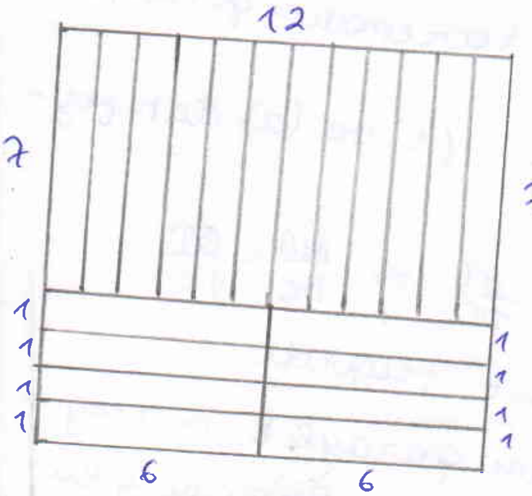
1. 11×12 talik tōgri tōrtburchakni 1×1 bōlgan kvadratchalarga ajratsak, 132 ta kvadrat bōladi. Kichik tōgri tōrtburchaklarda esa 6 ta va 7 ta kvadratcha bōladi. 1×6 tōgri-tōrtburchakdan x ta, 1×7 tōgri tōrtburchakdan y tasini qōyish kerak bōlsin. U holda 1×7 liklarni soni eng kōp bōlganda polosalar soni eng kam bōladi. Chunki u ōzining ichiga kōp kvadratchani oladi. Demak, $6x+7y=132$, $y=\frac{132-6x}{7}=18+\frac{6-6x}{7}$
Demak, $x=1$, $y=18$ bu tenglamani qanoatlantiradi. Ammo 11×12 tōgri tōrtburchakni 18 ta 1×7 va 1 ta 1×6 bilan qoplab bōlmaydi. Bundan keyingi yechim esa $x=8$; $y=12$. Bu holda quyidagicha bōladi. U holda jami polosalarning eng kichik soni $8+12=20$ ta bōladi.



Javob: 20 ta

2. Biz bu ifodaning eng kichik qiymatini topish uchun koshi – bunyakovskiy – shvarts tengsizligidan foydalanamiz

$$(a_1^2+a_2^2+\dots+a_n^2)\cdot(b_1^2+\dots+b_n^2)\geq(|a_1b_1|+|a_2b_2|+\dots+|a_nb_n|)^2.$$

Bu tengsizlikning isboti vektorlardan oson chiqadi.
$\bar{A}(a_1;a_2\dots;a_n)$; $\bar{B}(b_1;b_2;\dots;b_n)$ Biz bilamizki,

$|\bar{A}|\cdot|\bar{B}|\geq\bar{A}\cdot\bar{B}$. $\Rightarrow$ tengsizlikni olamiz.

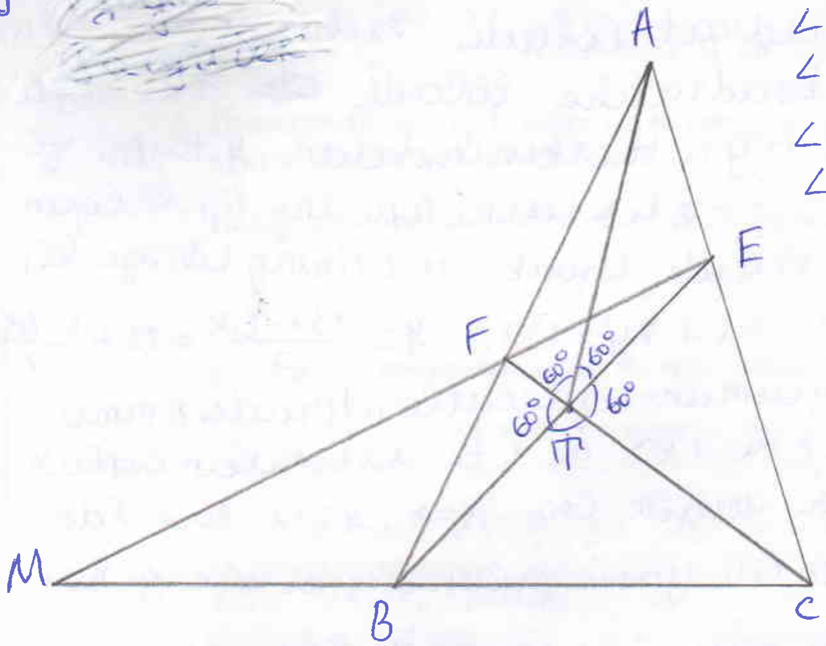
Endi masalaga qaytaylik.

$$\left(\frac{xy}{z}+\frac{zx}{y}+\frac{yz}{x}\right)\cdot\left(\frac{x}{yz}+\frac{y}{zx}+\frac{z}{xy}\right)\geq\left(\sqrt{\frac{xy}{z}\cdot\frac{z}{xy}}+\sqrt{\frac{zx}{y}\cdot\frac{y}{zx}}+\sqrt{\frac{yz}{x}\cdot\frac{x}{yz}}\right)^2=$$

$=3^2=9$ Demak, bu (teng) ifodaning eng kichik qiymati 9 ga teng. va $|x|=|y|=|z|$ bōlganda erishadi.

Javob: 9.

4



$\angle ETC = 60°$

$\angle ATE = 60°$

$\angle FTA = 60°$

$\angle FTB = 60°$

ΔATC da TE bissektrisa

Demak, $\frac{CE}{EA} = \frac{TC}{AT}$. (1)

ΔATB da TF Bissektrisa

Demak, $\frac{BT}{AT} = \frac{BF}{FA}$. (2)

ΔABC da E, F, M lar uchun meneloy teoremasini qōllaylik. (1) va (2) dan foydalansak,

$\frac{MB}{MC} \cdot \frac{CE}{EA} \cdot \frac{AF}{FB} = 1. \Rightarrow \frac{MB}{MC} = \frac{FB}{FA} \cdot \frac{CE}{EA}$.

$\frac{MB}{MC} = \frac{FB}{FA} \cdot \frac{CE}{EA} = \frac{BT}{\cancel{AT}} \cdot \frac{\cancel{AT}}{TC} = \frac{BT}{TC}. \Rightarrow \frac{MB}{MC} = \frac{BT}{TC}$.

Isbot tugadi.

5 Biz quyidagi $u^2 - 3v^2 = 1$ tenglamani qaraylik. Bu tenglama pell tenglamasi hisoblanadi va bu tenglamaning butun sonlarda, natural sonlarda cheksizta yechimi bor. Chunki $(u_0; v_0)$ yechim bōladigan bōlsa, $(u_0^2 + 3v_0^2; 2u_0v_0)$ ham

$(u_0^2 - 3v_0^2)^2 = 1 \Rightarrow (u_0^2 + 3v_0^2)^2 - 3 \cdot (2u_0v_0)^2 = 1 \Rightarrow$ yechim. Demak, cheksizta yechim bōladi. Bu tenglamada esa $(2;1)$ yechim. Biz $a = 2v - u$; $c = 2v + u$ va $b = 2v$ deb tanlasak,

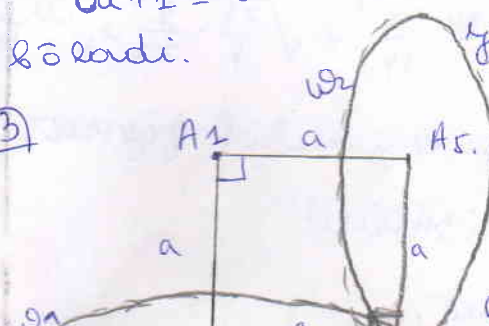
$ac + 1 = (2v - u) \cdot (2v + u) + 1 = 4v^2 - u^2 + 1 = v^2$

$bc + 1 = (2v + u) \cdot 2v + 1 = 4v^2 + 2uv + u^2 - 3v^2 = (u + v)^2$

$ba + 1 = (2v - u) \cdot 2v + 1 = 4v^2 - 2uv + u^2 - 3v^2 = (u - v)^2$ tōla kvadrat bōladi.

Jawob: Cheksizta.

3



Mavjud emas. Agar bu nuqtalar mavjud bōlsa ular (A_3, A_4 nuqtalar) ω_1 va ω_2 aylanalarda yotishi kerak. Ammo bu aylanalardagi ixtiyoriy 2 ta nuqtalarni qarasak, bu shartni bajarmaydi. Demak, bunday yopiq siniq chiziq mavjud emas

3) Davomi.

$A_2A_5=\sqrt{2}a$, $A_3A_5=\sqrt{2}a$, $A_2A_4=\sqrt{2}a$. Boshqa tomondan esa Agar A_3 va A_4 lar mavjud bo'lsa, A_2, A_3, A_4, A_5 lar bitta tekislikka tegishli bo'lishi kerak. Bu holatda esa ular o'zaro perpendikular bo'lmay qoladi. Demak, bunday yopiq siniq chiziq mavjud emas.

Javob: Mavjud emas.