2) $x, y, z \neq 0$

$$\left(\frac{xy}{z}+\frac{zx}{y}+\frac{yz}{x}\right)\left(\frac{x}{yz}+\frac{y}{zx}+\frac{z}{xy}\right)=\frac{x^2}{z^2}+\frac{y^2}{z^2}+1+$$

$$+\frac{x^2}{y^2}+1+\frac{z^2}{y^2}+1+\frac{y^2}{x^2}+\frac{z^2}{x^2}=3+\frac{x^2}{z^2}+\frac{z^2}{y^2}+\frac{y^2}{x^2}+$$

$$+\frac{y^2}{z^2}+\frac{x^2}{y^2}+\frac{z^2}{x^2}$$

Örta arifmetik örta geometrik qiymatdan kichik emasligiga köra,

$$+\begin{cases}\frac{x^2}{z^2}+\frac{z^2}{y^2}+\frac{y^2}{x^2}\geqslant 3\sqrt[3]{\frac{x^2}{z^2}\cdot\frac{z^2}{y^2}\cdot\frac{y^2}{x^2}}=3\\ \frac{y^2}{z^2}+\frac{x^2}{y^2}+\frac{z^2}{x^2}\geqslant 3\sqrt[3]{\frac{y^2}{z^2}\cdot\frac{x^2}{y^2}\cdot\frac{z^2}{x^2}}=3\end{cases}$$

$$\frac{x^2}{z^2}+\frac{z^2}{y^2}+\frac{y^2}{x^2}+\frac{y^2}{z^2}+\frac{x^2}{y^2}+\frac{z^2}{x^2}\geqslant 6$$

$\frac{x^2}{z^2}+\frac{z^2}{y^2}+\frac{y^2}{x^2}+\frac{y^2}{z^2}+\frac{x^2}{y^2}+\frac{z^2}{x^2}=A$ deb belgilash kiritamiz.
A ning eng kichik qiymati 6 ga teng. Demak $(A+3)$ ifodaning eng kichik qiymati 9 ga teng.

Javob: 9

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1) A, B, C... paketlar bölsin deyik. A paket ichida B, C... paketlar bölishi mumkin va B, C... paketlar ichida boshqa paket bölishi mumkin emas.
A paket ichida B paket, C paket ichida D paket, E paket ichida F paket, G paket ichida H paket va hokazo bölsin.
A paketda eng kamida 2 ta konfet bor. A paketda 2 ta bölsin. U holda B paketda 1 ta konfet bor.
C paketda eng kamida 4 ta konfet bor. C paketda 4 ta bölsa, D paketda 3 ta konfet bor.

E paketda eng kamida 6 ta konfet bor. E paketda 6 ta bólsa, F paketda 5 ta konfet bor.
G paketda eng kamida 8 ta konfet bor. G paketda 8 ta bólsa, H paketda 7 ta konfet bor.
Barcha konfetlar soni 20 ta bóldi.

A – 2 ta, B paket A paketni ichida – 1 ta

C – 4 ta, D paket C paketni ichida – 3 ta

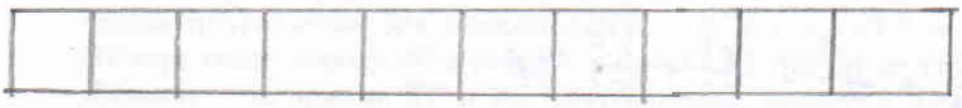
E – 6 ta, F paket E paketni ichida – 5 ta

G – 8 ta, H paket G paketni ichida – 7 ta

Paketlar soni eng kopi bilan 8 ta bólishi mumkin.

J: 8 ta

4)



Hosil bóladigan son tóla kvadrat bólsin deyaik. Petya birinchi yuradi demak u oxirgi katakka 1, 4, 5, 6, 9 raqamlarini yoki oxirgi 2 tasiga 00 ni qóyishi kerak. Agar u oxirgi 2 ta katakka 00 ni qóysi undan oldingi katakka 1, 4, 5, 6, 9 raqamlaridan birini qóyishi kerak. (Keyingi yurish) Agar oxiriga 5 qóysa, undan oldingi katakka 2 raqamini qóyish kerak. Keyingi yurish Vasyaniki bóladi. Vasya qolgan kataklarning biriga bitta raqamni qóyadi. Shundan keyin Petya qolgan kataklarni son tóla kvadrat bóla oladigan qilib raqamlar bilan tóldirib chiqishi mumkin. Demak Petyada galabaga erishish strategiyasi mavjud.

Javob: Petyada

Shartga kōra $\angle BTC = 120°$, $\angle ATB = \angle ATC = 120°$

$\angle FTB + \angle BTC = 180°$ Demak,

$\angle FTB = 60°$

$\triangle FTB$ da TK bissektrisa, Demak.

$$\frac{FT}{BT} = \frac{FK}{KB} \quad (4)$$

(3) va (4) tengliklardan $LF = FT$ ekanligi kelib chiqadi. Bundan esa $MB : MC = TB : TC$ degan xulosaga kelamiz. Isbotlandi.

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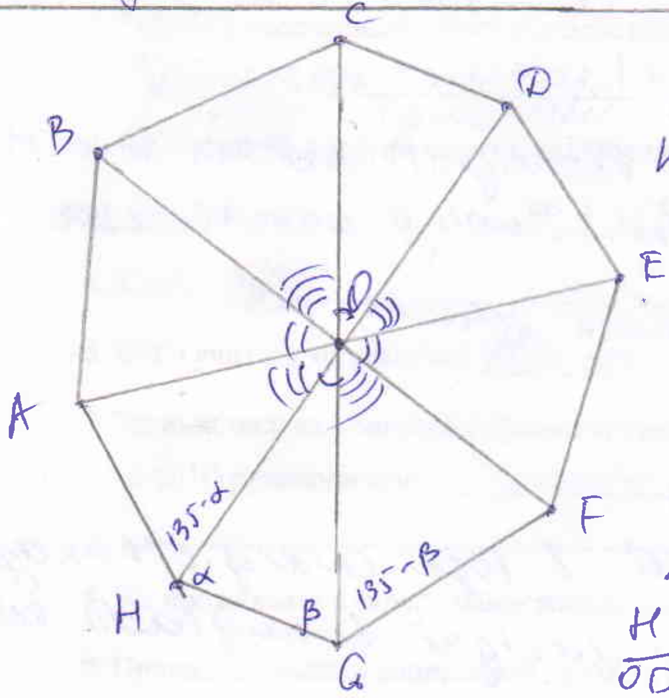
3) 

Qavariq sakkiz burchakning har bir burchagi $135°$ dan

$\angle OHG = \alpha$, $\angle OGH = \beta$

$\angle OHA = 135° - \alpha$ $\angle OGF = 135 - \beta$

$\angle FGO + \angle OCD = 135°$

$\angle AHO + \angle ODC = 135°$

Demak

$\angle ODC = \alpha$ $\angle OCD = \beta$

$$\frac{HO}{OD} = \frac{GO}{OC} \quad (1)$$

Haddi shunday, uchburchaklar oxshashligidan

$$\frac{GO}{OC} = \frac{FO}{OB} \quad (2)$$

$$\frac{FO}{OB} = \frac{EO}{OA} \quad (3)$$

$$\frac{EO}{OA} = \frac{BO}{OH} \quad (4)$$

(1), (2), (3) va (4) tengliklardan

$OD = OH$

$OC = OG$

$BO = OF$

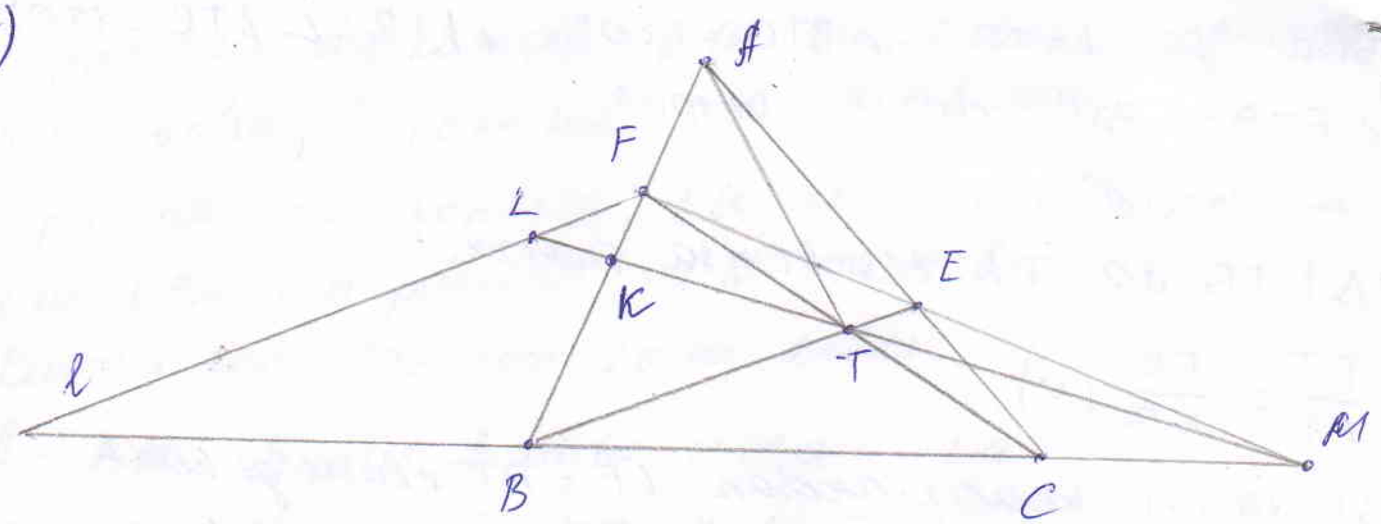
$AO = OE$ tengliklar kelib chiqadi.

Demak O qavariq kōpburchakning simmetriya markazi.

Ya'ni kōpburchakning simmetriya markazi bor.

Isbotlandi.

5)



Menelay teoremasiga ko'ra

$$\begin{cases} \dfrac{FE}{FM} \cdot \dfrac{MC}{BC} \cdot \dfrac{BT}{TE} = 1 \\ \dfrac{BC}{MB} \cdot \dfrac{ME}{EF} \cdot \dfrac{FT}{TC} = 1 \end{cases} \Rightarrow \quad \dfrac{MC}{MB} \cdot \dfrac{TB}{TC} \cdot \dfrac{ME \cdot FT}{FM \cdot TE} = 1$$

Bizdan $MB : MC = TB : TC$ ekanligini isbotlash so'ralmoqda.

Ya'ni $\dfrac{MC}{MB} \cdot \dfrac{TB}{TC} = 1$ ekanligini. Buning uchun esa

$\dfrac{ME \cdot FT}{FM \cdot TE} = 1$ ekanligini isbotlash yetarli.

$$\frac{ME}{TE} = \frac{FM}{FT} \quad (1)$$

F nuqtadan BE ga parallel l tog'ri chiziq o'tkazaylik.

MT tog'ri chiziq l tog'ri chiziqni L nuqtada kessin.

$LF \parallel TE$ bolganidan

$$\frac{ME}{TE} = \frac{FM}{FL} \quad (2)$$

(1) va (2) tengliklardan xulosa qilish mumkinki

$FL = FT$ ekanligini isbotlashimiz kerak.

MT tog'ri chiziq AB kesmani kesishish nuqtasini K deb belgilaylik.

$LF \parallel BT$ bolgani uchun

$$\frac{LF}{BT} = \frac{FK}{KB} \quad (3)$$