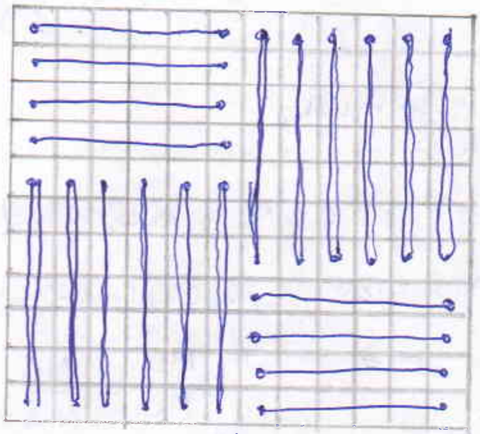
12 jadvalni (1) rasmdagi deb qilib jamlab olaylik. qarasak 1×7 polosa-ni qayerda olmaylik uning ichida 1 dan 7 gacha raqam turadi. raqamlarni sanasak

4	5	6	7	1	2	3	4	5	6	7	1
3	4	5	6	7	1	2	3	4	5	6	7
2	3	4	5	6	7	1	2	3	4	5	6
1	2	3	4	5	6	7	1	2	3	4	5
7	1	2	3	4	5	6	7	1	2	3	4
6	7	1	2	3	4	5	6	7	1	2	3
5	6	7	1	2	3	4	5	6	7	1	2
4	5	6	7	1	2	3	4	5	6	7	1
3	4	5	6	7	1	2	3	4	5	6	7
2	3	4	5	6	7	1	2	3	4	5	6
1	2	3	4	5	6	7	1	2	3	4	5

(1)

1 — 18 ta
2 — 18 ta
3 — 19 ta
4 — 20 ta
5 — 20 ta
6 — 19 ta
7 — 18 ta



(2)

Agar 18 (1; 18) yechim bo'lsin deb faraz qilaylik. 18 ta 1×7 polosa bo'lsa qolgan 6 ta katakchalar bor 2 ta 4, 2 ta 5, 1 ta 3, 1 ta 6.
shu qolgan 6 ta katakcha yonma-yon joylashmagan (chunki 2 ta 4 va 2 ta 5 bor) ⇒ 1×6 ni joylashtira olmaymiz. Demak (1; 18) yechim bo'lmas ekan.
qolgan u 3 ta javob dan x+y eng kichigi — (8; 12).
Bir bu javob bo'lsa olishini (2) rasmda ko'rsatganmiz!
Demak Javob: 20

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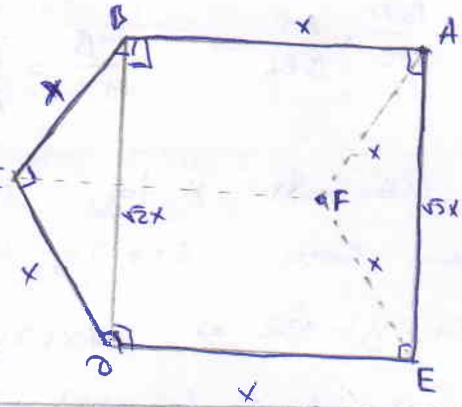
(3) Ha, bor, ABCDE — sozlangan yopiq siniq chiziq.



asoslari teng yonli to'g'ri burchakli Δ bo'lgan to'g'ri prizma!

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(5) a, $b = a + d$, $c = a + 2d$ deylik. Bir javob cheksiz ko'pligini isbotlaymiz:
$a = 1$, deb olaylik $1;\ 1+d;\ 1+2d$. $\Rightarrow \begin{cases} d+2 = x^2 \\ 2d+2 = y^2 \\ 2d^2+3d+2 = z^2 \end{cases} \Rightarrow \begin{cases} d+2 = x^2 \\ d+1 = 2y_1^2 \Rightarrow \\ 2d^2+3d+2 = z^2 \end{cases}$ $2y_1^2 + 1 = x^2$

$x^2 - 2y_1^2 = 1$ bu tenglama Pell ga ko'ra cheksiz ko'p yechimga ega

Javob: cheksiz ko'p.

2) Biz bu tengsizlikni yechayotganda Koshi-Bunyakovskiy-Shwarz tengsizligining $n=3$ holi dan foydalanamiz:

$$\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)\cdot\left(x^2+y^2+z^2\right) \geq \left(\frac{1}{x}\cdot x+\frac{1}{y}\cdot y+\frac{1}{z}\cdot z\right)^2=(1+1+1)^2=9. \quad (*)$$

$$\left(\frac{xy}{z}+\frac{zx}{y}+\frac{yz}{x}\right)\left(\frac{x}{yz}+\frac{y}{zx}+\frac{z}{xy}\right)=\left(\frac{xyz}{z^2}+\frac{xyz}{y^2}+\frac{xyz}{x^2}\right)\left(\frac{x^2}{xyz}+\frac{y^2}{xyz}+\frac{z^2}{xyz}\right)=$$

$$=xyz\cdot\left(\frac{1}{z^2}+\frac{1}{y^2}+\frac{1}{x^2}\right)\cdot\frac{1}{xyz}\cdot\left(x^2+y^2+z^2\right)=\left(x^2+y^2+z^2\right)\left(\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}\right)\overset{(*)}{\geq} 9.$$

Demak, Javob: 9.

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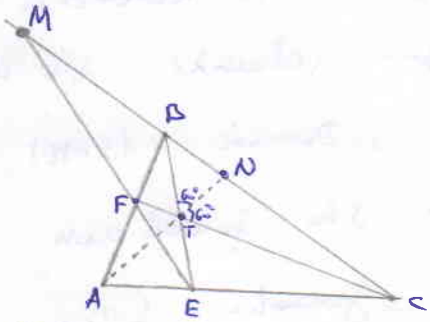
4) $AT \cap BC = N$ deb olaylik. $\angle ATB=180° \Rightarrow \angle BTN=60°$

$\angle ATC=180° \Rightarrow \angle CTN=60° \Rightarrow$ $TN - \angle BTC$ ning bissektrisasi $\Rightarrow \frac{TB}{TC}=\frac{BN}{NC}$.

Endi biz Ft $\frac{MB}{MC}=\frac{NB}{NC}$ ni isbotlaymiz.



1-usul AN, CF, BE — concurrent $\Rightarrow$

$\Rightarrow (MBNC)=-1 \Rightarrow \frac{MB}{MC}=\frac{NB}{NC}$ Q.E.D.

2-usul $\triangle ABC$ni EF tich kesyapti, Meneley ga ko'ra: $\frac{AF}{FB}\cdot\frac{BM}{MC}\cdot\frac{CE}{AE}=1$

$\triangle ABC$ ga Cheva ga ko'ra: $\frac{AF}{BF}\cdot\frac{BN}{NC}\cdot\frac{CE}{AE}=1$ bo'lib ~~ko'paytirib~~ yuboraylik

$\frac{BM}{MC}\cdot\frac{NC}{BN} \Rightarrow \frac{MB}{MC}=\frac{NB}{NC} \Rightarrow$ isbotlandi! ▲

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5) 1×6 dan x ta, 1×7 dan y ta bo'lin deylik. Barcha katakchalar soni $6x+7y$ ta boshqa tomondan $11\cdot12=132$ ta bor.

$6x+7y=132 \Rightarrow 7x+7y=133+x-1 \Rightarrow x-1=7(x+y)-7\cdot19 \Rightarrow x-1 \,\vdots\, 7 \Rightarrow$

$\Rightarrow x=7k+1 \; (k\in Z^{0}) \Rightarrow 7k=7(7k+1+y)-7\cdot19 \Rightarrow k=7k+1+y-19 \Rightarrow$

$\Rightarrow y=18-6k. \Rightarrow k=0,1,2,3 \Rightarrow (x;y)=(1;18);(8;12);(15;6);(22;0)$

yechimlar chiqadi.

Biz $(1;18)$ yechim bo'la olmasligini ko'rsatamiz: