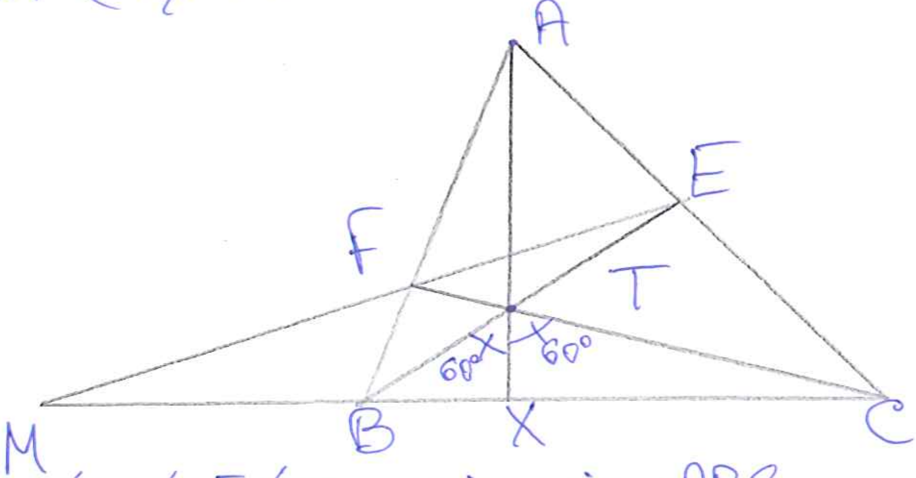


Let $X = AT \cap BC \Rightarrow \angle BTX = \angle CTX = 60^\circ$.

Suppose $EF \parallel BC \Rightarrow \frac{AF}{BF} = \frac{AE}{CE}$. From Ceva's theorem $\Rightarrow \frac{AF}{BF} \cdot \frac{BX}{CX} \cdot \frac{CE}{AE} = 1 \Leftrightarrow BX = CX \Rightarrow$

$\Rightarrow (TX)$ is median and angle bisector in $\triangle BTC \Rightarrow$

$\Rightarrow \triangle BTC$ is isosceles $\Rightarrow TX$ is the segment bisector of (BC). Because $A \in TX \Rightarrow AB = AC$ - contradiction.



$$\begin{cases} (AX, (BE, (CF \text{ - cevians in } \triangle ABC \\ M = EF \cap BC \end{cases} \Rightarrow$$

$\Rightarrow (M, X; B, C) = -1$. With unoriented segments $\Rightarrow$

$\Rightarrow \frac{BM}{BX} : \frac{CM}{CX} = 1 \Rightarrow \frac{MB}{MC} = \frac{BX}{CX} = \frac{TB}{TC}$ (from angle bisector theorem for $\triangle BTC$ and (TX) - angle bisector).