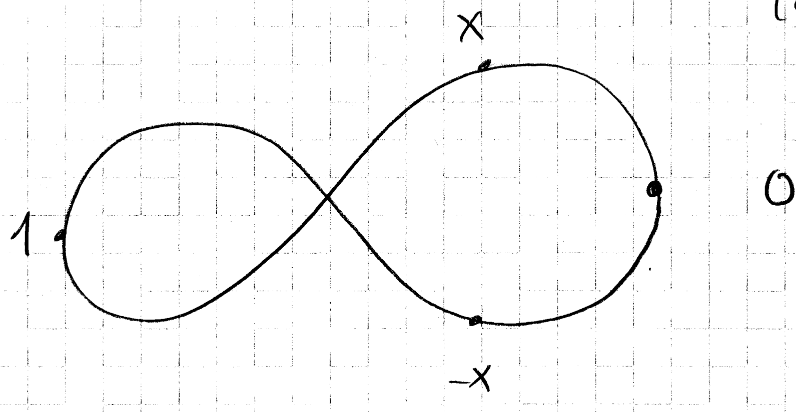
1.

total road length: 2 meters



Jerry speed: $S_1 \frac{m}{s}$

Tom speed: $S_2 \frac{m}{s}$

Jerry starts at $1 > X > 0$

Tom at $-X$

after 20 mins: Jerry at $X + 20S_1$

Tom at $-X + 20S_2$

So $X + 20S_1 + (-X + 20S_2) = 2$ because Tom is above Jerry

$$S_1 + S_2 = \frac{1}{10}$$

when they met (after T mintues):

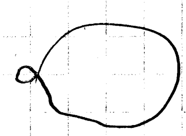
$$TS_1 = 2, \quad TS_2 = 2 + 2X$$

$$S_2 = S_1(X+1)$$

$$S_1 = \frac{1}{10(X+2)}$$

$$T = \frac{2}{S_1} = 20(X+2)$$

X can be anything between 0 to 1, and road can be something like



So every thing we did was okay. $40 < 20(X+2) < 60$

So answer: between 40 to 60 mintues.

2. First player wins.

For fixed first and last digits, there are 9 options to middle digits (for every option for 2nd digit there is exactly one option for 3rd digit).

We can see the game like this: every player gives a pair of digits, and the first digit in your pair is the last digit in the last pair, and each pair can be repeated no more than 9 times. (the pair represents the first and last digits of the number).

Strategy for first player:

* begin with $(1,1)$
* if you are given (a,b), return (b,a)

proof:

Lemma 1: All of the pairs given from first player end with a 1. by induction:

first pair is $(1,1)$

if 1st player gave $(a,1)$, second player gave $(1,b)$ then 1st player gave $(b,1)$.

Lemma 2: (1) At the end of first player turn, amount of used $(1,1)$ is odd.

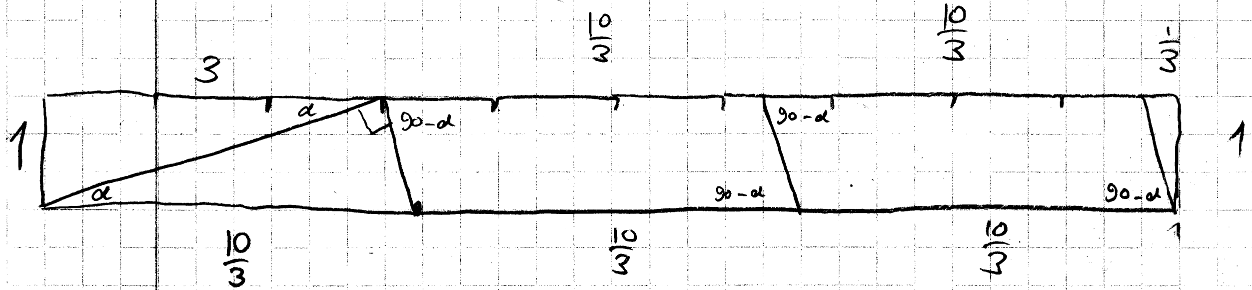
(2) Amount of used $(a,1)$ is equal to amount of used $(1,a)$ at the end of 1st player turn $(a\neq 1)$

(3) no other pairs are ever used.

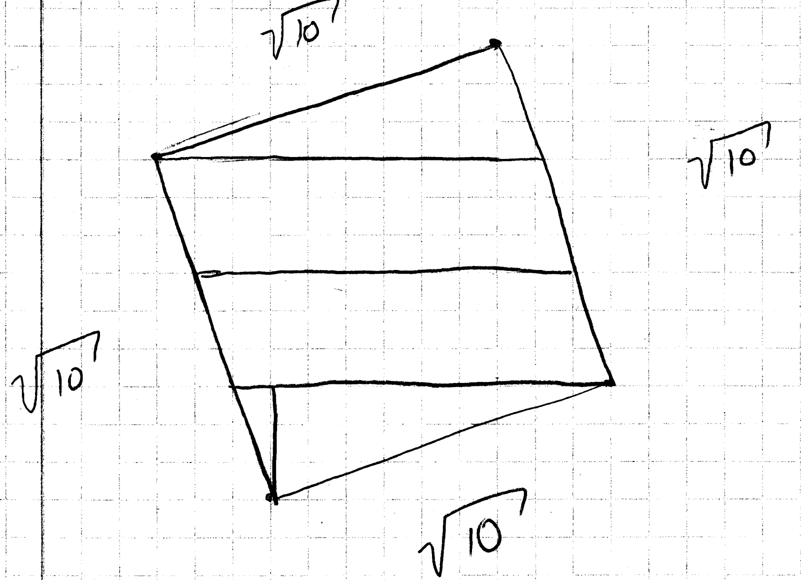
By induction. The details are obvious, so I will not write them. ((3) is by Lemma 1)

from Lemma2 we can see that if player 1 used a pair for the 10th time, then there must have been an illegal move earlier in the game (if it was $(1,1)$ and if it was $(a,1), a\neq 1$) the amount of moves is finite, so player 1 wins.

3.

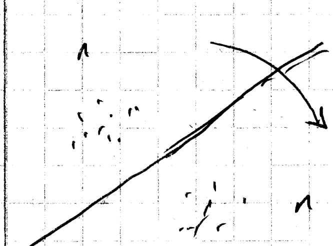


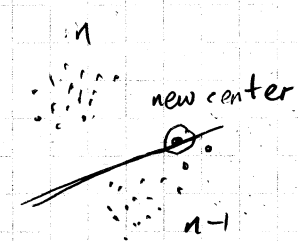
where $tg\alpha = \frac{1}{3}$

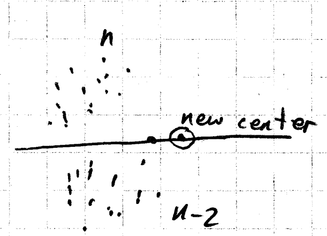


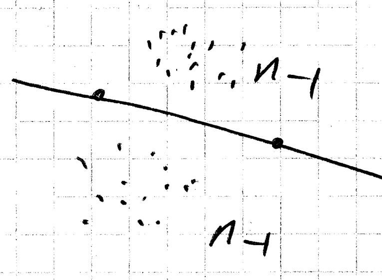
4. Lemma: given $2n$ points on the plane so that no 3 of them lie on the same line, there is a line passing on 2 points so that he divides the plane into two parts each containing $n-1$ points. proof:

place a line far left from all the points. now move him right continuosly. By continuety, there is a line that divides the plane into 2 parts each containing n points. choose a point on the line and rotate him* continuosly until it hits a point. Choose that point as the new rotation center, and keep rotating until you hit another point. If the two points are from the same side, choose the one that was achieved last as the new center, and keep rotating. At every part, no side of the plane lost or gained points (except when they lied on the line). Because of that, the line never returned to how it was before, So between 2 specific points he could have passed only once. So he can't keep running to pairs of points from the same side forever. And we proved the Lemma.









* All rotations are clock wise

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Take our $2n+1$ points. Choose one, O, and so inverse the plane in respect to O.
Other points turned to a group of $2n$ points, and O went to the infinity point.*
By our Lemma, Construct a line ℓ passing on 2 of the $2n$ points, so that both sides of ℓ have $n-1$ of the points. ℓ also passes through the point at infinity, O.

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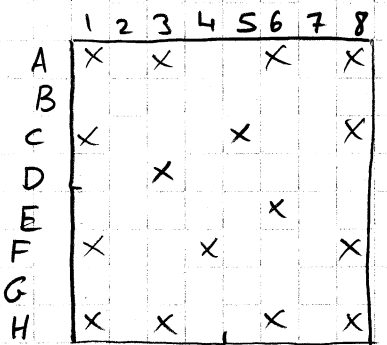
*proof that Lemma condition holds: if a line passes on 3 of the $2n$ points, he also passes through infinity point so in the original picture he was a line or a circle passing on 4 points, which is impossible.

---

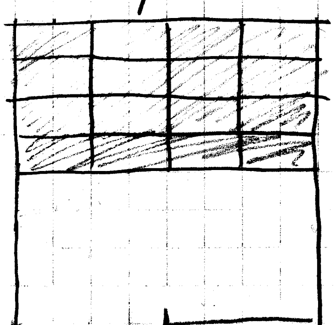
now, reverse the inversion. ℓ passes through 3 points, so in the original picture he was a circle and not a line. both sides of ℓ still have $n-1$ points each, So ℓ as a circle has exactly $n-1$ points inside and $n-1$ points outside.

5. $S > N$

We proof by showing that for each tiling in 32 dominoes, there is a unique tiling in 16 dominoes, but not all 16-tailings are achieved this way:
Take the 32-tailing. now, erase all dominoes that pass through one of the points:



we are left with a 16 tailing. Not all 16-tiling can be achieved, for example this one:



for obvious reasons.

why is this unique? because given the 16-tiling that came from the 31-tailing, we can reconstruct the 32 tiling: Let us look on such a 16-tiling. look at one of the four middle squares that a domino was taken from.* No other missing domino can reach squares next to it, so exactly one square next to it will be missing and therefore we know where that missing domino was originally. A similiar thing works for A3, A6, C1, C8, F1, F8, H3, H6: (next page)

*C5, D3, E6, F4

5. (page 2)

two of the squares around them cannot be reached by any other missing domino. if one of those squares are missing, we know where the removed domino was originally. if they are both full, we also know where the removed domino was originally.

with all that information, we obviously know how the removed corner dominoes were placed originally. therefore, we uniquely detected the original 32-tiling from the 16-tiling that came from it.

Therefore, $S \geq N$.