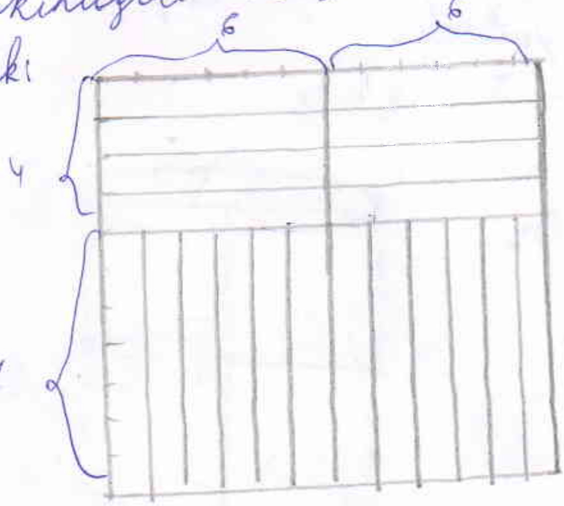
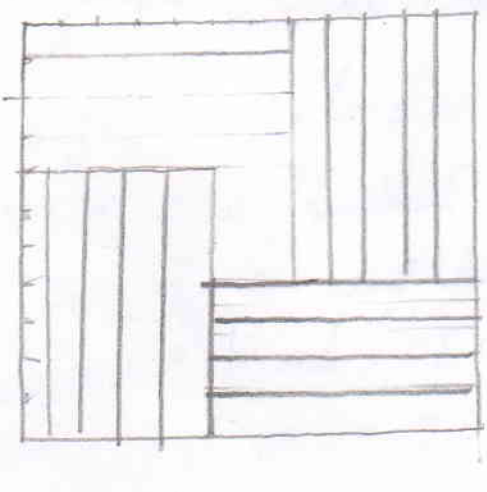
11 x 12 tōg'ri tōrtburchak 1 x 6 va 1 x 7 polosalarga qirqilgan. Eng kamida nechta polosa bōlishi mumkin?

Yechish: 1) 11 x 12 tōg'ri tōrtburchak 1 x 1 kvadratlarga ajratilgan deylik, u holda 1x6 va 1x7 polosalar mos ravishda 6 ta va 7 ta kvadratni ōz ichiga oladi. U holda x ta 1x6 polosa, y ta 1x7 polosaga qirqildi desak, $6 \cdot x + 7 \cdot y = 132$ bōladi.

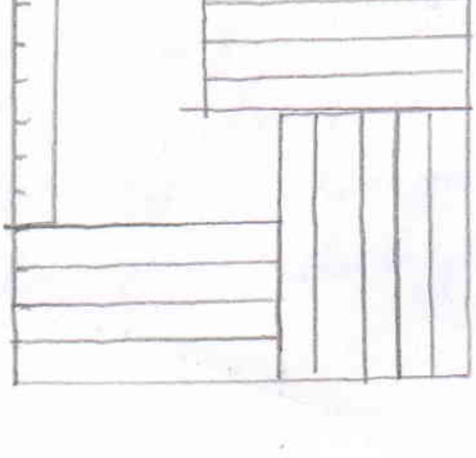
Bu tenglama natural sonlarda $x_1 = 1$; $y_1 = 18$; $x_2 = 8$; $y_2 = 12$; $x_3 = 15$; $y_3 = 6$ yechimlarga ega. U holda $x+y$ soni $x_1 = 1$ va $y_1 = 18$ da eng kichik qiymatga erishadi, lekin tōg'ri tōrtburchakni shunday qirqish mumkinligini kōrsatish kerak. $x_2 = 8$ va $y_2 = 12$ holni qaraylik:



Demak, bu holda kesish mumkin. Oldingi holni qaraylik, bironta 1x7 polosa 12 ga teng tomondan ~~qirqishga~~ qirqilsa, qolgan 5 ga teng joydan 5 ta 7x1 lik polosa vertikal ravishda qirqib olish zarur, keyingi tomonda ham shunday, va hokazo. U holda tōg'ri tōrtburchak quyidagi kōrinishga keladi:



Agar 11 lik tomondan kesib boshlansa quyidagi holga keladi:



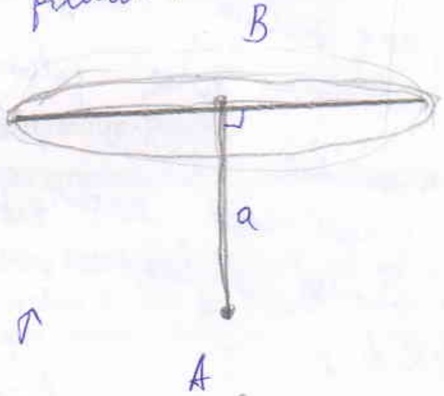
Demak, bu holda 18 ta 1×7 polosa va 1 ta 2×3 polosa h
bōladi, ya'ni 1 ta 1×6 polosani butun kesib olishning ilo
yoq. U holda $x+y$ ning eng kichik qiymati $x_2+y_2 =$
$= 8+12 = 20$ bōladi.

Javob: 20 ta bōlishi mumkin

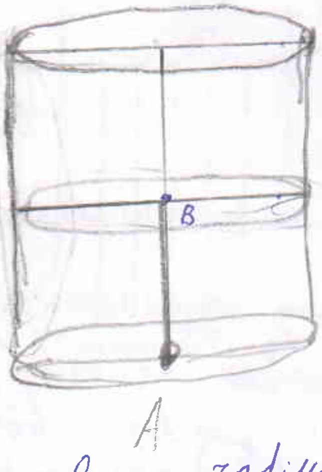
3. Barcha bōlaklari teng hamda ixtiyoriy ikki qōshni bōlak perpendikulyar bōlgan beshbōlakli yassi bōlmagan yopiq siniq chiziq mavjudmi?

Yechish: 1) Siniq chiziq uchlarini A; B; C; D; E deylik. $AB=BC=\ldots=DA=a$ deylik. U holda A va B nuqtalarni fikrlasak, C nuqta a radiusli, markazi B da bōlgan va aylana tekisligi AB ga perpendikulyar bōlgan aylanada yotadi. D nuqta esa silindr hosil qiladi:

1)



2)



Bunda D nuqta silindr asosidagi aylana radiusi $\sqrt{2}a$; balandligi esa $2a$ ga teng

Noldan farqli $x; y; z$ haqiqiy sonlar uchun

$$\left(\frac{xy}{z}+\frac{zx}{y}+\frac{yz}{x}\right)\cdot\left(\frac{x}{yz}+\frac{y}{zx}+\frac{z}{xy}\right)$$ ifodaning eng

kichik qiymatini toping.

Yechish: 1) $A=\left(\frac{xyz}{z^2}+\frac{xyz}{y^2}+\frac{xyz}{x^2}\right)\cdot\left(\frac{x^2}{xyz}+\frac{y^2}{xyz}+\frac{z^2}{xyz}\right)$ deb

yozib olish mumkin, bunda $xyz\neq 0$ bo'lgani uchun

$A=\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)\cdot(x^2+y^2+z^2)$ bo'ladi. $x; y; z\in R$

bo'lgani uchun $x^2>0$; $y^2>0$; $z^2>0$. U holda

Cauchy tengsizligini qo'llash mumkin, chunki $x^2; y^2; z^2$ musbat sonlar $\Rightarrow$

$$\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\geq 3\cdot\sqrt[3]{\frac{1}{x^2y^2z^2}}$$

$$x^2+y^2+z^2\geq 3\cdot\sqrt[3]{x^2y^2z^2}$$

$$A=\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)\cdot(x^2+y^2+z^2)\geq 3\cdot\sqrt[3]{\frac{1}{x^2y^2z^2}}\cdot 3\cdot\sqrt[3]{x^2y^2z^2}=9$$

Ya'ni $A\geq 9$ bo'lib qoldi.

Javob: eng kichik qiymat 9 ga teng.

4. ABC uchburchakning barcha burchaklari $120°$ dan kichik, $AB\neq AC$. Uchburchak ichidagi T nuqta uchun BTC, CTA va ATB burchaklar $120°$ ga teng. BT to'g'ri chiziq AC tomonni E nuqtada, CT to'g'ri chiziq esa AB tomonni F nuqtada kesadi. EF va BC to'g'ri chiziqlar M nuqtada kesishsa, $MB:MC=TB:TC$ ekanligini isbotlang.

1. B-n; $\triangle ABC$

$\angle A < 120^\circ$; $\angle B < 120^\circ$; $\angle C < 120^\circ$

$AB \neq AC$

$T \in \triangle ABC$.

$\angle ATC = \angle BTC = \angle ATB = 120^\circ$

$BT \cap AC = E$

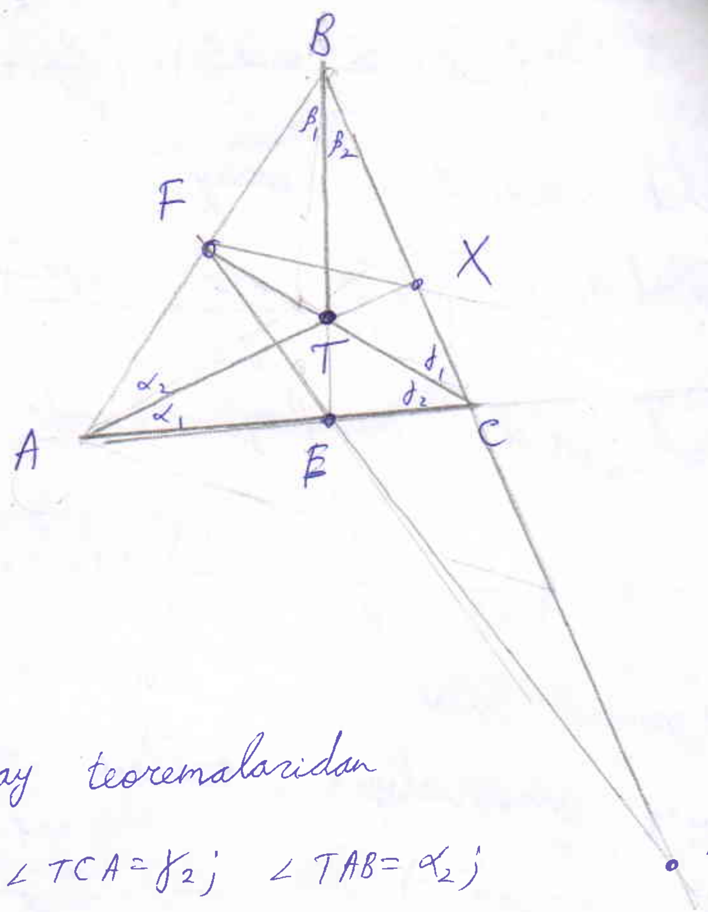
$CT \cap AB = F$

$EF \cap BC = M$

Is. q. k-k:

$$\frac{MB}{MC} = \frac{TB}{TC}$$

chizma:



Isbot. Cheva va Menelay teoremalaridan foydalanamiz. $\angle TAC = \alpha_1$; $\angle TCA = \gamma_2$; $\angle TAB = \alpha_2$; $\angle TBA = \beta_1$; $\angle TBC = \beta_2$; $\angle TCB = \gamma_1$; $\angle TCA = \gamma_2$ deb belgilab olamiz. U holda Cheva teoremasiga kōra

$$R = \frac{\sin\alpha_1}{\sin\alpha_2} \cdot \frac{\sin\beta_1}{\sin\beta_2} \cdot \frac{\sin\gamma_1}{\sin\gamma_2} = 1 = \frac{AF}{FB} \cdot \frac{BX}{XC} \cdot \frac{CE}{EA}$$

Menelay teoremasiga kōra:

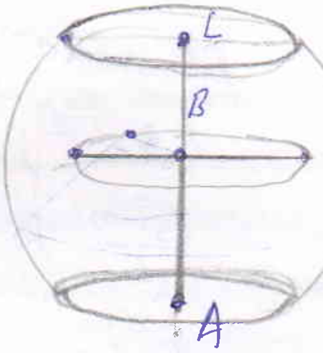
$$\frac{AF}{FB} \cdot \frac{BM}{MC} \cdot \frac{CE}{EA} = 1 = \frac{AF}{FB} \cdot \frac{BX}{XC} \cdot \frac{CE}{EA} \text{ ya'ni}$$

$\frac{BM}{MC} = \frac{BX}{XC}$, bunda $AT \cap BC = X$ deb olindi. U holda $\frac{BX}{XC} = \frac{BT}{TC}$ ni isbotlash yetarli.

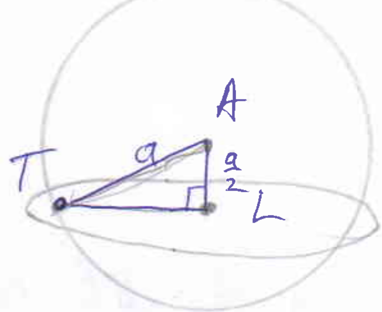
Masala shartiga kōra, $\angle ATB = \angle ATC = 120^\circ$, demak, $\angle BTX = \angle CTX = 60^\circ$, chunki ular qōshni burchaklar. U holda TX – bissektrisa ($\triangle BTC$ uchun) va bissektrisa xossasiga kōra $\frac{BT}{TC} = \frac{BX}{XC}$. Buni $\triangle BTX$ va $\triangle TCX$da sinuslar teoremasini qōllab ham kōrsatish mumkin: $\frac{BT}{BX} = \frac{CT}{CX} = \frac{\sin TXB}{\sin 60^\circ}$

Da'vo isbotlandi.

Yechish: Siniq chiziq uchlarini A, B, C, D, E deb belgilab, A va B nuqtani fiksirlaymiz, bunda $AB = BC = \ldots = EA = a$ deb olamiz. C nuqtalar tōplami aylanani, D nuqtalar tōplami esa kesik sharni hosil qiladi:



Bunda shar radiusi $\sqrt{2}\,a$ ga, BL esa a ga teng, ya'ni B – shar markazi. Shu sharda biror D_0 nuqta topiladiki, bunda $AD = \sqrt{2}\,a$ bōladi, bu D_0 nuqtaga mos C_0 nuqta aniq mavjud. Demak, A markazli va $\sqrt{2}\,a$ radiusli shar b-n D nuqtalar tōplami kesishgandagi ixtiyoriy nuqta bu shartni qanoatlantiradi, bu kesishma esa aylanadan iborat. Bu aylana radiusi $\frac{\sqrt{7}\,a}{2}$ ga teng; uning markazi AB ning ōrtasida yotadi. Demak, D nuqtalar tōplami ana shu aylanadir. U holda E nuqtalar tōplami ham kesik shar hosil qiladi, bunda $\frac{AB}{2} \equiv X$ desak, shar markazi X; $AE = a$ bōlgani radiusi $\sqrt{\frac{7a^2}{4} + a^2} = \frac{\sqrt{11}\,a}{2}$ bōladi. E nuqtalar tōplami uchun A markazli shar b-n biror nuqtada kesishsa, siniq chiziq mavjud bōladi. Bunday nuqtalar esa mavjud emas, chunki, BA nurda $XL = a$ bōlgan AB nurda $XQ = a$ bōlgan Q nuqta va L nuqta olsak, E nuqtalar tōplami hosil qilgan kesik shar chegaralari Q va L markazli, $\frac{\sqrt{7}\,a}{2}$ radiusli aylanalar bilan chegaralangan bōladi, A markazli a radiusli ~~aylana~~ shar esa shu aylanani kesib ōtadi. Agar L markazli aylana A markazli shar ichida yotganda, bunday nuqtalar tōplami cheksiz kōp bōlar edi:



Chizmada: $TA = a$; $LA = \frac{a}{2}$; demak, $LT = \frac{\sqrt{3}a}{2} < \frac{\sqrt{7}a}{2}$.

A markazli shar E nuqtalar tōplami b-n kesishmaydi.

Javob: mavjud emas

Izoh: $\angle ABC = \angle BCD = \ldots = 90^\circ$ bōlgani uchun kichik diagonallarning bari $\sqrt{2}a$ ga teng bōladi.

5. Arifmetik progressiyani tashkil qilgan natural sonlardan iborat (a, b, c) uchlik $(a < b < c)$ uchun $ab+1$, $bc+1$, $ca+1$ sonlar tōla kvadrat bōladi. Bunday uchliklar nechta?

Yechish: 1) Cheksiz kōp. $a+d = b$; $c = a+2d$ deylik, bunda $d \in N$. d ni tanlash bilan topamiz. Tōla kvadratlar a^2+2a+1; a^2+4a+4; a^2+6a+9; $a^2+8a+16$; ... kōrinishda bōladi. Shulardan ōzaro mos 3 tasini a^2+2a; a^2+4a+3; a^2+6a+8... ab; bc; ac tanlaymiz; bunda va hokazolardan biri bōladi.

$b = a+2$; $c = a+4$ deb tanlasak, a^2+4a+1 tōla kvadrat bōladigan a mavjud emas. $b = a+6$ $c = a+12$ deylik,